

NCERT Solutions for Class-XI Physics

Chapter-7 NCERT Physics Class 11

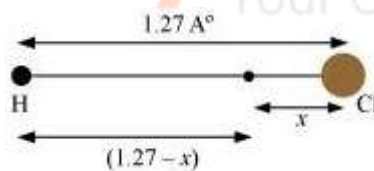
1. Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

1. Geometric centre; No

The centre of mass (C.M.) is a point where the mass of a body is supposed to be concentrated. For the given geometric shapes having a uniform mass density, the C.M. lies at their respective geometric centres. The centre of mass of a body need not necessarily lie within it. For example, the C.M. of bodies such as a ring, a hollow sphere, etc., lies outside the body.

2. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 \AA ($1 \text{ \AA} = 10^{-10} \text{ m}$). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus.

2. Let, Mass of the Hydrogen = m
Mass of the Chlorine = $35.5m$
and given that, Separation between atoms = 1.27 \AA
 $= 1.27 \times 10^{-10} \text{ m}$



Let centre of mass for the molecule is a ' x ' \AA distance from the Cl atom then distance from Hydrogen atom be $(1.27 - x) \text{ \AA}$.

(As shown in the above diagram)

For one dimension, centre of mass CM can be calculated by,

$$CM = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} m$$

Where, m_1 = mass of the first body,

m_2 = mass of the second body,

x_1 = Distance from centre of mass to the first body,

x_2 = Distance from centre of mass to the second body,

Suppose Centre of mass of the molecule lies at origin (Zero) then,

(\because It is in 1 dimension)

$$CM = 0 = \frac{[m(1.27 - x) + 35.5mx]}{(m + 35.5m)}$$

$$\Rightarrow [m(1.27 - x) + 35.5mx] = 0$$

$$\Rightarrow 1.27m - mx + 35.5mx = 0$$

$$\Rightarrow 1.27 - x = -35.5x$$

$$\therefore x = \frac{1.27}{-34.5} \text{ \AA} = -0.037 \text{ \AA} = -0.037 \times 10^{-10} \text{ m}$$

But, negative sign just indicates that CM lies left to chlorine (as assumed in figure) and it lies at a distance of $0.037 \times 10^{-10} \text{ m}$.

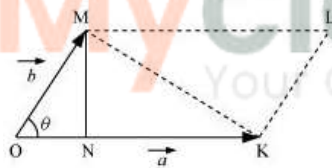
3. A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?

3. No change

The child is running arbitrarily on a trolley moving with velocity v . However, the running of the child will produce no effect on the velocity of the centre of mass of the trolley. This is because the force due to the boy's motion is purely internal. Internal forces produce no effect on the motion of the bodies on which they act. Since no external force is involved in the boy-trolley system, the boy's motion will produce no change in the velocity of the centre of mass of the trolley.

4. Show that the area of the triangle contained between the vectors \vec{a} and \vec{b} is one half of the magnitude of $\vec{a} \times \vec{b}$.

4. Consider two vectors \vec{a} (OK) and \vec{b} (OM) are making angle θ which each other as shown in following figure,



Now,

In $\triangle OMN$ we can write the equation,

$$\sin\theta = \frac{MN}{OM} = \frac{MN}{|\vec{b}|}$$

$$\Rightarrow MN = |\vec{b}| \sin\theta$$

We know that magnitude of cross product of vectors \vec{a} and \vec{b} is,

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin\theta$$

$$= OK \times MN$$

$$(\because |\vec{b}| \sin\theta = MN)$$

$$= \frac{2}{2} \times OK \times MN$$

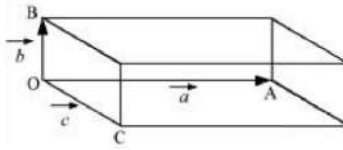
$$(\because \frac{2}{2} = 1)$$

$$= 2 \times \text{Area of } \triangle OMK$$

$$\therefore \text{Area of } \triangle OMK = \frac{1}{2} |\vec{a} \times \vec{b}|$$

Hence Proved.

