AREAS OF TRIANGLES AND QUADRILATERALS

CHAPTER 14

EXERCISE 14

Answer1 : Given:Base = 24 cm
Height = 14.5 cm

Area of triangle=1/2×Base×Height=1/2×24×14.5=174 cm^2

Answer2 : Let the height of the triangle be *h* m.
∴ Base = 3*h* m
Area of the triangle =Total Cost/Rate =783/58=13.5 ha =135000 m^2
Area of triangle = 135000 m^2⇒1/2×Base×Height =135000⇒1/2×3*h*×*h* =135000

⇒h^2=90000
⇒*h* = √90000
⇒*h* =300 m
Thus, we have:
Height = *h* = 300 m
Base = 3*h* = 900 m

Answer3 : Let, *a*=42 cm, *b* = 34 cm and *c*=20 cm

∴*s*= (a+b+c)/2=(42+34+20)/2=48 cm

By Heron's formula,

Area of triangle =√[*s*(*s*−*a*)(*s*−*b*)(*s*−*c*)]

=√[48(48−42)(48−34)(48−20)]

=√(48×6×14×28)

=4×2×6×7

=336 cm^2
We know that the longest side is 42 cm.

Thus, we can find out the height of the triangle corresponding to 42 cm.
Area of triangle =

$\Rightarrow 336\ cm^2 = \frac{1}{2} \times Base \times Height$

$\Rightarrow Height = \frac{336 \times 2}{42} = 16cm$

Answer4: Let: a=18 cm, b = 24 cm and c=30 cm

$\therefore s = (a+b+c)/2 = (24+18+30)/2 = 36$ cm

By Heron's formula,

Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$

$= \sqrt{[36(36-18)(36-24)(36-30)]}$

$= \sqrt{(36 \times 18 \times 12 \times 6)}$

$= 12 \times 3 \times 6$

$= 216\ cm^2$

We know that the smallest side is 18 cm.
Thus, we can find out the height of the triangle corresponding t18 cm.
Area of triangle

$\Rightarrow 216\ cm^2 = \frac{1}{2} \times Base \times Height$

$\Rightarrow Height = \frac{2 \times 216}{18} = 24cm$

Answer5: Let: a=91 m, b = 98 m and c=105 m

$\therefore s = (a+b+c)/2 = (91+98+105)/2 = 147$ m

By Heron's formula,

Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[147(147-91)(147-98)(147-105)]}$

$=\sqrt{(147\times56\times49\times42)}$

$=7\times7\times7\times2\times3\times2$

$=4116\ m^2$

We know that the longest side is 105 m.
Thus, we can find out the height of the triangle corresponding to 42 cm.
Area of triangle = 4116 $m^2 \Rightarrow 1/2\times Base\times Height = 4116 \Rightarrow Height = 78.4$ m

Answer6: Let the sides of the triangle be $5x$ m, $12x$ m and $13x$ m.
Perimeter = Sum of all sides
or, $150 = 5x + 12x + 13x$
or, $30x = 150$
or, $x = 5$
Thus, we obtain the sides of the triangle.
$5\times5 = 25$ m
$12\times5 = 60$ m
$13\times5 = 65$ m

Now ATQ,
Let: a=25 m, b = 60 m and c=65 m

$\therefore s=150/2=75$ m

By Heron's formula,

Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[75(75-25)(75-60)(75-65)]}=\sqrt{(75\times50\times15\times10)}$

$=15\times5\times10$

$=750\ m^2$

Answer7 : Let the sides of the triangle be $25x$ m, $17x$ m and $12x$ m.
Perimeter = Sum of all sides
or, $540 = 25x + 17x + 12x$

or, $54x = 540$
or, $x = 10$
Thus, we obtain the sides of the triangle.
$25\times10 = 250$ m
$17\times10 = 170$ m
$12\times10 = 120$ m
Let, a=250 m, b =170 m and c=120 m

$\therefore s$=540/2 =270 m

By Heron's formula,

Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[270(270-250)(270-170)(270-120)]}$

$=\sqrt{(270\times20\times100\times150)}$

$=30\times3\times20\times5$

$=9000\ m^2$

Cost of ploughing 10 m^2 field = Rs 18.80
Cost of ploughing 1 m^2 field = Rs18.8/10
Cost of ploughing 9000 m^2 field =18.8/10×9000= Rs 16920

Answer8:(i)Let, a=85 m and b = 154 m

Given: Perimeter = 324 m
or, $a+b+c$ =324
$\Rightarrow c = 324 - a - b$
$\Rightarrow c=324-85-154=85$ m
$\therefore s=\frac{324}{2}$ =162 m
By Heron's formula,
Area of triangle $=\sqrt{[s(s-a)(s-b)(s-c)]}$
$=\sqrt{[162(162-85)(162-154)(162-85)]}$
$=\sqrt{(162\times77\times8\times77)}$
$=\sqrt{(1296\times77\times77)}$
$=\sqrt{(36\times77\times77\times36)}$

$=36\times77$
$=2772\ m^2$

(ii) We can find out the height of the triangle corresponding to 154 m in the following manner:
Area of triangle
$\Rightarrow 2772\ m^2 = \frac{1}{2}\times\text{Base}\times\text{Height}$
$\Rightarrow\text{Height} = (2772\times2)/154 = 36$ m

Answer9: *Given :* $a=13$ cm and $b=20$ cm

$\therefore$Area of isosceles triangle$=\frac{b}{4}\times\sqrt{(4a^2-b^2)}=\frac{20}{4}\times\sqrt{[4(13)^2-20^2]}$

$\Rightarrow 5\times\sqrt{[(4\times13\times13)-(20\times20)]}$

$\Rightarrow 5\times\sqrt{(676-400)} = 5\times\sqrt{276} = 5\times16.6$

$=83.06\ cm^2$

Answer10: Let ΔPQR be an isosceles triangle and $PX\perp QR$.
Area of triangle $=360\ cm^2 \Rightarrow 1/2\times QR\times PX = 360 \Rightarrow h = 9$ cm

