EXERCISE 1.3

Find which of the variables x, y, z and u represent rational numbers and which irrational numbers:

(i) $x^2 = 5$
(ii) $y^2 = 9$
(iii) $z^2 = .04$
(iv) $u^2 = 17/4$

Solution:

(i) $x^2 = 5$
On solving, we get
$\Rightarrow x = \pm \sqrt{5}$
Hence, x is an irrational number.

(ii) $y^2 = 9$
On solving, we get
$\Rightarrow y = \pm 3$
Hence, y is a rational number.

(iii) $z^2 = .04$
On solving, we get
$\Rightarrow z = \pm 0.2$
Hence, z is a rational number.

(iv) $u^2 = 17/4$
On solving, we get
$\Rightarrow u = \pm \sqrt{17}/2$
$\sqrt{17}$ is irrational.
Hence, u is an irrational number

2. Find three rational numbers between

(i) –1 and –2
(ii) 0.1 and 0.11
(iii) 5/7 and 6/7
(iv) 1/4 and 1/5

Solution:

(i) –1 and –2
Three rational numbers between –1 and –2 are –1.1, –1.2 and –1.3.

(ii) 0.1 and 0.11
Three rational numbers between 0.1 and 0.11 are 0.101, 0.102 and 0.103.

(iii) 5/7 and 6/7
5/7 can be written as $(5 \times 10)/(7 \times 10) = 50/70$
Similarly,

6/7 can be written as (6 × 10)/(7 × 10) = 60/70
Three rational numbers between 5/7 and 6/7 = three rational numbers between 50/70 and 60/70.
Three rational numbers between 5/7 and 6/7 are 51/70, 52/70, 53/70.

(iv) 1/4 and 1/5
Here, according to the question,
LCM of 4 and 5 is 20.
Let us make the denominators common, 80.
(4 × 20) = 80 and (5 × 16) = 80
Hence,
1/4 can be written as (1 × 20)/(4 × 20) = 20/80
Similarly,
1/5 can be written as (1 × 16)/(5 × 16) = 16/80
Three rational numbers between 1/4 and 1/5 = three rational numbers between 16/80 and 20/80.
Therefore, the three rational numbers are 17/80, 18/80 and 19/80.

3. Insert a rational number and an irrational number between the following:

(i) 2 and 3
(ii) 0 and 0.1
(iii) 1/3 and 1/2
(iv) – 2/5 and 1/2
(v) 0.15 and 0.16
(vi) √2 and √3
(vii) 2.357 and 3.121
(viii) .0001 and .001
(ix) 3.623623 and 0.484848
(x) 6.375289 and 6.375738.

Solution:

(i) 2 and 3
So, rational number between 2 and 3 = 2.5
And, irrational number between 2 and 3 = 2.040040004...

(ii) 0 and 0.1
So, rational number between 0 and 0.1 = 0.05
And, irrational number between 0 and 0.1 = 0.007000700007…

(iii) 1/3 and 1/2
LCM of 3 and 2 is 6.
1/3 = 0.33
1/3 can be written as (1 × 20)/(3 × 20) = 20/60
½ = 0.5
1/2 can be written as (1 × 30)/(2 × 30) = 30/60
So, rational number between 1/3 and 1/2 = 25/60
And, irrational number between 1/3 and 1/2 = irrational number between 0.33 and 0.5 = 0.414114111...

(iv) – 2/5 and 1/2
LCM of 5 and 2 is 10.
-2/5 = -0.4
-2/5 can be written as (-2 × 2)/(5 × 2) = -4/10
1/2 = 0.5
1/2 can be written as (1 × 5)/(2 × 5) = 5/10
So, rational number between -2/5 and 1/2 = rational number between -4/10 and 5/10 = 1/10
And, irrational number between -2/5 and 1/2 = irrational number between -0.4 and 0.5 = 0.414114111...

(v) 0.15 and 0.16
Rational number between 0.15 and 0.16 = 0.151
Irrational number between 0.15 and 0.16 = 0.151551555...

(vi) √2 = 1.41 and √3 = 1.732
Rational number between √2 and √3 = rational number between 1.41 and 1.732 = 1.5
Irrational number between √2 and √3 = irrational number between 1.41 and 1.732 = 1.585585558...

