Exercise 12(B)

Solution 1:

According to equal intercept theorem since CD=DE

Therefore AB=BC and EF=GF

(i)BC=AB=7.2cm

(ii)GE=EF+GF=2EF= 2×4 =8cm

Since B,D,F are the midpoint and AE||BF||CG

Therefore AE=2BD and CG=2DF

(iii)AE=2BD= 2×4.1 =8.2

(iv) $DF=\frac{1}{2}CG=\frac{1}{2}\times11=5.5cm$

Solution 2:

Given that AD=AP=PB as 2AD=AB and p is the midpoint of AB

(i)From triangle DPR, A and Q are the midpoint of DP and DR.

Therefore AQ||PR

Since PR||BS ,hence AQ||BS

(ii)From triangle ABC, P is the midpoint and PR||BS

Therefore R is the midpoint of BC

$From\ \Delta BRS\ and\ \Delta QRC$

$\angle BRS = \angle QRC$

$BR = RC$

$\angle RBS = \angle RCQ$

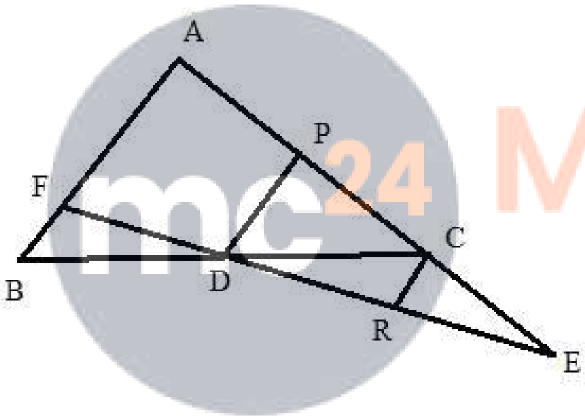
$\therefore\ \Delta BRS \cong \Delta QRC$

$\therefore\ QR = RS$

DS=DQ+QR+RS=QR+QR+RS=3RS

Solution 3:

Consider the figure:



Here *D* is the midpoint of *BC* and *DP* is parallel to *AB*, therefore *P* is the midpoint of *AC* and $PD = \frac{1}{2}AB$

(i)

Again from the triangle *AEF* we have *AE* ||*PD*||*CR* and $AP = \frac{1}{3}AE$

Therefore $DF = \frac{1}{3}EF$ or we can say that 3DF = EF .

Hence it is shown.

(ii)

From the triangle *PED* we have *PD*||*CR* and C is the midpoint of *PE* therefore $CR = \frac{1}{2}PD$

Now

$PD = \frac{1}{2}AB$

$\frac{1}{2}PD = \frac{1}{4}AB$

$CR = \frac{1}{4}AB$

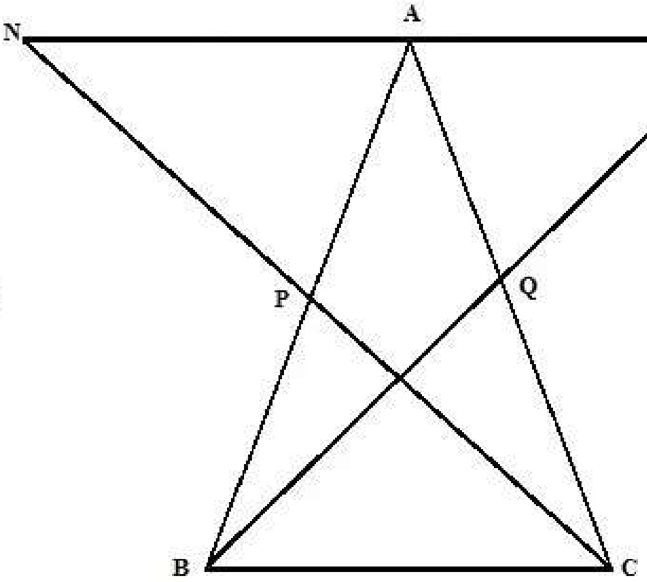
$4CR = AB$

Hence it is shown.