Now, $QX = 1/2\times80 = 40$ cm and $PX = 9$ cm
Also,
$PQ=\sqrt{(QX^2+PX^2)} = \sqrt{(40^2+9^2)}$

$\Rightarrow\sqrt{[(40\times40)+(9\times9)]}$

$\Rightarrow \sqrt{(1600+81)} = \sqrt{1681} = 41$ cm
$\therefore$ Perimeter $= 80 + 41 + 41 = 162$ cm

Answer11: The ratio of the equal side to its base is 3 : 2.
$\Rightarrow$Ratio of sides = 3 : 3 : 2.
Let the three sides of triangle be $3y$, $3y$, $2y$.
The perimeter of isosceles triangle = 32 cm.
$\Rightarrow 3y+3y+2y=32$ cm

$\Rightarrow 8y=32$

$\Rightarrow y = \frac{32}{8}$

$\Rightarrow y = 4$ cm

Therefore, the three side of triangle are $3y$, $3y$, $2y$ = 12 cm, 12 cm, 8 cm.

Let S be the semi-perimeter of the triangle. Then,

$S = \frac{1}{2}(12+12+8)$

$\Rightarrow s = \frac{1}{2} \times 32$

$\Rightarrow s = 16$

Area of the triangle will be

$= \sqrt{[S(S-a)(S-b)(S-c)]}$

$= \sqrt{[16(16-12)(16-12)(16-8)]}$

$= \sqrt{(16 \times 4 \times 4 \times 8)}$

$\Rightarrow 4 \times 4 \times 2\sqrt{2} = 32\sqrt{2}$ cm^2

Answer12: Let ABC be any triangle with perimeter 50 cm.

Let the smallest side of the triangle be z.

Then the other sides be $z + 4$ and $zx - 6$.

Now,

$z + z + 4 + 2z - 6 = 50$

$\Rightarrow 4z - 2 = 50$

$\Rightarrow 4z = 50 + 2$

$\Rightarrow 4z = 52$

$\Rightarrow z = 13$

∴ The sides of the triangle are of length 13 cm, 17 cm and 20 cm.

∴ Semi-perimeter of the triangle is

$s = (13+17+20)/2 = 25$ cm

∴ By Heron's formula,

Area of $\Delta ABC = \sqrt{[s(s-a)(s-b)(s-c)]}$

$= \sqrt{[25(25-13)(25-17)(25-20)]}$

$=\sqrt{[25(12)(8)(5)]}$

$=\sqrt{5 \times 5 \times 3 \times 4 \times 4 \times 2 \times 5}$

$=20\sqrt{30}$ cm^2

Hence, the area of the triangle is $20\sqrt{30}$ cm^2

Answer13: The sides of the triangle are of length 13 m, 14 m and 15 m.
∴ Semi-perimeter of the triangle is
$s=(13+14+15)/2=21$ m
∴ By Heron's formula,
Area of $\Delta=\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[21(21-13)(21-14)(21-15)]}$

$=\sqrt{[21(8)(7)(6)]}$

$=\sqrt{7 \times 3 \times 4 \times 2 \times 7 \times 3 \times 2}$

$=84$ m^2

Now,
The rent of advertisements per m^2 per year = Rs 2000
The rent of the wall with area 84 m^2 per year = Rs 2000 × 84
= Rs 168000
The rent of the wall with area 84 m^2 for 6 months = Rs $\frac{168000}{2}$
= Rs 84000

Hence, the rent paid by the company is Rs 84000.

Answer14: Let the equal sides of the isosceles triangle be a cm each.
∴ Base of the triangle, b = 32a cm
(i) Perimeter = 42 cm
or, $a + a + 3/2a = 42$
a=12
So, equal sides of the triangle are 12 cm each.

Base $=\frac{3}{2}a = \frac{3}{2} \times 12 = 18$ cm

(ii)Area of isosceles triangle $= b/4\sqrt{(4a^2 - b^2)} = \frac{18}{4} \times \sqrt{[4(12)^2 - 18^2]}$

$= 4.5\sqrt{(4 \times 144 - 324)}$

$= 4.5\sqrt{(576 - 324)}$

$= 4.5 \times \sqrt{252}$

$= 4.5 \times 15.87$

$= 71.42$ cm^2

(iii)Area of triangle $= 71.42$ cm^2

$\Rightarrow 1/2 \times \text{Base} \times \text{Height} = 71.42$

$\Rightarrow$Height $= 7.94$ cm

Answer15:

Area of equilateral triangle is $36\sqrt{3}$ is given.

Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{Side})^2$

$\Rightarrow \frac{\sqrt{3}}{4} \times (\text{Side})^2 = 36\sqrt{3}$

$\Rightarrow (\text{Side})^2 = 36\sqrt{3} \times \frac{4}{\sqrt{3}} = 36 \times 4 = 72$

$\Rightarrow$Side$=12$ cm

Thus, we have:
Perimeter $= 3 \times$ Side

$\Rightarrow 3 \times 12 = 36$ cm

Answer16: Area of equilateral triangle $=\frac{\sqrt{3}}{4}\times(\text{Side})^2$

$\Rightarrow \frac{\sqrt{3}}{4}\times(\text{Side})^2 = 81\sqrt{3}$

$\Rightarrow (side^2) = 81\sqrt{3} \times \frac{4}{\sqrt{3}}$

$\Rightarrow(\text{Side})^2=324$

$\Rightarrow$Side=18 cm

Now, we have:
Height $=\frac{\sqrt{3}}{2}\times side = \frac{\sqrt{3}}{2}\times 18 = 9\sqrt{3}$cm.