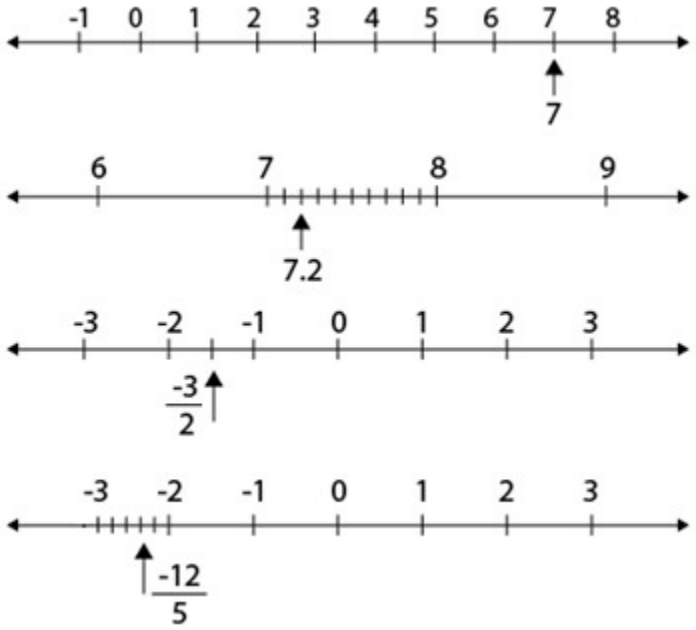
(vii) 2.357 and 3.121
Rational number between 2.357 and 3.121 = 3
Irrational number between 2.357 and 3.121 = 3.101101110...

(viii) .0001 and .001
Rational number between .0001 and .001 = 0.00011
Irrational number between .0001 and .001 = 0.0001131331333...

(ix) 3.623623 and 0.484848
Rational number between 3.623623 and 0.484848 = 1
Irrational number between 3.623623 and 0.484848 = 1.909009000...

(x) 6.375289 and 6.375738.
Rational number between 6.375289 and 6.375738 = 6.3753
Irrational number between 6.375289 and 6.375738 = 6.375414114111...

4. Represent the following numbers on the number line:
7, 7.2, −3/2 , −12/5
Solution:

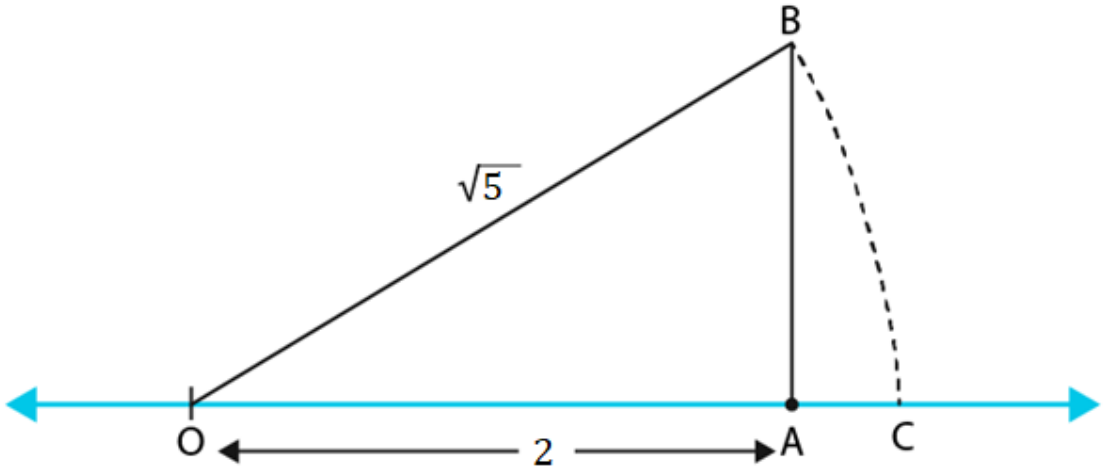


5. Locate √5, √10 and √17 on the number line.

Solution:

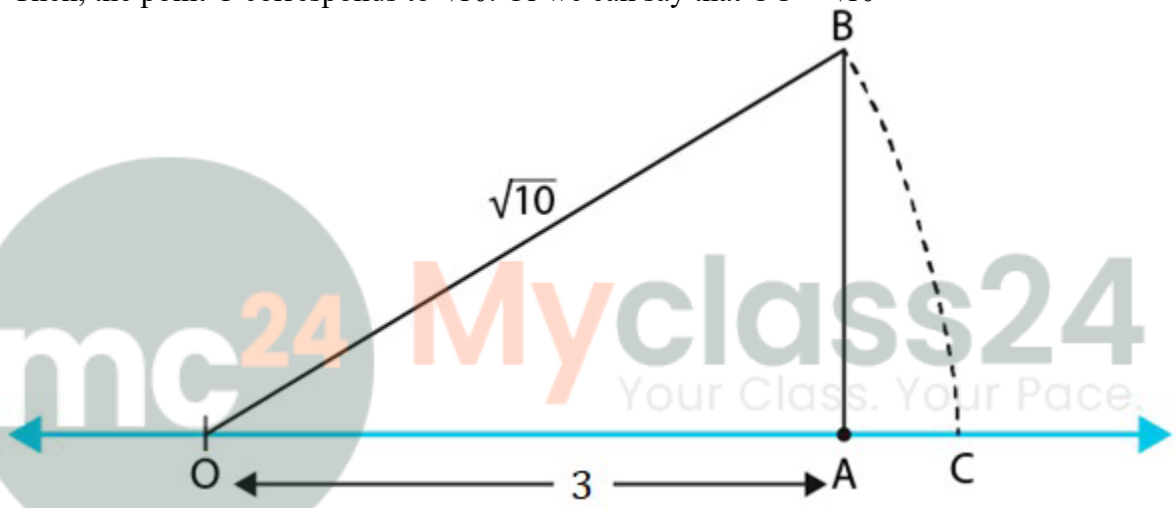
√5 on the number line:

5 can be written as the sum of the square of two natural numbers:
i.e., $5 = 1 + 4 = 1^2 + 2^2$
On the number line,
Take OA = 2 units.
Perpendicular to OA, draw BA = 1 unit.
Join OB.
Using Pythagoras theorem,
We have, OB= √5
Draw an arc with centre O and radius OB using a compass such that it intersects the number line at the point C.
Then, we get, C corresponds to √5. Or we can say that OC = √5



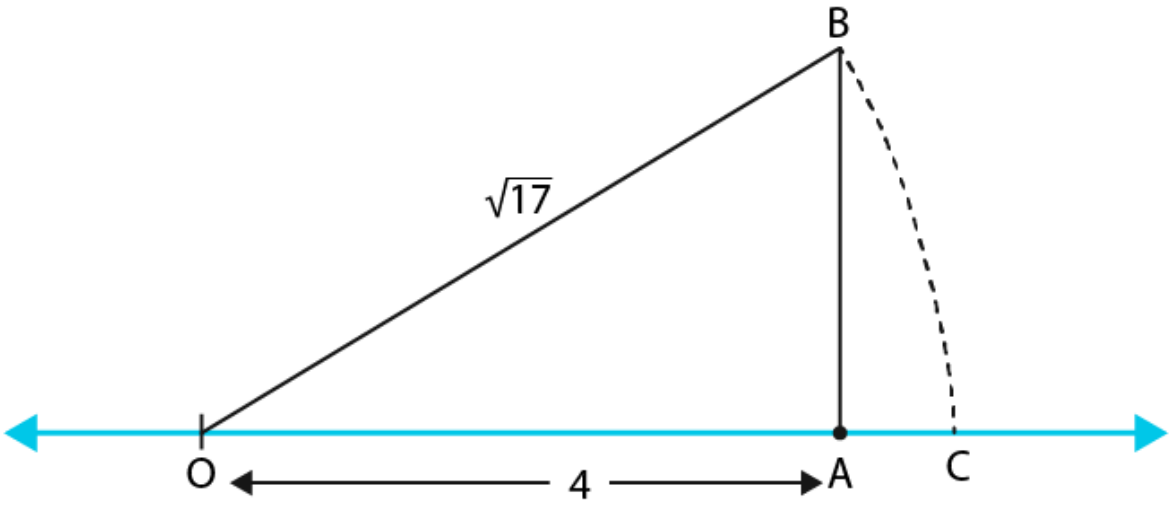
√10 on the number line:

10 can be written as the sum of the square of two natural numbers:
i.e., 10 =1+ 9 =$1^2 + 3^2$
On the number line,
Take OA = 3 units.
Perpendicular to OA, draw BA = 1 unit.
Join OB.
Using Pythagoras theorem,
We have, OB= √10
Draw an arc with centre O and radius OB using a compass such that it intersects the number line at the point C.
Then, the point C corresponds to √10. Or we can say that OC = √10



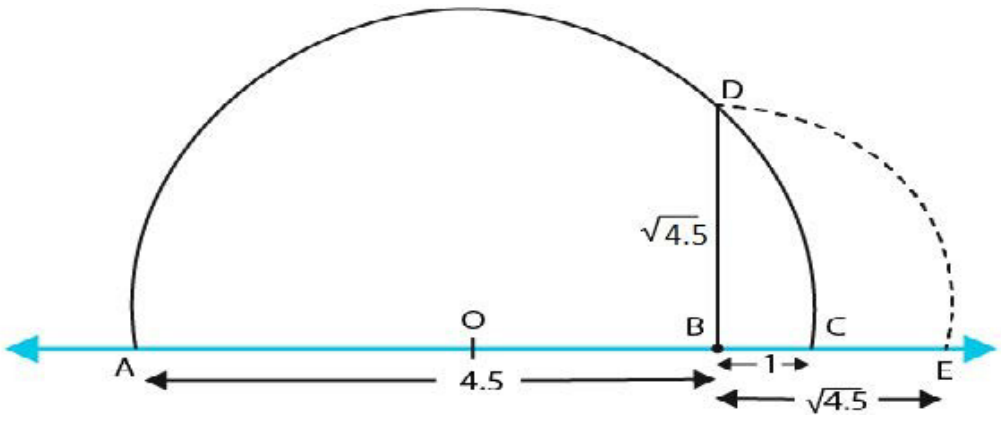
√17 on the number line:

17 can be written as the sum of the square of two natural numbers:
i.e., 17 =1+ 16 =$1^2 + 4^2$
On the number line,
Take OA = 4 units.
Perpendicular to OA, draw BA = 1 unit.
Join OB.
Using Pythagoras theorem,
We have, OB= √17
Draw an arc with centre O and radius OB using a compass such that it intersects the number line at the point C.
Then, the point C corresponds to √17. Or, we can say that OC = √17



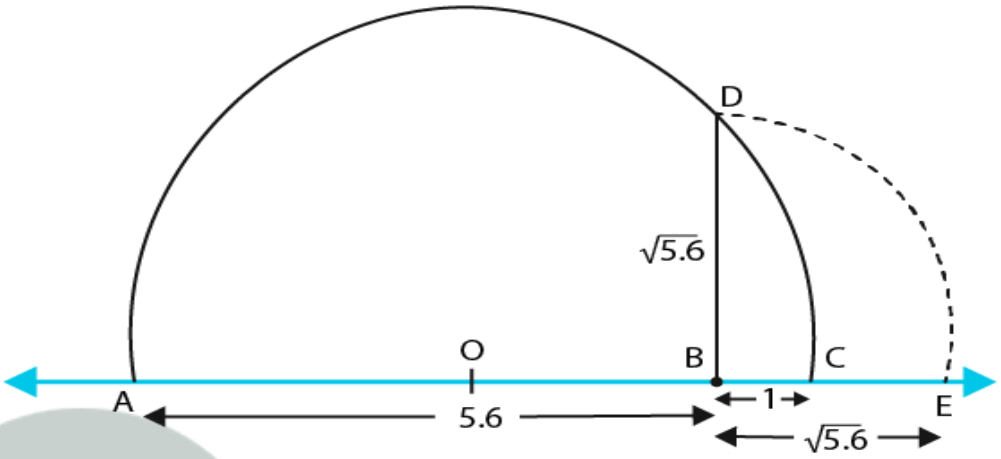
6. Represent geometrically the following numbers on the number line:
(i) √4.5
(ii) √5.6
(iii) √8.1
(iv) √2.3
Solution:

(i) √4.5
Draw a line segment such that AB = 4.5 units.
Mark C at a distance of 1 unit from B.
Mark O, the mid-point of AC.
Draw a semicircle with centre O and radius OC.
Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.
Now, BD = √4.5.
Draw an arc with centre B and radius BD, meeting AC produced at E.
Then BE = BD = √4.5 units.

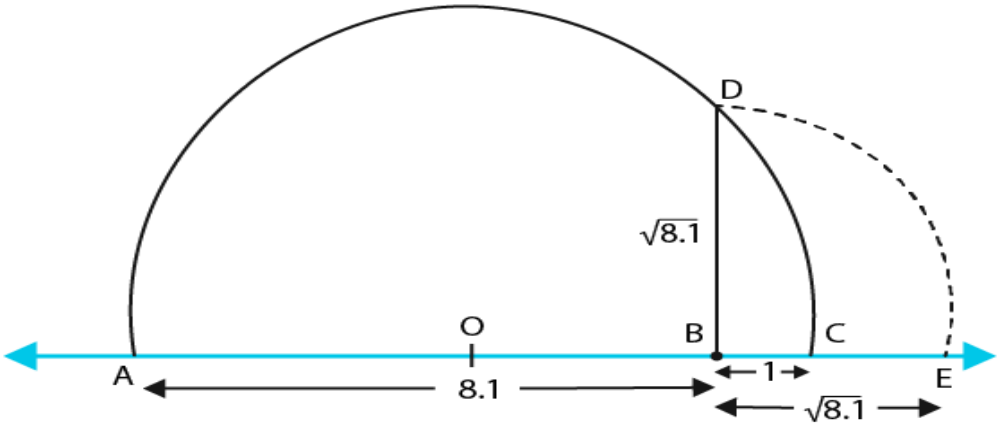


(ii) √5.6
Draw a line segment such that AB = 5.6 units.

Mark C at a distance of 1 unit from B.
Mark O, the mid-point of AC.
Draw a semicircle with centre O and radius OC.
Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.
Now, BD = √5.6
Draw an arc with centre B and radius BD, meeting AC produced at E.
Then BE = BD = √5.6 units.

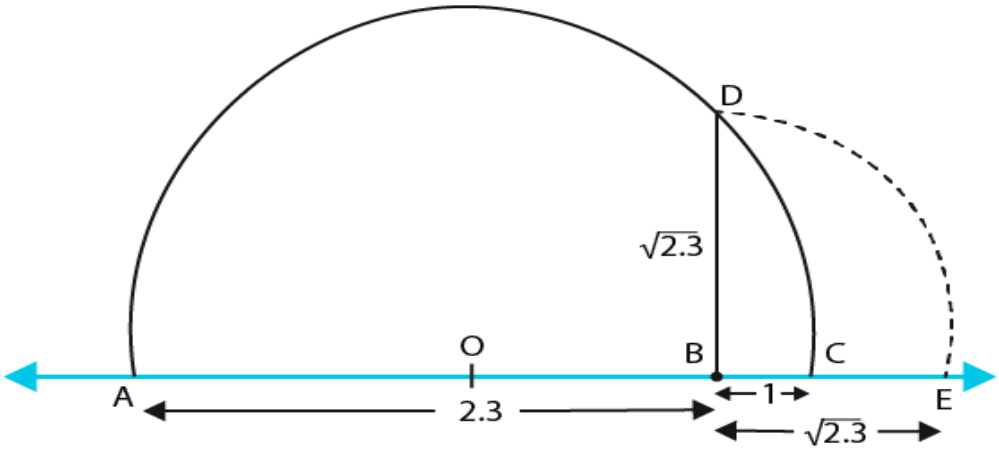


(iii) √8.1
Draw a line segment such that AB = 8.1 units.
Mark C at a distance of 1 unit from B.
Mark O, the mid-point of AC.
Draw a semicircle with centre O and radius OC.
Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.
Now, BD = √8.1.
Draw an arc with centre B and radius BD, meeting AC produced at E.
Then BE = BD = √8.1 units.