5. Show that $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ is equal in magnitude to the volume of the parallelepiped formed on the three vectors \mathbf{a} , \mathbf{b} and \mathbf{c} .
5. A parallelepiped with origin O and sides \mathbf{a} , \mathbf{b} , and \mathbf{c} is shown in the following figure.



Volume of the given parallelepiped = abc

$$\overline{OC} = \mathbf{a}$$

$$\overline{OB} = \mathbf{b}$$

$$\overline{OC} = \mathbf{c}$$

Let \hat{n} be a unit vector perpendicular to both \mathbf{b} and \mathbf{c} . Hence, \hat{n} and \mathbf{a} have the same direction.

$$\mathbf{b} \times \mathbf{c} = bc \sin \theta \hat{n}$$

$$= bc \sin 90^\circ \hat{n}$$

$$= bc \hat{n}$$

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$$

$$= \mathbf{a} \cdot (bc \hat{n})$$

$$= abc \cos \theta$$

$$= abc \cos 0^\circ$$

$$= abc$$

$$= \text{Volume of the parallelepiped}$$

6. Find the components along the x, y, z axes of the angular momentum \mathbf{l} of a particle, whose position vector is \mathbf{r} with components x, y, z and momentum is \mathbf{p} with components p_x , p_y and p_z . Show that if the particle moves only in the x-y plane the angular momentum has only a z-component.

6. From the given data,

$$\text{Linear momentum vector, } \vec{p} = p_x \hat{i} + p_y \hat{j} + p_z \hat{k}$$

$$\text{Position vector, } \vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$$

$$\text{We have, Angular momentum, } \vec{I} = \vec{r} \times \vec{p}$$

$$= (x \hat{i} + y \hat{j} + z \hat{k}) \times (p_x \hat{i} + p_y \hat{j} + p_z \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$= \begin{vmatrix} x & y & z \\ p_x & p_y & p_z \end{vmatrix}$$

$$= \begin{vmatrix} p_x & p_y & p_z \end{vmatrix}$$

$$\therefore I_x \hat{i} + I_y \hat{j} + I_z \hat{k} = \hat{i}(yp_z - zp_y) - \hat{j}(xp_z - zp_x) + \hat{k}(xp_y - yp_x)$$

By comparing respective components we get,

$$I_x = (yp_z - zp_y)$$

$$I_y = (xp_z - zp_x)$$

$$I_z = (xp_y - yp_x)$$

Since, the particle moves in x-y plane, then the \hat{k} vectors of both position and linear momentum vectors be Zero.

Thus,

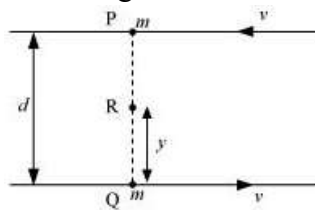
$$I_x = 0$$

$$I_y = 0$$

$$I_z = (xp_y - yp_x)$$

Hence, the particle moves in x-y plane the angular momentum acts towards z-direction.

- Two particles, each of mass m and speed v , travel in opposite directions along parallel lines separated by a distance d . Show that the vector angular momentum of the two particle system is the same whatever be the point about which the angular momentum is taken.
- Let at a point of time particles P and Q are in some position (exactly collinear, perpendicular to paths) as shown in figure below,



Angular momentum, $I = mvr$

Where,

- m = mass of the particle,
- v = velocity of the particle,
- r = distance from rotating point.

Thus,

$$\text{Angular momentum about P, } I_P = mv \times 0 + mv \times d = mvd \quad \dots(1)$$

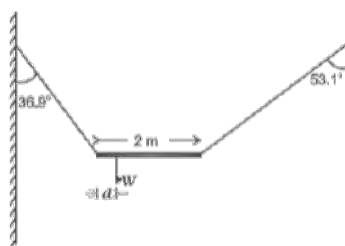
$$\text{Angular momentum about Q, } I_Q = mv \times d + mv \times 0 = mvd \quad \dots(2)$$

If the rotating point is R as shown in figure above,

$$I_R = [mv \times (d - y)] + mv \times y = mvd \quad \dots(3)$$

From equations 1, 2 and 3 we can conclude that angular momentum of system doesn't depend on the point at which it is taken.