Answer17: Side of the equilateral triangle = 8 cm

(i)Area of equilateral triangle $=\frac{\sqrt{3}}{4}\times(\text{Side})^2=\frac{\sqrt{3}}{4}\times(8)^2 = \frac{\sqrt{3}}{4}\times 64 = 27.71$ cm^2
(ii)Height $=\frac{\sqrt{3}}{2}\times\text{Side}=\frac{\sqrt{3}}{2}\times 8=6.93$ cm

Answer18: Height of the equilateral triangle = 9 cm
Thus, we have:
Height $=\frac{\sqrt{3}}{2}\times\text{Side}$

$\Rightarrow 9 =\frac{\sqrt{3}}{2}\times$S9ide

$\Rightarrow$Side $= \frac{9\times2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = 18\times\frac{\sqrt{3}}{3} = 6\sqrt{3}$cm

Also,
Area of equilateral triangle $=\frac{\sqrt{3}}{4}\times(\text{Side})^2=\frac{\sqrt{3}}{4}\times(6\sqrt{3})^2$

$\Rightarrow\frac{\sqrt{3}}{4}\times 36\times 3$

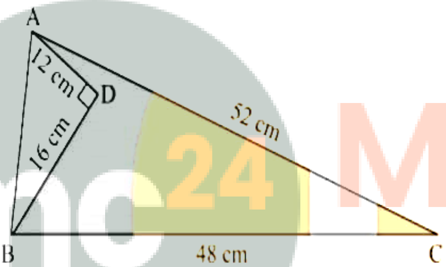
$\Rightarrow 27\sqrt{3} = 46.76$ cm^2

Answer19: Let $\triangle PQR$ be a right-angled triangle and $PQ \perp QR$.
Now,
$PQ = \sqrt{(PR^2 - QR^2)} = \sqrt{(50^2 - 48^2)} = \sqrt{(2500 - 2304)}$

$\Rightarrow \sqrt{196} = \sqrt{14} \times 14 = 14$ cm

Area of triangle $= \frac{1}{2} \times QR \times PQ = \frac{1}{2} \times 48 \times 14 = 336 \text{cm}^2$

Answer 20:



In right angled ΔABD,
$AB^2 = AD^2 + DB^2$

$\Rightarrow AB^2 = 12^2 + 16^2$
$\Rightarrow AB^2 = 144 + 256$
$\Rightarrow AB^2 = 400$

$\Rightarrow AB = \sqrt{400}$
$\Rightarrow AB = 20$ cm

Area of $\Delta ADB = \frac{1}{2} \times DB \times AD$
$= \frac{1}{2} \times 16 \times 12 = 16 \times 6$
$= 96 \text{ cm}^2$ ....(1)

In ΔACB,
The sides of the triangle are of length 20 cm, 52 cm and 48 cm.
∴ Semi-perimeter of the triangle is
$s = \frac{(20+52+48)}{2} = \frac{120}{2} = 60$ cm

∴ By Heron's formula,
Area of $\Delta ACB = \sqrt{[s(s-a)(s-b)(s-c)]}$

$= \sqrt{[60(60-20)(60-52)(60-48)]}$

$= \sqrt{[60(40)(8)(12)]}$

$= \sqrt{6 \times 10 \times 4 \times 10 \times 4 \times 2 \times 6 \times 2}$

$= 480 \text{ cm}^2$ ...(2)

Now,
Area of the shaded region = Area of ΔACB − Area of ΔADB
= 480 − 96
= 384 cm^2

Hence, the area of the shaded region in the given figure is 384 cm^2.

Answer21 : In right angled ΔABC, by Pythagoras theorem
$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 6^2 + 8^2$
$\Rightarrow AC^2 = 36 + 64$
$\Rightarrow AC^2 = 100$

$\Rightarrow AC = \sqrt{100}$
$\Rightarrow AC = 10$ cm
Area of $\Delta ABC = \frac{1}{2} \times AB \times BC$
$= \frac{1}{2} \times 6 \times 8$
$= 24 \text{ cm}^2$ ....(1)
In ΔACD,
The sides of the triangle are of length 10 cm, 12 cm and 14 cm.
∴ Semi-perimeter of the triangle is
$s = \frac{(10+12+14)}{2} = 18$ cm

∴ By Heron's formula,
Area of $\Delta ACD = \sqrt{[s(s-a)(s-b)(s-c)]} = \sqrt{[18(18-10)(18-12)(18-14)]}$

$= \sqrt{[18(8)(6)(4)]}$

$= \sqrt{9 \times 2 \times 4 \times 2 \times 6 \times 4}$

$= 24\sqrt{6}$ cm^2

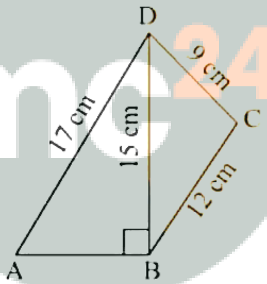
$= 24(2.45)$ cm^2

$= 58.8$ cm^2 ...(2)

Thus,
Area of quadrilateral *ABCD* = Area of Δ*ABC* + Area of Δ*ACD*
= (24 + 58.8) cm^2
= 82.8 cm^2

Hence, the area of quadrilateral *ABCD* is 82.8 cm^2.

Answer 22



We know that Δ*ABD* is a right-angled triangle.

By Pythagoras theorem

$\therefore AB^2 = \sqrt{(AD^2 - DB)^2} = \sqrt{(17^2 - 15^2)} = \sqrt{(289 - 225)} = \sqrt{64} = 8$cm

Now, Area of triangle *ABD* = $\frac{1}{2}$ × Base × Height

$\Rightarrow \frac{1}{2} \times AB \times BD = \frac{1}{2} \times 8 \times 15 = 60$ cm^2

Let, $a = 9$ cm, $b = 15$ cm and $c = 12$ cm

$S = 18$ cm

By Heron's formula,

Area of triangle $DBC = \sqrt{[s(s-a)(s-b)(s-c)]}$

$= \sqrt{[18(18-9)(18-15)(18-12)]}$

$= \sqrt{[(8\times9\times3\times6)}$

$= \sqrt{(6\times3\times3\times3\times3\times6)}$

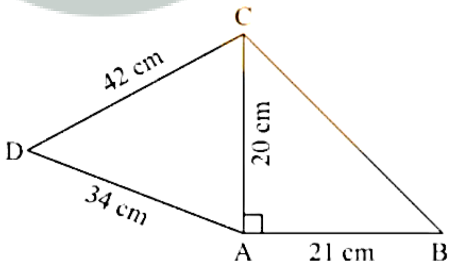
$= 6\times3\times3$

$= 54\ cm^2$

Now,
Area of quadrilateral $ABCD$ = Area of $\triangle ABD$ + Area of $\triangle BCD$
$= (60 + 54)\ cm^2 = 114\ cm^2$