Solution 4:

The figure is shown below

(i)



From triangle BPC and triangle APN

$\angle BPC = \angle APN$ [Opposite angle]

$BP = AP$

$PC = PN$

$\therefore \Delta BPC \cong \Delta APN$ [SAS postulate]

$\therefore \angle PBC = \angle PAN$ ..... (1)

And BC=AN ......(3)

Similarly $\angle QCB = \angle QAN$ .....(2)

And BC=AM ......(4)

Now

$\angle ABC + \angle ACB + \angle BAC = 180°$

$\angle PAN + \angle QAM + \angle BAC = 180°$ [(1),(2) we get]

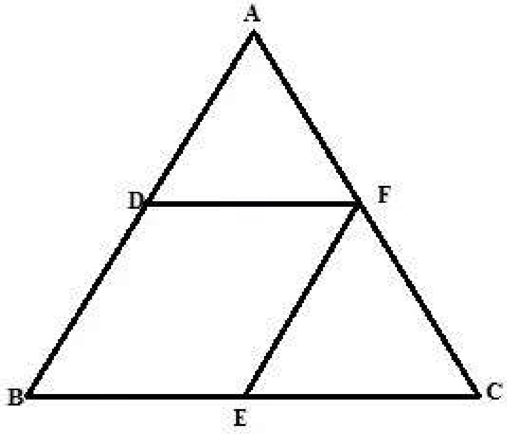
Therefore M,A,N are collinear

(ii) From (3) and (4) MA=NA

Hence A is the midpoint of MN

Solution 5:

The figure is shown below



From the figure EF||AB and E is the midpoint of BC.

Therefore F is the midpoint of AC.

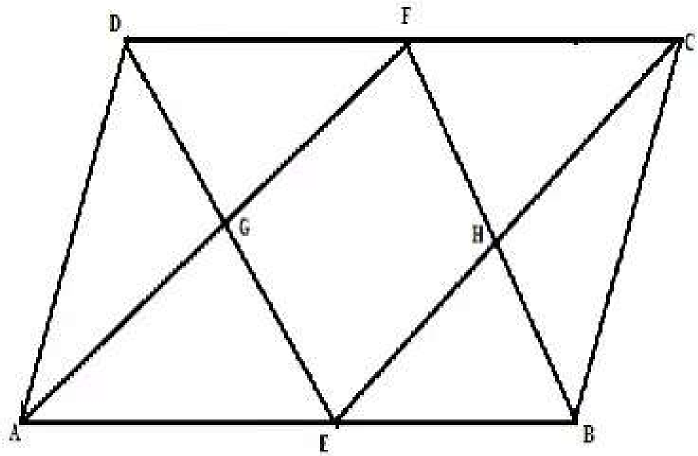
Here EF||BD, EF=BD as D is the midpoint of AB

BE||DF, BE=DF as E is the midpoint of BC.

Therefore BEFD is a parallelogram.

Solution 6:

The figure is shown below



(i)

$From\ \Delta HEB\ and\ \Delta FHC$

$BE = FC$

$\angle EHB = \angle FHC \quad$ [Opposite angle]

$\angle HBE = \angle HFC$

$\therefore\ \Delta HEB \cong \Delta FHC$

$\therefore\ EH = CH, BH = FH$

(ii)

Similarly AG=GF and EG=DG .....(1)

For triangle ECD, F and H are the midpoint of CD and EC.

Therefore HF||DE and $HF = \frac{1}{2}DE$ .....(2)

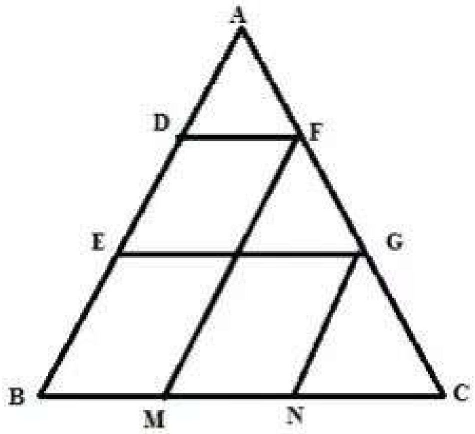
(1),(2) we get, HF=EG and HF||EG

Similarly we can show that EH=GF and EH||GF

Therefore GEHF is a parallelogram.

Solution 7:

The figure is shown below



For triangle AEG

D is the midpoint of AE and DF||EG||BC

Therefore F is the midpoint of AG.

AF=GF .....(1)

Again DF||EG||BC DE=BE, therefore GF=GC .....(2)

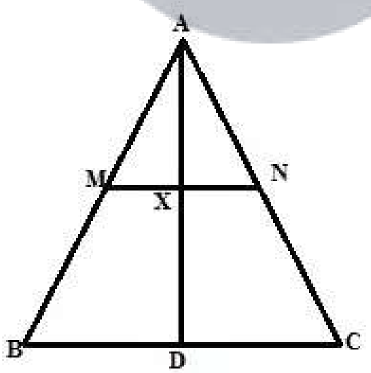
(1),(2) we get AF=GF=GC.