- A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Figure. The angles made by the strings with the vertical are 36.9° and 53.1° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.

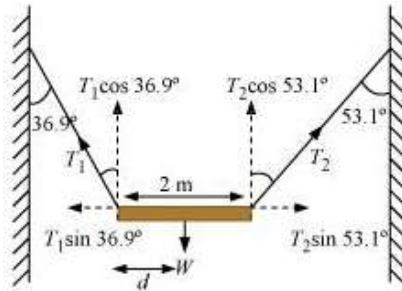


- Given,

Length of the bar, $l = 2 \text{ m}$

Let T_1 and T_2 be the tensions produced by the strings.

The free body diagram can be drawn as,



For translational equilibrium we have,

$$T_1 \sin 36.9 = T_2 \sin 53.1$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\sin 53.1}{\sin 36.9} = \frac{4}{3}$$

$$\therefore T_1 = \frac{4}{3} T_2$$

For rotational equilibrium, on taking the torque about the centre of gravity. We have,

$$T_1 (\cos 36.9) \times d = T_2 (\cos 53.1) \times (2 - d)$$

$$\Rightarrow T_1 \times 0.8 \times d = T_2 \times 0.6 \times (2 - d)$$

$$\Rightarrow \frac{4}{3} T_2 \times 0.8 \times d = T_2 [(0.6 \times 2) - (0.6 \times d)]$$

$$\Rightarrow 1.067d + 0.6d = 1.2$$

$$\therefore d = \frac{1.2}{1.67} \text{ m} = 0.72 \text{ m}$$

Hence, Centre of gravity lies at a distance of 0.72 m from the left side of the bar.

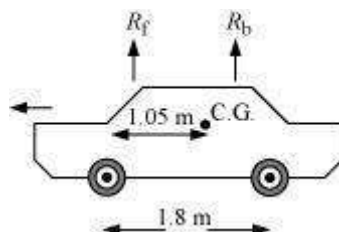
9. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel.

9. Mass of the car, $m = 1800 \text{ kg}$

Distance between the front and back axles, $d = 1.8 \text{ m}$

Distance between the C.G. (centre of gravity) and the back axle = 1.05 m

The various forces acting on the car are shown in the following figure.



R_f and R_b are the forces exerted by the level ground on the front and back wheels respectively.

At translational equilibrium:

$$R_f + R_b = mg$$

$$= 1800 \times 9.8$$

$$= 17640 \text{ N ... (i)}$$

For rotational equilibrium, on taking the torque about the C.G., we have:

$$R_f(1.05) = R_b(1.8 - 1.05)$$

$$R_f \times 1.05 = R_b \times 0.75$$

$$\frac{R_f}{R_b} = \frac{0.75}{1.05} = \frac{5}{7}$$

$$\frac{R_b}{R_f} = \frac{7}{5}$$

$$\frac{R_b}{R_f} = \frac{7}{5}$$

$$R_b = 1.4R_f$$

Solving equations (i) and (ii), we get:

$$1.4R_f + R_f = 17640$$

$$R_f = \frac{17640}{2.4} = 7350\text{N}$$

$$\therefore R_b = 17640 - 7350 = 10290\text{ N}$$

Therefore, the force exerted on each front wheel = $\frac{7350}{2} = 3675\text{N}$, and

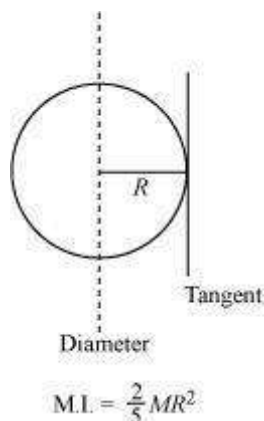
The force exerted on each back wheel = $\frac{10290}{2} = 5145\text{N}$

10. Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $\frac{2MR^2}{5}$, where M is the mass of the sphere and R is the radius of the sphere.