And,
Perimeter of quadrilateral $ABCD = AB + BC + CD + AD = 17 + 8 + 12 + 9 = 46$ cm

Answer 23



In right angled ΔABC,
$BC^2 = AB^2 + AC^2$

$\Rightarrow BC^2 = 21^2 + 20^2$
$\Rightarrow BC^2 = 441 + 400$
$\Rightarrow BC^2 = 841$

$\Rightarrow BC = \sqrt{841}$
$\Rightarrow BC = 29$ cm

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$
$= \frac{1}{2} \times 21 \times 20$
$= 210$ cm^2 ....(1)

In ΔACD, The sides of the triangle are of length 20 cm, 34 cm and 42 cm.
∴ Semi-perimeter of the triangle is
$s = \frac{20+34+42}{2} = 48$ cm

∴ By Heron's formula,

Area of ΔACD

$= \sqrt{[s(s-a)(s-b)(s-c)]}$

$= \sqrt{[48(48-20)(48-34)(48-42)]}$

$= \sqrt{[48(28)(14)(6)]}$

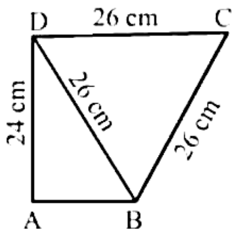
$= \sqrt{6 \times 4 \times 2 \times 7 \times 4 \times 7 \times 2 \times 6}$

$= 336$ cm^2 ...(2)

Thus,
Area of quadrilateral $ABCD$ = Area of ΔABC + Area of ΔACD
$= (210 + 336)$ cm^2
$= 546$ cm^2
Also,
Perimeter of quadrilateral $ABCD = (34 + 42 + 29 + 21)$ cm
$= 126$ cm

Hence, the perimeter and area of quadrilateral $ABCD$ is 126 cm and 546 cm^2, respectively.

Answer 24:



We know that $\triangle BAD$ is a right-angled triangle.

$\therefore AB = \sqrt{(BD^2 - AD)} = \sqrt{(26^2 - 24)^2}$

$\Rightarrow \sqrt{(676-576)} = \sqrt{100} = 10$ cm

Now, Area of triangle $BAD = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times AB \times AD = \frac{1}{2} \times 10 \times 24 = 120 \text{ cm}^2$

Also, we know that $\triangle BDC$ is an equilateral triangle.

$\therefore$Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times (\text{Side})^2 = \frac{\sqrt{3}}{4} \times (26)^2 = \frac{\sqrt{3}}{4} \times 676$

$\Rightarrow 169\sqrt{3} = 292.37 \text{ cm}^2$

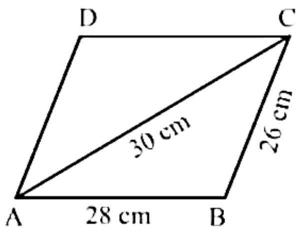
Now,

Area of quadrilateral $ABCD$ = Area of $\triangle ABD$ + Area of $\triangle BDC$

$= (120 + 292.37) \text{ cm}^2 = 412.37 \text{ cm}^2$

Perimeter of ABCD = AB + BC + CD + DA = 10 + 26+ 26 + 24 = 86 cm

Answer 25:



Let, a=26 cm, b =30 cm and c=28 cm

$\Rightarrow s = \frac{(a+b+c)}{2} = \frac{(30+26+28)}{2} = 42\text{cm}$

By Heron's formula,

Area of triangle $ABC = \sqrt{[s(s-a)(s-b)\sqrt{(s-c)}]}$

$= \sqrt{[42(42-26)(42-30)(42-28)]}$

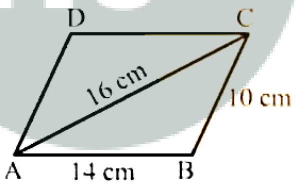
$= \sqrt{(42\times16\times12\times14)}$

$= \sqrt{(14\times3\times4\times4\times2\times2\times3\times14)}$

$= \sqrt{(14\times4\times2\times3)} = 336\ \text{cm}^2$

We know that a diagonal divides a parallelogram into two triangles of equal areas.
∴ Area of parallelogram $ABCD$ = 2(Area of triangle ABC) = 2×336=672 cm^2

Answer 26



Let, a=10 cm, b =16 cm and c=14 cm

$s = \frac{(a+b+c)}{2} = \frac{(10+16+14)}{2} = \frac{40}{2} = 20cm$

By Heron's formula,

Area of triangle $ABC = \sqrt{[s(s-a)(s-b)(s-c)]}$

$= \sqrt{20(20-10)(20-16)(20-14)}$

$= \sqrt{20 \times 10 \times 4 \times 6}$

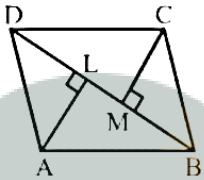
$=\sqrt{(10\times2\times10\times2\times2\times3\times2)}$

$= 10 \times 2 \times 2 \times \sqrt{3}$

$=69.2\ cm^2$

We know that a diagonal divides a parallelogram into two triangles of equal areas.
$\therefore$ Area of parallelogram *ABCD* = 2(Area of triangle *ABC*) = $2\times69.2\ cm^2=138.4\ cm^2$

Answer 27:



Area of *ABCD*=Area of $\triangle ABD$+Area of $\triangle BDC$

$=\frac{1}{2}\times BD\times AL+\frac{1}{2}\times BD\times CM$

$=\frac{1}{2}\times BD(AL+CM)$

$=\frac{1}{2}\times64(16.8+13.2)$

$=32\times30$

$=960\ cm^2$

Answer 28: Let the length of *CD* be *y*.
Then, the length of *AB* be $y + 4$.
Area of trapezium $=\frac{1}{2}\times$sum of parallel sides$\times$height
$\Rightarrow 475=\frac{1}{2}\times(y+y+4)\times19$

