Exercise 1.3

Express each of the following integers as a product of its prime.

(i) 420

Solution:

2	420
2	210
5	105
7	21
3	3
	1

420 = 2 x 2 x 3 x 5 x 7

∴ 420 = 2^2x 3 x 5 x 7

(ii) 468

Solution:

2	468
2	234
3	117
3	39
13	13
	1

468 = 2 x 2 x 3 x 3 x 13

∴ 468 = 2^2x 3^2 x 13

(iii) 945

Solution:

5	945
3	189
3	63
3	21
7	7
	1

945 = 3 x 3 x 3 x 5 x 7

∴ 945 = 3^3x 5 x 7

(iv) 7325

Solution:

5	7325
5	1465
293	293
	1

7325 = 5 x 5 x 293

∴ 7325 = 5^2 x 293

2. Determine the prime factorization of each of the following positive integer :

(i) 20570

Solution:

5	20570
2	4114
11	2057
11	187
17	17
	1

20570 = 2 x 5 x 11 x 11 x 17

∴ 20570 = 2 x 5 x 11^2x 17

(ii) 58500

Solution:

5	58500
5	11700
5	2340
2	468
2	234
3	117
3	39
13	13
	1

58500 = 2 x 2 x 3 x 3 x 5 x 5 x 5 x 13

∴ 58500 = 2^2 x 3^2x 5^3x 13

(iii) 45470971

Solution:

7	45470971
7	6495853
13	927979
13	71383
17	5491
17	323
19	19
	1

45470971 = 7x7x13x13x17x17x19

∴ 45470971 = 7^2x13^2x17^2x19

3. Explain why 7 x 11 x 13 + 13 and 7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 are composite numbers.

Solution:

So, basically there are two types of numbers i.e., prime numbers and composite numbers.

Understanding that,

Prime numbers are those numbers having 1 and the number itself as factors. And,

Composite numbers are those numbers having factors other than 1 and itself.

It's seen that,

7 x 11 x 13 + 13 = 13 x (7 x 11 + 1) [taking 13 out as common]

= 13 x (77 + 1)

= 13 x 78

= 13 x 13 x 6

So, the given expression has 6 and 13 as its factors. Therefore, we can conclude that it is a composite number.

Similarly,

7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 = 5 x (7 x 6 x 4 x 3 x 2 x 1 + 1) [taking 5 out- common]

= 5 x (1008 + 1)

= 5 x 1009

Since, 1009 is a prime number the given expression has 5 and 1009 as its factors other than 1 and the number itself. Hence, it is also a composite number.

4. Check whether 6^n can end with the digit 0 for any natural number n.

Solution:

In order to check whether 6^n can end with the digit 0 for any natural number n, let us find the factors of 6.

It's seen that the factors of 6 are 2 and 3.

So, $6^n = (2 \times 3)^n$

$6^n = 2^n \times 3^n$

Since, the prime factorization of 6 does not contain 5 and 2 as its factor, together. We can thus conclude that 6^n can never end with the digit 0 for any natural number n.

5. Explain why $3 \times 5 \times 7 + 7$ is a composite number.

Solution:

So, basically there are two types of numbers i.e., prime numbers and composite numbers.

Understanding that,

Prime numbers are those numbers having 1 and the number itself as factors. And,

Composite numbers are those numbers having factors other than 1 and itself.

It's seen that,

$$3 \times 5 \times 7 + 7 = 7 \times (3 \times 5 + 1) = 7 \times (15 + 1) = 7 \times 16$$

Since, the given expression has 7 and 16 as its factors we can conclude that it is a composite number.