Given the moment of inertia of a disc of mass M and radius R about any of its diameters to be $\frac{MR^2}{4}$, find its moment of inertia about an axis normal to the disc and passing through a point on its edge.

10. $\frac{7}{5}MR^2$

The moment of inertia (M.I.) of a sphere about its diameter = $\frac{2}{5}MR^2$



According to the theorem of parallel axes, the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes.

The M.I. about a tangent of the sphere $= \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$

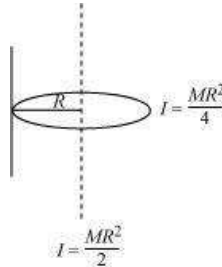
(b) $\frac{3}{2}MR^2$

The moment of inertia of a disc about its diameter $= \frac{1}{4}MR^2$

According to the theorem of perpendicular axis, the moment of inertia of a planar body (lamina) about an axis perpendicular to its plane is equal to the sum of its moments of inertia about two perpendicular axes concurrent with perpendicular axis and lying in the plane of the body.

The M.I. of the disc about its centre $= \frac{1}{4}MR^2 + \frac{1}{4}MR^2 = \frac{1}{2}MR^2$

The situation is shown in the given figure.



Applying the theorem of parallel axes:

The moment of inertia about an axis normal to the disc and passing through a point on its edge $= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

11. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?
11. Let m and r be the respective masses of the hollow cylinder and the solid sphere.

The moment of inertia of the hollow cylinder about its standard axis, $I_1 = mr^2$

The moment of inertia of the solid sphere about an axis passing through its centre,

$$I_{II} = \frac{2}{5}mr^2$$

We have the relation:

$$\tau = I\alpha$$

Where, α = Angular

acceleration τ = Torque

I = Moment of inertia

For the hollow cylinder, $\tau_1 = I_1\alpha_1$

For the solid sphere, $\tau_{II} = I_{II}\alpha_{II}$

As an equal torque is applied to both the bodies, $\tau_1 = \tau_2$

$$\therefore \frac{\alpha_{II}}{\alpha_1} = \frac{I_1}{I_{II}} = \frac{mr^2}{\frac{2}{5}mr^2} = \frac{5}{2}$$

$$\alpha_{II} > \alpha_1 \quad \dots(i)$$

Now, using the relation:

$$\omega = \omega_0 + \alpha t$$

Where,

ω_0 = Initial angular velocity

t = Time of rotation ω =

Final angular velocity For

equal ω_0 and t, we have:

$$\omega \propto \alpha \dots(ii)$$

From equations (i) and (ii), we can write:

$$\omega_{II} > \omega_I$$

Hence, the angular velocity of the solid sphere will be greater than that of the hollow cylinder.

12. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1} . The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis?

12. We need to find Kinetic energy of the rotating cylinder

We know that, Rotational kinetic energy,

$$KE = \frac{1}{2} I \omega^2$$

Where

I is moment of inertia

ω is angular velocity

Given

Mass of cylinder, $M = 20 \text{ kg}$

Radius of cylinder, $R = .25 \text{ m} = \frac{1}{4} \text{ m}$

Angular speed of cylinder, $\omega = 100 \text{ rad s}^{-1}$

Moment of inertia of a solid cylinder, $I = \frac{1}{2} MR^2$

$$\text{I.e. } I = \frac{1}{2} \times 20 \times \left(\frac{1}{4}\right)^2$$

$$= \frac{5}{8} \text{ kg m}^2$$

$$\therefore KE = \frac{1}{2} I \omega^2$$

$$= \frac{5}{16} (100)^2$$

$$= 3125 \text{ J}$$

$$= 3.125 \text{ kJ}$$

We need to find angular momentum of rotating cylinder about its axis.

Angular momentum, $L = I \omega$

We know that,

Moment of inertia of cylinder, $I = \frac{5}{8} \text{ kg m}^2$

Angular speed of cylinder, $\omega = 100 \text{ rad s}^{-1}$

$$\therefore \text{Angular momentum, } L = \frac{5}{8} \times 100$$

$$= 62.5 \text{ kg m s}^{-1}$$

$$= 62.5 \text{ J s}$$

13. A. A child stands at the centre of a turntable with his two arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his

moment of inertia to $2/5$ times the initial value? Assume that the turntable rotates without friction.