$\Rightarrow 475\times2 = 19(2y+4)$

$\Rightarrow 950=38y+76$

$\Rightarrow 38y=950-76$

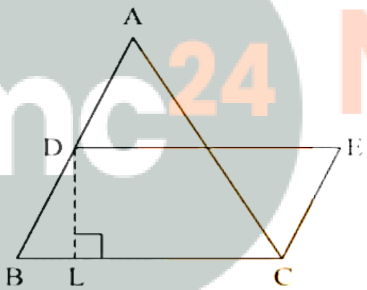
$\Rightarrow 38x=874$

$\Rightarrow y = \frac{874}{38}$

$\Rightarrow x=23$

∴ The length of *CD* is 23 cm and the length of *AB* is 27 cm.
Hence, the lengths of two parallel sides is 23 cm and 27

Answer 29:



In ΔABC,
The sides of the triangle are of length 7.5 cm, 6.5 cm and 7 cm.
∴ Semi-perimeter of the triangle is
$s= (7.5+6.5+7)/2=10.5$ cm
∴ By Heron's formula,
Area of $\Delta ABC=\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[10.5(10.5-7.5)(10.5-6.5)(10.5-7)]}$

$=\sqrt{[10.5(3)(4)(3.5)]}$

$=\sqrt{441}$

$=21$ cm^2 ...(2)

Now,
Area of parallelogram $DBCE$ = Area of ΔABC
= 21 cm^2
Also,
Area of parallelogram $DBCE$ = base × height
$\Rightarrow 21 = BC \times DL$

$\Rightarrow 21 = 7 \times DL$

$\Rightarrow DL = \frac{21}{3} = 3cm$

Hence, the height DL of the parallelogram is 3 cm.

Answer 30: In right angled ΔADE,
$AD^2 = AE^2 + ED^2$

$\Rightarrow 100^2 = (90 - 30)^2 + ED^2$
$\Rightarrow 10000 = 3600 + ED^2$
$\Rightarrow ED^2 = 10000 - 3600$
$\Rightarrow ED^2 = 6400$
$\Rightarrow ED = \sqrt{6400} = 80m$

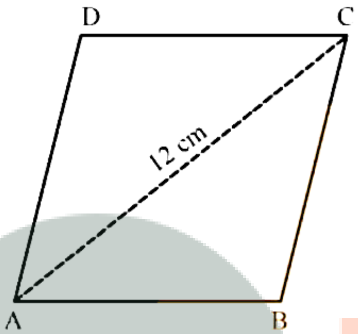
Thus, the height of the trapezium = 80 m ...(1)
Now,
Area of trapezium $= \frac{1}{2} \times$ sum of parallel sides × height
$= \frac{1}{2} \times (90 + 30) \times 80$

$= \frac{1}{2} \times 120 \times 80 = 60 \times 80$
= 4800 m^2
The cost to plough per m^2 = Rs 5
The cost to plough 4800 m^2 = Rs 5 × 4800
= Rs 24000
Hence, the total cost of ploughing the field is Rs 24000.

Answer31 : Let $ABCD$ be a rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front.

According to the laws, the length of the inner rectangle = 40 − 3 − 3 = 34 m and the breath of the inner rectangle = 15 − 2 − 2 = 11 m.
∴ Area of the inner rectangle *PQRS* = Length × Breath
= 34 × 11
= 374 m^2
Hence, the largest area where house can be constructed is 374 m^2.

Answer 32: Let the sides of rhombus be of length x cm.



Perimeter of rhombus = $4x$
⇒ $40 = 4x$
⇒ $x = 10$ cm
Now,
In ΔABC,
The sides of the triangle are of length 10 cm, 10 cm and 12 cm.
∴ Semi-perimeter of the triangle is
$s=(10+10+12)/2=16$ cm
∴ By Heron's formula,
Area of $\Delta ABC=\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[16(16-10)(16-10)(16-12)]}$

$=\sqrt{[16(6)(6)(4)]}$

$=\sqrt{4 \times 4 \times 6 \times 6 \times 2 \times 2}$

$=48$ cm^2 ...(1)
In ΔADC, The sides of the triangle are of length 10 cm, 10 cm and 12 cm.
∴ Semi-perimeter of the triangle is
$s=(10+10+12)/2=16$ cm
∴ By Heron's formula,
Area of $\Delta ABC=\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[16(16-10)(16-10)(16-12)]}$

$=\sqrt{[16(6)(6)(4)]}$

$=\sqrt{2304}$

$=48\text{ cm}^2$ ..(2)

∴ Area of the rhombus = Area of ΔABC + Area of ΔADC

= 48 + 48

= 96 cm^2

The cost to paint per cm^2 = Rs 5

The cost to paint 96 cm^2 = Rs 5 × 96

= Rs 480

The cost to paint both sides of the sheet = Rs 2 × 480

= Rs 960

Hence, the total cost of painting is Rs 960.

Answer33: Let the semi-perimeter of the triangle be s.

Let the sides of the triangle be a, b and c.

Given: $s - a = 8$, $s - b = 7$ and $s - c = 5$ ....(1)

Adding all three equations

$3s - (a + b + c) = 8 + 7 + 5$

$\Rightarrow 3s - (a + b + c) = 20$

$\Rightarrow 3s - 2s = 20$

$\Rightarrow s = 20$ cm ....(2)

∴ By Heron's formula,

Area of $\Delta=\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[20(8)(7)(5)]}$

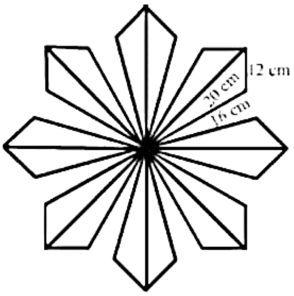
$=\sqrt{2\times5\times2\times2\times2\times2\times7\times5}$

$=20\sqrt{14}\text{cm}^2$

Hence, the area of the triangle is $20\sqrt{14}\text{ cm}^2$

.

