EXERCISE 15.4 PAGE NO: 15.24

1. In each of the following, there are three positive numbers. State if these numbers could possibly be the lengths of the sides of a triangle:
(i) 5, 7, 9
(ii) 2, 10, 15
(iii) 3, 4, 5
(iv) 2, 5, 7
(v) 5, 8, 20

Solution:
(i) Given 5, 7, 9
Yes, these numbers can be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side.
Here, $5 + 7 > 9$, $5 + 9 > 7$, $9 + 7 > 5$

(ii) Given 2, 10, 15
No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case.
Here, $2 + 10 < 15$

(iii) Given 3, 4, 5
Yes, these numbers can be the lengths of the sides of a triangle because the sum of any two sides of triangle is always greater than the third side.
Here, $3 + 4 > 5$, $3 + 5 > 4$, $4 + 5 > 3$

(iv) Given 2, 5, 7
No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case.
Here, $2 + 5 = 7$

(v) Given 5, 8, 20
No, these numbers cannot be the lengths of the sides of a triangle because the sum of any two sides of a triangle is always greater than the third side, which is not true in this case.

Here, 5 + 8 < 20

2. In Fig. 46, P is the point on the side BC. Complete each of the following statements using symbol '=',' > 'or '< 'so as to make it true:
(i) AP… AB+ BP
(ii) AP… AC + PC
(iii) AP…. ½ (AB + AC + BC)

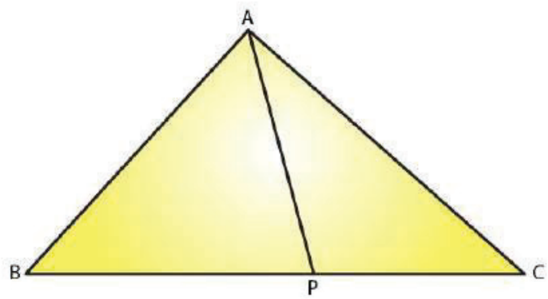

Fig. 46

Solution:
(i) In △APB, AP < AB + BP because the sum of any two sides of a triangle is greater than the third side.

(ii) In △APC, AP < AC + PC because the sum of any two sides of a triangle is greater than the third side.

(iii) AP < ½ (AB + AC + BC)
In △ABP and △ACP, we can write as
AP < AB + BP… (i) (Because the sum of any two sides of a triangle is greater than the third side)
AP < AC + PC … (ii) (Because the sum of any two sides of a triangle is greater than the third side)
On adding (i) and (ii), we have:
AP + AP < AB + BP + AC + PC
2AP < AB + AC + BC (BC = BP + PC)
AP < ½ (AB + AC + BC)

3. P is a point in the interior of △ABC as shown in Fig. 47. State which of the following statements are true (T) or false (F):
(i) AP + PB < AB
(ii) AP + PC > AC

(iii) BP + PC = BC

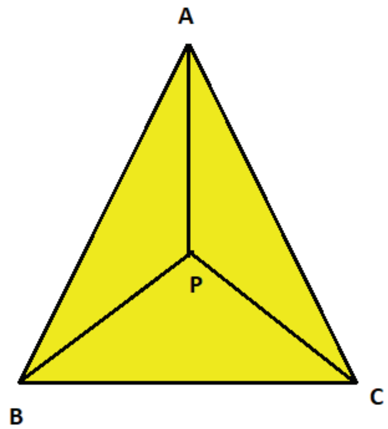

Fig. 47

Solution:

(i) False

Explanation:

We know that the sum of any two sides of a triangle is greater than the third side, it is not true for the given triangle.

(ii) True

Explanation:

We know that the sum of any two sides of a triangle is greater than the third side, it is true for the given triangle.

(iii) False

Explanation:

We know that the sum of any two sides of a triangle is greater than the third side, it is not true for the given triangle.

4. O is a point in the exterior of △ABC. What symbol '>','<' or '=' will you see to complete the statement OA+OB….AB? Write two other similar statements and show that OA + OB + OC > ½ (AB + BC +CA)

Solution:

We know that the sum of any two sides of a triangle is always greater than the third side, in △OAB, we have,

OA + OB > AB ….. (i)

In △OBC we have

OB + OC > BC …… (ii)
In △OCA we have
OA + OC > CA ….. (iii)
On adding equations (i), (ii) and (iii) we get:
OA + OB + OB + OC + OA + OC > AB + BC + CA
2(OA + OB + OC) > AB + BC + CA
OA + OB + OC > (AB + BC + CA)/2
Or
OA + OB + OC > ½ (AB + BC +CA)
Hence the proof.

5. In △ABC, ∠A = 100°, ∠B = 30°, ∠C = 50°. Name the smallest and the largest sides of the triangle.

Solution:
We know that the smallest side is always opposite to the smallest angle, which in this case is 30°, it is AC.
Also, because the largest side is always opposite to the largest angle, which in this case is 100°, it is BC.