B. Show that the child's new kinetic energy of rotation is more than the initial for this increase in kinetic energy?

13. A

Given

Angular speed of child, $\omega = 40 \text{ rev min}^{-1}$

Angular momentum of child = $I \omega$

Where I is moment of inertia of child

ω is angular velocity of child

When the child folds his hand his moment of inertia decreases, but angular momentum is conserved. Therefore, angular speed increases. We need to find the new angular speed (ω')

Let new moment of inertia be I'

And new angular velocity be ω'

Given, $I' = 2/5 I$

Since Angular momentum is conserved,

$$I \omega = I' \omega'$$

$$\omega' = I \omega / I'$$

$$= 5/2 \times \omega$$

$$= 5/2 \times 40$$

$$= 100 \text{ rev min}^{-1}$$

$$= 5/3 \text{ rev s}^{-1}$$

$$= 1.66 \text{ rev s}^{-1}$$

B

Kinetic energy of rotation,

$$KE_{\text{rot}} = \frac{1}{2} I \omega^2$$

Let moment of inertia of child before folding hands be I

Given, angular velocity before folding hand, $\omega = 100 \text{ rev min}^{-1}$

$$\therefore KE = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} I (40)^2$$

$$= 800I$$

Let moment of inertia after folding hands be I' , Then,

$$I' = 2/5 I$$

Angular speed after folding hands, $\omega' = 10\pi / 3$

$$\therefore KE' = \frac{1}{2} I' \omega'^2$$

$$= \frac{1}{2} (2I/5) \times (100)^2$$

$$= 2000I$$

$$KE'/KE = 2000I / 800I$$

$$= 5/2$$

i.e. KE' is greater than KE . When the child folded the hands, Kinetic energy increased

14. A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N? What is the linear acceleration of the rope? Assume that there is no slipping.

14. We need to find angular acceleration of cylinder when force was applied tangential to hollow cylinder (by pulling rope wound in it).

Torque produced by action of force on a body,

$$\tau = F r$$

Where

F is force applied on body

r is perpendicular distance of point of application of force with axis of rotation.

Given

$$F = 30 \text{ N}$$

$$r = 40 \text{ Cm} = .4 \text{ m}$$

$$\begin{aligned} \therefore \tau &= 30 \times .4 \\ &= 12 \text{ Nm} \end{aligned}$$

On action of this torque (or tangential force), body gains an angular acceleration, say, α .

In terms of α , Torque, $\tau = I \alpha$

Where

I is moment of inertia of body

α is angular acceleration

Moment of inertia of hollow cylinder,

$$I = Mr^2$$

Where

M is mass of cylinder

r is radius of cylinder

Given

$$M = 3 \text{ kg}$$

$$r = 40 \text{ Cm} = .4 \text{ m}$$

$$\begin{aligned} \therefore I &= 3 \times (.4)^2 \\ &= .48 \text{ kg m}^2 \end{aligned}$$

$$\tau = I \alpha = 12 \text{ Nm}$$

$$I = .48 \text{ kg m}^2$$

$$\begin{aligned} \therefore \text{Angular acceleration, } \alpha &= \tau / I \\ &= 12 / .48 \\ &= 25 \text{ rad s}^{-2} \end{aligned}$$

Linear acceleration on point P,

$$a = r \alpha$$

Where

α is angular acceleration

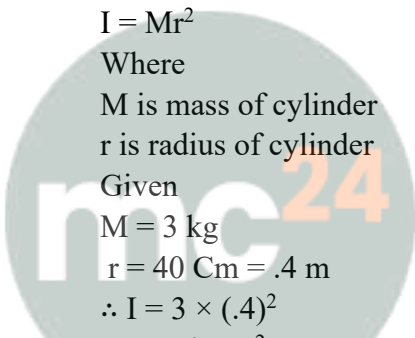
r is perpendicular distance of point P from axis of rotation