Answer34



Let, a=16 cm, b = 12 cm and c=20 cm

s= (a+b+c)/2=(16+12+20)/2=24 cm

By Heron's formula,

:Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$=\sqrt{24(24-16)(24-12)(24-20)}$

$=\sqrt{24\times8\times12\times4}$

$=\sqrt{6\times4\times4\times4\times4\times6}$

$=6\times4\times4$

$=96\ cm^2$

Now,
Area of 16 triangular-shaped tiles = $16\times96=1536\ cm^2$
Cost of polishing tiles of area 1 cm^2 = Rs 1
Cost of polishing tiles of area 1536 cm^2 = 1×1536 =Rs 1536

Answer 35

We know that the triangle is an isosceles triangle.
Thus, we can find out the area of one triangular piece of cloth.
Area of isosceles triangle $= b/4\sqrt{(4a^2 - b^2)}$

$= 20/4 \times \sqrt{[4(50)^2 - 20^2]}$

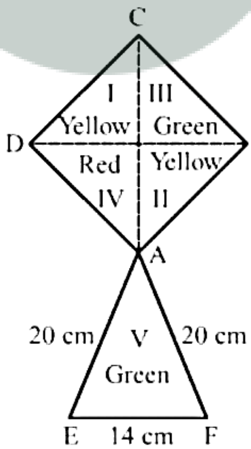
$= 5 \times \sqrt{(10000 - 400)}$

$\Rightarrow 5 \times \sqrt{9600} = 5 \times 40\sqrt{6}$

$\Rightarrow 200\sqrt{6} = 490 \text{ cm}^2$

Now,
Area of 1 triangular piece of cloth = 490 cm^2
Area of 12 triangular pieces of cloth = 12×490 = 5880 cm^2

Answer 36:



In the given figure, *ABCD* is a square with diagonal 44 cm.
∴ $AB = BC = CD = DA$. ....(1)

In right angled ΔABC,
$AC^2 = AB^2 + BC^2$

$\Rightarrow 44^2 = 2AB^2$
$\Rightarrow 1936 = 2AB^2$
$\Rightarrow AB^2 = 1936/2$
$\Rightarrow AB^2 = 968$

$\Rightarrow AB = \sqrt{968}$
$\Rightarrow AB = 22\sqrt{2}$ cm ...(2)
$\therefore$ Sides of square $AB = BC = CD = DA = 22\sqrt{2}$ cm
Area of square $ABCD$ = $(\text{side})^2$
$= (22\sqrt{2})^2$
$= 968 \text{ cm}^2$ ...(3)
Area of red portion $= \frac{968}{4} = 242 \text{ cm}^2$
Area of yellow portion $= \frac{968}{2} = 484 \text{ cm}^2$
Area of green portion $= \frac{968}{4} = 242 \text{ cm}^2$
Now, in ΔAEF,
The sides of the triangle are of length 20 cm, 20 cm and 14 cm.
$\therefore$ Semi-perimeter of the triangle is
$s = (20+20+14)/2 = 27$ cm
$\therefore$ By Heron's formula,
Area of $\Delta AEF = \sqrt{[s(s-a)(s-b)(s-c)]}$

$= \sqrt{[27(27-20)(27-20)(27-14)]}$

$= \sqrt{[27(7)(7)(13)]}$

$= \sqrt{2139}$

$= 131.04 \text{ cm}^2$ ...(4)
Total area of the green portion = 242 + 131.04 = 373.04 cm^2
Hence, the paper required of each shade to make a kite is red paper 242 cm^2, yellow paper 484 cm^2 and green paper 373.04 cm^2.

Answer37 : Area of rectangle $ABCD$ = Length × Breath
= 75 × 4
= 300 m^2
Area of rectangle $PQRS$ = Length × Breath
= 60 × 4
= 240 m^2

Area of square $EFGH = (\text{side})^2$
$= (4)^2$
$= 16\ m^2$
$\therefore$ Area of the footpath = Area of rectangle $ABCD$ + Area of rectangle $PQRS$ − Area of square $EFGH$
$= 300 + 240 - 16$
$= 524\ m^2$
The cost of gravelling the road per m^2 = Rs 50
The cost of gravelling the roads 524 m^2 = Rs 50 × 524
= Rs 26200
Hence, the total cost of gravelling the roads at Rs 50 per m^2 is Rs 26200.

Answer38 : Given, 10m wide at the top, and 6m wide at the bottom
Let the height of the trapezium be h.

Area of trapezium $=\frac{1}{2}\times$sum of parallel sides$\times$height

$\Rightarrow 640 = \frac{1}{2} \times (10+6) \times h$
$\Rightarrow 640 = \frac{1}{2} \times 16 \times h = 8h$
$\Rightarrow h = \frac{640}{8} = 80$ m
Hence, the depth of the canal is 80 m.

Answer39: In ΔBCE, The sides of the triangle are of length 15 m, 13 m and 14 m.
$\therefore$ Semi-perimeter of the triangle is
$s = (15+13+14)/2 = 21$ m
$\therefore$ By Heron's formula,
Area of $\Delta BCE = \sqrt{[s(s-a)(s-b)(s-c)]}$

$= \sqrt{[21(21-15)(21-13)(21-14)]}$

$= \sqrt{[21(6)(8)(7)]}$

$= \sqrt{7 \times 3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 7}$

$= 84\ m^2$ ...(1)
Also,

Area of $\Delta BCE = \frac{1}{2} \times \text{Base} \times \text{Height}$

$\Rightarrow 84 = \frac{1}{2} \times 14 \times Height$

$\Rightarrow 84 = 7 \times \text{Height}$

$\Rightarrow \text{Height} = \frac{84}{7}$

$\Rightarrow$Height=12 m

$\therefore$ Height of ΔBCE = Height of the parallelogram $ABED$ = 12 m

Now,

Area of the parallelogram $ABED$ = Base × Height

= 11 × 12

= 132 m^2 ...(2)

$\therefore$ Area of the trapezium = Area of the parallelogram $ABED$ + Area of the triangle BCE

= 132 + 84

= 216 m^2

Hence, the area of a trapezium is 216 m^2.

