

EXERCISE 22.4

Question. 1

Solution:

From the question it is given that,

Function, $y = \log x$

Now, differentiate with respect x ,

$$dy/dx = 1/x$$

By cross multiplication we get,

$$x (dy/dx) = 1$$

Therefore, $y = \log x$ is a solution of the equation.

Then, $x = 1$

$$\text{So, } y = \log (1) = 0$$

$$\text{Hence, } y(1) = 0$$

Question. 2

Solution:

From the question it is given that,

Function, $y = e^x$

Now, differentiate with respect x ,

$$dy/dx = e^x$$

$$dy/dx = y \quad [\text{given } y = e^x]$$

Therefore, $y = e^x$ is a solution of the equation.

Then, $x = 0$

$$\text{So, } y = e^0 = 1$$

$$\text{Hence, } y(0) = 1$$

Question. 3

Solution:

From the question it is given that,

Function, $y = \sin x$... [equation (i)]

Now, differentiate with respect x ,

$$dy/dx = \cos x \quad \dots [\text{equation (ii)}]$$

Then, the above equation is again differentiating with respect to x we get,

$$d^2y/dx^2 = -\sin x$$

From equation (i) $y = \sin x$

So,

$$d^2y/dx^2 = -y$$

Transposing we get,

$$d^2y/dx^2 + y = 0$$

Therefore, $y = \sin x$ is a solution of the equation.

Then, substitute $x = 0$ in equation (i)

$$\text{So, } y = \sin(0)$$

$$y = 0$$

$$\text{Hence, } y(0) = 0$$

Now, substitute $x = 0$ in equation (ii)

$$dy/dx = \cos(0)$$

$$dy/dx = 1$$

$$\text{Hence, } (dy/dx)(0) = 1$$

Question. 4

Solution:

From the question it is given that,

$$\text{Function, } y = e^x + 1$$

... [equation (i)]

Now, differentiate with respect to x ,

$$dy/dx = e^x$$

$$\text{From equation (i), } y = e^x + 1$$

$$\text{Then, } e^x = y - 1$$

$$dy/dx = y - 1$$

... [equation (ii)]

Then, the above equation is again differentiating with respect to x we get,

$$d^2y/dx^2 = dy/dx$$

Transposing we get,

$$(d^2y/dx^2) - (dy/dx) = 0$$

Therefore, $y = e^x + 1$ is a solution of the equation.

Then, substitute $x = 0$ in equation (i)

$$\text{So, } y = e^0 + 1$$

$$y = 1 + 1$$

$$y = 2$$

$$\text{Hence, } y(0) = 2$$

Now, substitute $x = 0$ in equation (ii)

$$dy/dx = e^0 = 1$$

$$dy/dx = 1$$

$$\text{Hence, } (dy/dx)(0) = 1$$

Question. 5

Solution:

From the question it is given that,

Function, $y = e^{-x} + 2$

... [equation (i)]

Now, differentiate with respect x,

$$dy/dx = -e^{-x}$$

From equation (i), $y = e^{-x} + 2$

$$\text{Then, } e^{-x} = y - 2$$

$$dy/dx = -(y - 2)$$

$$dy/dx = -y + 2$$

Transposing we get,

$$(dy/dx) + y = 2$$

Then, the above equation is again differentiating with respect to x we get,

$$d^2y/dx^2 = dy/dx$$

Transposing we get,

$$(d^2y/dx^2) - (dy/dx) = 0$$

Therefore, $y = e^{-x} + 2$ is a solution of the equation.

Then, substitute $x = 0$ in equation (i)

$$\text{So, } y = e^0 + 2$$

$$y = 1 + 2$$

$$y = 3$$

$$\text{Hence, } y(0) = 3$$