We need to find linear acceleration of rope, which is at distance of .4 m from axis of rotation,

$$\text{Thus, } r = .4 \text{ m}$$

$$\alpha = 25 \text{ s}^{-2}$$

$$\begin{aligned} \therefore \text{Linear acceleration, } a &= r \alpha \\ &= .4 \times 25 = 10 \text{ m s}^{-2} \end{aligned}$$



15. To maintain a rotor at a uniform angular speed of 200 rad s^{-1} , an engine needs to transmit a torque of 180 N m . What is the power required by the engine?
(Note: uniform angular velocity in the absence of friction implies zero torque. In practice, applied torque is needed to counter frictional torque). Assume that the engine is 100% efficient.

15. Angular speed of the rotor, $\omega = 200 \text{ rad/s}$

Torque required, $\tau = 180 \text{ Nm}$

The power of the rotor (P) is related to torque and angular speed by the relation:

$$P = \tau\omega$$

$$= 180 \times 200 = 36 \times 10^3$$

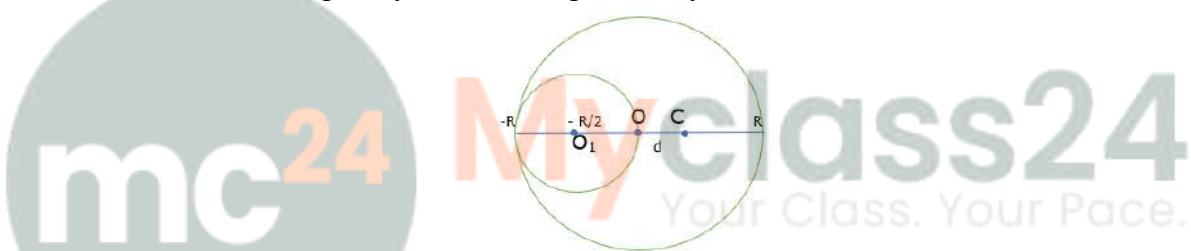
$$= 36 \text{ kW}$$

Hence, the power required by the engine is 36 kW .

16. From a uniform disk of radius R , a circular hole of radius $R/2$ is cut out. The centre of the hole is at $R/2$ from the centre of the original disc. Locate the centre of gravity of the resulting flat body.

16. From a big uniform disc of radius R with centre O , a smaller circular hole of radius $R/2$ with its centre O_1 (where $OO_1 = R/2$) is cut out.

Let the centre of gravity of remaining flat body be C and $OC = d$.



If Ω is mass per unit area, then mass of the whole disk of Radius R is,

$$M_1 = \pi R^2 \Omega$$

and mass cut out part

$$M_2 = \pi (R/2)^2 \Omega = \pi R^2 \Omega / 4 = M_1 / 4$$

Let centre of mass of original disc be origin (O)

It can be considered as centre of mass of system consisting of smaller carved out disc and remaining portion.

Thus we have,

$$M_1 \times (0) = (M_1 - M_2) \times d + M_2 \times (OO_1)$$

$$0 = (M_1 - M_2) \times d + M_2 \times (-R/2)$$

$$\text{(Since, } M_2 = M_1/4)$$

$$d = (M_2 R/2) / (M_1 - M_2)$$

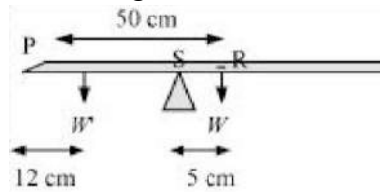
$$= (M_1/4) \times (R/2) / (M_1 - (M_1/4))$$

$$= R/6$$

$\therefore C$ is at a distance $R/6$ from the centre of disc on diametrically opposite side of the centre of hole.

17. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5 g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm . What is the mass of the metre stick?

17. Let W and W' be the respective weights of the metre stick and the coin.



The mass of the metre stick is concentrated at its mid-point, i.e., at the 50 cm mark.