Answer40 : Let the length of the parallel sides be l and $l - 8$.

The height of the trapezium = 24 cm

Area of trapezium =1/2×sum of parallel sides×height

$\Rightarrow 312 = \frac{1}{2} \times (l + l - 8) \times 24$

$\Rightarrow 312 = 12(2l - 8)$

$\Rightarrow 2l - 8 = 312/12$

$\Rightarrow 2l - 8 = 26$

$\Rightarrow 2l = 26 + 8$

$\Rightarrow 2l = 34$

$\Rightarrow l = \frac{34}{2}$

$\Rightarrow l = 17$ cm

Hence, the lengths of the parallel sides are 17 cm and 9 cm.

Answer41: Diagonals d_1 and d_2 of the rhombus measure 120 m and 44 m, respectively.

Base of the parallelogram = 66 m

Now,

Area of the rhombus = Area of the parallelogram

$\Rightarrow \frac{1}{2} \times d_1 \times d_2 = \text{Base} \times \text{Height}$

$\Rightarrow \frac{1}{2} \times 120 \times 44 = 66 \times \text{Height}$

$\Rightarrow 60 \times 44 = 66 \times \text{Height}$

$\Rightarrow \text{Height} = \frac{60 \times 44}{66} = \frac{2640}{66}$

$\Rightarrow$Height=40 m
Hence, the measure of the altitude of the parallelogram is 40 m.

Answer42 : It is given that,
Sides of the square = 40 m
Altitude of the parallelogram = 25 m
Now,
Area of the parallelogram = Area of the square
$\Rightarrow \text{Base} \times \text{Height} = (\text{side})^2$

$\Rightarrow \text{Base} \times 25 = (40)^2$

$\Rightarrow \text{Base} \times 25 = 1600$

$\Rightarrow \text{Base} = \frac{1600}{25}$

$\Rightarrow$Base = 64 m
Hence, the length of the corresponding base of the parallelogram is 64 m.

Answer43: It is given that,
The sides of rhombus = 20 cm.
One of the diagonal = 24 cm.
In ΔABC,

The sides of the triangle are of length 20 cm, 20 cm and 24 cm.
∴ Semi-perimeter of the triangle is
s= (20+20+24)/2
⇒ 64/2=32 cm

∴ By Heron's formula,
Area of $\Delta ACD = \sqrt{[s(s-a)(s-b)(s-c)]}$
$= \sqrt{[32(32-20)(32-20)(32-24)]}$

$=\sqrt{[32(12)(12)(8)]}$

$=192\ cm^2$ ..(1)

In ΔACD,
The sides of the triangle are of length 20 cm, 20 cm and 24 cm.
∴ Semi-perimeter of the triangle is
$s= (20+20+24)/2=64/2=32$ cm
∴ By Heron's formula,
Area of $\Delta ACD=\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[32(32-20)(32-20)(32-24)]}$
$=\sqrt{[32(12)(12)(8)]}$
$= 12 \times 8 \times 2$
$=192\ cm^2$ ...(2)
∴ Area of the rhombus = Area of ΔABC + Area of ΔACD
= 192 + 192
= 384 cm^2
Hence, the area of a rhombus is 384 cm^2.

Answer44 : It is given that,
Area of rhombus = 480 cm^2.
One of the diagonal = 48 cm.
(i) Area of the rhombus = $\frac{1}{2}\times d_1\times d_2$
$\Rightarrow 480=\frac{1}{2}\times 48\times d_2$

$\Rightarrow 480=24\times d_2$
$\Rightarrow d_2= \frac{480}{24} = \frac{6\times 8\times 10}{6\times 4}$
$\Rightarrow d_2=20$ cm
Hence, the length of the other diagonal is 20 cm.
(ii) We know that the diagonals of the rhombus bisect each other at right angles.
In right angled ΔABO,
$AB^2 = AO^2 + OB^2$
$\Rightarrow AB^2 = 24^2 + 10^2$
$\Rightarrow AB^2 = 576 + 100$
$\Rightarrow AB^2 = 676$
$\Rightarrow AB = \sqrt{676}$
$\Rightarrow AB = 26$ cm
Hence, the length of each of the sides of the rhombus is 26 cm.
(iii) Perimeter of the rhombus = 4 × side
= 4 × 26

= 104 cm

Hence, the perimeter of the rhombus is 104 cm.

MULTIPLE-CHOICE QUESTIONS

Answer1: (b) 30 cm^2

Area of triangle = $\frac{1}{2}$×Base×Height

Area of $\Delta ABC = \frac{1}{2} \times 12 \times 5 = 30 cm^2$

Answer2: (a)96 cm^2

Let, a=20 cm, b = 16 cm and c=12 cm

s= (a+b+c)/2=(20+16+12)/2=24 cm
By Heron's formula, we have:
Area of triangle =√[$s(s-a)(s-b)(s-c)$]
=√[24(24−20)(24−16)(24−12)]
=√(24×4×8×12)
=√(6×4×4×4×4×6)
=6×4×4
=96 cm^2

Answer3: (b) 16√3 cm^2

Area of equilateral triangle = √3/4(Side) ^{2}=√3/4(8) ^{2}=√3/4(64) =16√3 cm^2

Answer4: (b) 8√5 cm^2

Area of isosceles triangle = b/4×√($4a^2-b^2$)

a= 6 cm and b=8 cm

Thus

$=8/4\times\sqrt{[4(6)^2-8^2]}$

$=8/4\times\sqrt{(144-64)}$

$=8/4\times\sqrt{80}$

$=8/4\times4\sqrt{5}$

$=8\sqrt{5}\text{ cm}^2$

Answer5: (c) 4 cm

Height of isosceles triangle = $\frac{1}{2}\times\sqrt{(4a^2-b^2)}$

$=\frac{1}{2}\times\sqrt{[4(5)^2-6^2]}$

$=\frac{1}{2}\times\sqrt{(100-36)}$

$=\frac{1}{2}\times\sqrt{64}$

$=\frac{1}{2}\times8$

$=4$ cm

Answer6: (b) 50 cm^2

Here, the base and height of the triangle are 10 cm and 10 cm, respectively.