Similarly Since GN||FM||AB and AF=GF ,therefore BM=MN=NC

Hence proved

Solution 8:

The figure is shown below



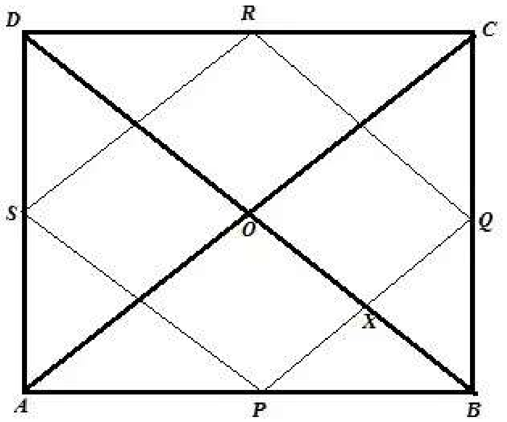
Since M and N are the midpoint of AB and AC, MN||BC

According to intercept theorem Since MN||BC and AM=BM,

Therefore AX=DX. Hence proved

Solution 9:

The figure is shown below



Let ABCD be a quadrilateral where P,Q,R,S are the midpoint of AB,BC,CD,DA.PQRS is a rectangle. Diagonal AC and BD intersect at point O. We need to show that AC and BD intersect at right angle.

Proof:

PQ||AC, therefore $\angle AOD = \angle PXO$ $\left[Corresponding\ angle\right]$ ....(1)

||Again BD||RQ, therefore $\angle PXO = \angle RQX = 90°\left[\text{Corresponding angle and angle of rectangle}\right]$ ...(2)

From (1) and (2) we get ,

$\angle AOD = 90°$

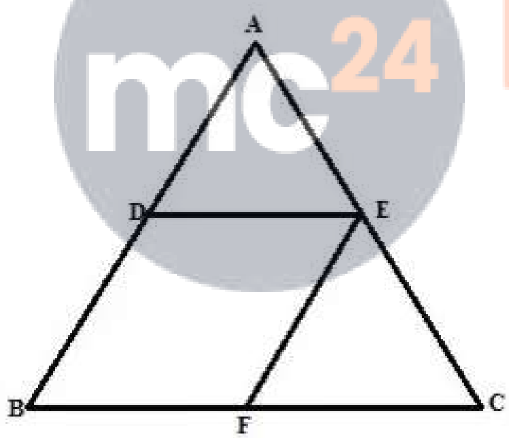
Similarly $\angle AOB = \angle BOC = \angle DOC = 90°$

Therefore diagonals AC and BD intersect at right angle

Hence proved

Solution 10:

The figure is shown below



From figure since E is the midpoint of AC and EF||AB

Therefore F is the midpoint of BC and 2DE=BC or DE=BF

Again D and E are midpoint ,therefore DE||BF and EF=BD

Hence BDEF is a parallelogram.

Now

$$BD = EF = \frac{1}{2}AB = \frac{1}{2}\times 16 = 8cm$$

$$BF = DE = \frac{1}{2}BC = \frac{1}{2}\times 18 = 9cm$$

Therefore perimeter of BDEF=2(BF+EF)= $2(9+8) = 34cm$

Solution 11:

Given AD and CE are medians and DF || CE.
We know that from the midpoint theorem, if two lines are parallel and the starting point of segment is at the midpoint on one side, then the other point meets at the midpoint of the other side.
Consider triangle BEC. Given DF || CE and D is midpoint of BC.
So F must be the midpoint of BE.

So $FB = \frac{1}{2}BE$ but $BE = \frac{1}{2}AB$

Substitute value of BE in first equation, we get

$FB = \frac{1}{4}AB$

Hence Prove

Solution 12:

Given ABCD is parallelogram, so AD = BC, AB = CD.
Consider triangle APB, given EC is parallel to AP and E is midpoint of side AB. So by midpoint theorem, C has to be the midpoint of BP.
So BP = 2BC, but BC = AD as ABCD is a parallelogram.
Hence BP = 2AD
Consider triangle APB, AB || OC as ABCD is a parallelogram. So by midpoint theorem, O has to be the midpoint of AP.
Hence Proved

Solution 13:

Consider trapezium ABCD.
Given E and F are midpoints on sides AD and BC, respectively.

We know that AB = GH = IJ

From midpoint theorem, $EG = \frac{1}{2}DI$, $HF = \frac{1}{2}JC$

Consider LHS,
AB + CD = AB + CJ + JI + ID = AB + 2HF + AB + 2EG
So AB + CD = 2(AB + HF + EG) = 2(EG + GH + HF) = 2EF
AB + CD = 2EF
Hence Proved

Solution 14:

Given Δ ABC
AD is the median. So D is the midpoint of side BC.
Given DE || AB. By the midpoint theorem, E has to be midpoint of AC.
So line joining the vertex and midpoint of the opposite side is always known as median. So BE is also median of Δ ABC.