Mass of the meter stick = m'

Mass of each coin, $m = 5$ g

When the coins are placed 12 cm away from the end P, the centre of mass gets shifted by 5 cm from point R toward the end P. The centre of mass is located at a distance of 45 cm from point P.

The net torque will be conserved for rotational equilibrium about point R.

$$10 \times g(45 - 12) - m'g(50 - 45) = 0$$

$$\therefore m' = \frac{10 \times 33}{5} = 66 \text{ g}$$

Hence, the mass of the metre stick is 66 g.

18. A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination. (a) Will it reach the bottom with the same speed in each case? (b) Will it take longer to roll down one plane than the other? (c) If so, which one and why?

18. Both spheres are identical.

\therefore Total energy,

$$TE = KE + PE$$

Where

KE is kinetic energy

PE is potential energy

Since bodies are starting from rest, there initial KE is zero,

$$TE_i = PE_i = mgh$$

Where

m is mass of the bodies

g is acceleration due to gravity

h is height of inclined plane

On reaching bottom, the potential energy drops to zero ($PE_f = 0$)

Therefore, total energy of bodies at bottom are KE of bodies.

$$TE_f = KE_f = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega^2$$

Where

m is mass of body

v_f is velocity attained by body

ω is angular velocity

I is moment of inertia

By Law of conservation of energy, we have,

$$TE_i = TE_f$$

$$\text{I.e. } mgh = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} m v_f^2 + \frac{1}{2} (2mR^2/5) \omega^2$$

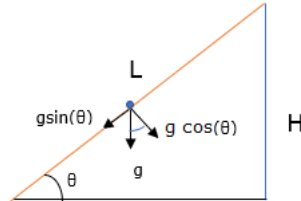
$$V = R\omega$$

$$\therefore mgh = \frac{1}{2} mv_f^2 + mv_f^2/5$$

$$v_f = (10gh/7)^{1/2}$$

The final velocity is independent of angle of inclination. Therefore both bodies will have same final speed on reaching ground.

B & C



When a body is moving on an inclined plane of inclination θ , acceleration acting on body is g (acceleration due to gravity vertically downward)

The acceleration along inclined plane is $g \sin(\theta)$ component of acceleration.

Let height from which body fall be H .

Then length of inclined plane of inclination θ ,

$$L = H/\sin(\theta)$$

We know the equation,

$$s = u t + \frac{1}{2} at^2$$

Where

u is initial velocity

s is distance covered

a is acceleration acting on body

t is time

For case of motion along plane,

$u = 0$ (since body starts from rest)

$$s = L = H/\sin(\theta)$$

$$a = g \sin(\theta)$$

t is time

Substituting the values,

$$H/\sin(\theta) = 0 \times t + \frac{1}{2} g \sin(\theta) t^2$$

\therefore time for reaching ground,

$$t = (2H/g)^{1/2} / \sin(\theta)$$

From above equation, we can infer that time taken by a body to roll down an inclined plane is inversely proportional to \sin of angle of inclination (θ).

Sine increases with increase of angle, thus time for fall decreases for greater inclination ($t \propto 1/\sin(\theta)$)

Thus greater the angle of inclination, lower the time needed for reaching ground.

19. A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it?
19. Radius of the hoop, $r = 2$ m
Mass of the hoop, $m = 100$ kg
Velocity of the hoop, $v = 20$ cm/s = 0.2 m/s
Total energy of the hoop = Translational KE + Rotational KE

$$E_R = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

Moment of inertia of the hoop about its centre, $I = mr^2$

$$E_r = \frac{1}{2}mv^2 + \frac{1}{2}(mr^2)\omega^2$$

But we have the relation, $v = r\omega$

$$\begin{aligned} \therefore E_r &= \frac{1}{2}mv^2 + \frac{1}{2}mr^3\omega^2 \\ &= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2 \end{aligned}$$

The work required to be done for stopping the hoop is equal to the total energy of the hoop.

$$\therefore \text{Required work to be done, } W = mv^2 = 100 \times (0.2)^2 = 4 \text{ J}$$

- 20.** The oxygen molecule has a mass of 5.30×10^{-26} kg and a moment of inertia of 1.94×10^{-46} kg m² about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule.