Thus,

Area of triangle $=\frac{1}{2}\times\text{Base}\times\text{Height}=\frac{1}{2}\times10\times10=50\text{ cm}^2$

Answer7: (b) $5\sqrt{3}$cm

Height of equilateral triangle$=\sqrt{3}/2\times$ (Side) $=\sqrt{3}/2\times10=5\sqrt{3}$ cm

Answer8:(a) $12\sqrt{3}\text{ cm}^2$

Height of equilateral triangle $=\sqrt{3}/2\times$ (Side)

$\Rightarrow 6=\sqrt{3}/2\times$ (Side)

$\Rightarrow$Side$=(12/\sqrt{3})\times(\sqrt{3}/\sqrt{3})=(12/3)\times\sqrt{3}=4\sqrt{3}$ cm

Now,

Area of equilateral triangle = $\sqrt{3}/4 \times (\text{Side})^2 = \sqrt{3}/4 \times (4\sqrt{3})^2 = \sqrt{3}/4 \times 48 = 12\sqrt{3}$ cm^2

Answer9: (c) 384 m^2
Let, a=40 m, b = 24 m and c=32 m

s= (a+b+c)/2=(40+24+32)/2=48 m

By Heron's formula,

Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[48(48-40)(48-24)(48-32)]}$

$=\sqrt{(48\times8\times24\times16)}$

$=\sqrt{(24\times2\times8\times24\times8\times2)}$

$=24\times8\times2$

=384 m^2

Answer10:(b) 750 cm^2
Let the sides of the triangle be $5x$ cm, $12x$ cm and $13x$ cm.
Perimeter = Sum of all sides
or, $150 = 5x + 12x + 13x$
or, $30x = 150$
or, $x = 5$
Thus, the sides of the triangle are 5×5 cm, 12×5 cm and 13×5 cm, i.e., 25 cm, 60 cm and 65 cm.
Now,
Let: a=25 cm, b = 60 cm and c=65 cm

s= 150/2=75 cm

By Heron's formula,

Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[75(75-25)(75-60)(75-65)]}$

$=\sqrt{(75\times50\times15\times10)}$

$=\sqrt{(15\times5\times5\times10\times15\times10)}$

$=15\times5\times10$

$=750\ cm^2$

Answer11:(a)24 cm

Let, a=30 cm, b = 24 cm and c=18 cm

$s=(a+b+c)/2=(30+24+18)/2=36$ cm

On applying Heron's formula, we get

Area of triangle = $\sqrt{[s(s-a)(s-b)(s-c)]}$

$=\sqrt{[36(36-30)(36-24)(36-18)]}$

$=\sqrt{(36\times6\times12\times18)}$

$=\sqrt{(12\times3\times12\times6\times3)}$

$=12\times3\times6$

$=216\ cm^2$

The smallest side is 18 cm.

Hence, the altitude of the triangle corresponding to 18 cm is given by:

Area of triangle = 216 cm^2

$\Rightarrow\frac{1}{2}\times\text{Base}\times\text{Height} = 216$

$\Rightarrow\text{Height} = (216\times2)/18=24$ cm

Answer12:(b) 36 cm

Let $\triangle PQR$ be an isosceles triangle and $PX\perp QR$.

Now,

Area of triangle =48 cm^2

$\Rightarrow\frac{1}{2}\times QR\times PX = 48$

$\Rightarrow h = 96/16=6$ cm

Also,

$QX = 1/2 \times 24 = 12$ cm and $PX = 12$ cm
$PQ = \sqrt{(QX^2 + PX^2)}$

$a = \sqrt{(8^2+6^2)} = \sqrt{(64+36)} = \sqrt{100} = 10$ cm
∴ Perimeter = (10 + 10 + 16) cm = 36 cm

Answer13: (a)36 cm
Area of equilateral triangle = $\sqrt{3}/4 \times (\text{Side})^2 \Rightarrow \sqrt{3}/4 \times (\text{Side})^2 = 36\sqrt{3}$

$(\text{Side})^2 = 144$

Side =12cm
Now,
Perimeter = 3 × Side = 3 × 12 = 36 cm

Answer14:(c) 60 cm^2
Area of isosceles triangle = $\frac{b}{4} \times \sqrt{4(a)^2 - (b)^2}$ Here,

a = 13 cm and b=24 cm

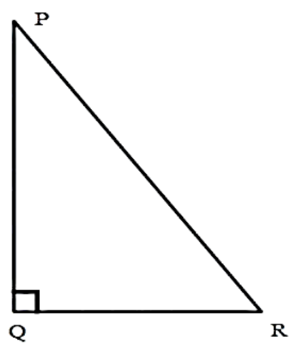
Thus, $\frac{24}{4} \times \sqrt{(4(13)^2 - 24^2}$

$= 6 \times \sqrt{(676 - 576)}$

$= 6 \times \sqrt{100}$

$\Rightarrow 6 \times 10 = 60$ cm^2

Answer15 :(c) 336 cm^2



Let $\triangle PQR$ be a right-angled triangle and PQ⊥QR.
Now,
$PQ=\sqrt{PR^2-QR^2}$

$=\sqrt{(50^2-48^2)}$

$=\sqrt{(2500-2304)}$

$=\sqrt{196}$ $=14$ cm
∴Area of triangle $=1/2\times QR\times PQ=\frac{1}{2}\times 48\times 14=336cm^2$

Answer16 :(a) $9\sqrt{3}$cm
Area of equilateral triangle = $81\sqrt{3}$ cm2

$\Rightarrow\sqrt{\frac{3}{4}}(side)^2=81\sqrt{3}$
$\Rightarrow(\text{Side})^2=81\times4$
$\Rightarrow(\text{Side})^2=324$
⇒Side=18 cm
Now,
Height = $\sqrt{3}/2\times\text{Side}=\sqrt{3}/2\times18=9\sqrt{3}$ cm.