(iv) √2.3
Draw a line segment such that AB = 2.3 units.
Mark C at a distance of 1 unit from B.
Mark O, the mid-point of AC.
Draw a semicircle with centre O and radius OC.

Draw a line perpendicular to AC, passing through B and intersecting the semicircle at D.
Now, BD = √2.3.
Draw an arc with centre B and radius BD, meeting AC produced at E.
Then BE = BD = √2.3 units.



7. Express the following in the form p/q, where p and q are integers and q ≠ 0 :
(i) 0.2
(ii) 0.888...
(iii) $5.\overline{2}$
(iv) $0.\overline{001}$
(v) 0.2555...
(vi) $0.1\overline{34}$
(vii) .00323232...
(viii) .404040...
Solution:

(i) 0.2

We know that,
0/2 can be written as,
0.2 = 2/10 = 1/5

(ii) 0.888...

Assume that x = 0.888 …
⇒ x = 0.8..... Eq.(1)
Multiply L.H.S and R.H.S by 10,
We get
10 x = 8.8 .....Eq.(2)
Subtracting equation (1) from (2),
We get
10 x − x = 8.8 − 0.8
⇒ 9x = 8
⇒ x = 8/9

(iii) $5.\overline{2}$

Assume that $x = 5.2$.....Eq.(1)
Multiply L.H.S and R.H.S by 10,
We get
$10x = 52.2$ .....Eq. (2)
Subtracting equation (1) from (2),
We get
$10x - x = 52.2 - 5.2$
$\Rightarrow 9x = 47$
$\Rightarrow x = 47/9$

(iv) $0.\overline{001}$

Assume that $x = 0.001$ .....Eq. (1)
Multiply L.H.S and R.H.S by 1000,
We get
$1000x = 1.001$ .....Eq. (2)
Subtracting equation (1) from (2),
We get
$1000x - x = 1.001 - 0.001$
$\Rightarrow 999x = 1$
$\Rightarrow x = 1/999$

(v) 0.2555...

Assume that $x = 0.2555$ …
$\Rightarrow x = 0.25$ ..... Eq. (1)
Multiply L.H.S and R.H.S by 10,
We get
$10x = 2.5$ .....Eq. (2)
Multiply L.H.S and R.H.S by 100,
We get
$100x = 25.5$.....Eq. (3)
Subtracting equation (2) from (3),
We get
$100x - 10x = 25.5 - 2.5$
$\Rightarrow 90x = 23$
$\Rightarrow x = 23/90$

(vi) $0.1\overline{34}$

Let $x = 0.134$ ..... Eq. (1)
Multiply L.H.S and R.H.S by 10,
We get
$10x = 1.34$ .....Eq. (2)
Multiply L.H.S and R.H.S by 1000,
We get
$1000x = 134.34$ .....Eq. (3)
Subtracting equation (2) from (3),

We get
$1000\,x - 10x = 134.34 - 1.34$
$\Rightarrow 990x = 133$
$\Rightarrow x = 133/990$

(vii) .00323232...

Let x = 0.00323232 …
⇒ x = 0.0032 ..... Eq. (1)
Multiply L.H.S and R.H.S by 100,
We get,
100x = 0.32.....Eq. (2)
Multiply L.H.S and R.H.S by 10000,
We get
10000 x = 32.32.....Eq. (3)
Subtracting equation (2) from (3),
We get
10000 x-100x = 32.32 – 0.32
$\Rightarrow 9900x = 32$
$\Rightarrow x = 32/9900 = 8/2475$

(viii) .404040...

Let x = 0.404040 …
$\Rightarrow x = 0.40$..... (1)
Multiply L.H.S and R.H.S by 100,
We get
$100\,x = 40.40$ …… (2)
Subtracting equation (1) from (2),
We get
$100\,x - x = 40.40 - 0.40$
$\Rightarrow 99x = 40$
$\Rightarrow x = 40/99$