- 20.** Mass of an oxygen molecule, $m = 5.30 \times 10^{-26}$ kg

Moment of inertia, $I = 1.94 \times 10^{-46}$ kg m²

Velocity of the oxygen molecule, $v = 500$ m/s

The separation between the two atoms of the oxygen molecule = $2r$

Mass of each oxygen atom = $\frac{m}{2}$

Hence, moment of inertia I , is calculated as:

$$\left(\frac{m}{2}\right)r^2 + \left(\frac{m}{2}\right)r^2 = mr^2$$

$$r = \sqrt{\frac{I}{m}}$$

$$\sqrt{\frac{1.94 \times 10^{-46}}{5.36 \times 10^{-26}}} = 0.60 \times 10^{-10} \text{ m}$$

It is given that:

$$KE_{\text{rot}} = \frac{2}{3} KE_{\text{trans}}$$

$$\frac{1}{2}I\omega^2 = \frac{2}{3} \times \frac{1}{2} \times mv^2$$

$$mr^2\omega^2 = \frac{2}{3}mv^2$$

$$\omega = \sqrt{\frac{2}{3}} \frac{v}{r}$$

$$= \sqrt{\frac{2}{3}} \times \frac{500}{0.6 \times 10^{-10}}$$

$$= 6.80 \times 10^{12} \text{ rad/s}$$

21. A solid cylinder rolls up an inclined plane of angle of inclination 30° . At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m/s.
- A. How far will the cylinder go up the plane?
 B. How long will it take to return to the bottom?

21. At bottom of inclined plane centre of mass has speed 5 m s^{-1} let this be v .

We need to find how far the solid cylinder moves up on the plane.

By law of conservation of energy, energy is conserved.

Initial energy of body is the completely Kinetic (at bottom), On moving up the plane its kinetic energy gradually decreases. At top most point, Kinetic energy is zero and energy is completely gravitational potential energy.

The axis of rotation of cylinder passes through point B, we know that velocity of point O (axis of cylinder) is v . If ω is angular velocity of cylinder then,

$$v = R \omega$$

(R is Distance between axis of rotation and axis of cylinder)

Initial KE,

$$KE_i = KE_r + KE_t$$

Where

KE_r is rotational kinetic energy

KE_t is translational kinetic energy

$$KE_t = \frac{1}{2} mv^2$$

$$KE_r = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} (mR^2/2) \omega^2 = \frac{1}{4} mv^2 \text{ (since } v = R \omega \text{)}$$

$$\therefore \text{Total kinetic energy, } KE_i = \frac{1}{4} mv^2 + \frac{1}{2} mv^2$$

$$= \frac{3mv^2}{4}$$

Let h be the maximum height attained.

Then potential energy at that point,

$$PE_f = mgh$$

By law of conservation of energy,

$$KE_i = PE_f$$

$$\frac{3mv^2}{4} = mgh$$

$$h = \frac{3v^2}{4g}$$

We know,

$$v = 5 \text{ ms}^{-1}$$

$$g = 9.8 \text{ ms}^{-2}$$

Thus,

$$h = \frac{3(5)^2}{4 \times 9.8}$$

$$= 1.91 \text{ m}$$

Thus maximum height the body would reach is 1.91 m from ground.

By law of conservation of energy,

$$\frac{3mv^2}{4} = mgh$$

$$v = (4gh/3)^{1/2}$$

Let h be maximum height attained and θ be angle of inclination.

Then, Length travelled along plane to reach maximum height,

$$L = h/\sin(\theta)$$

When v is the velocity, the times taken to reach maximum height,

$$\begin{aligned} T &= L/v \\ &= h/\sin(\theta) \div (4gh/3)^{1/2} \\ &= (3h/4g)^{1/2} \div \sin(\theta) \\ &= (3 \times 1.91 / 4 \times 9.8)^{1/2} / \sin(30) \\ &= 0.765 \text{ s} \end{aligned}$$

the time taken for the body to move up and reach the bottom twice this time that is,



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