NCERT Solutions for Class 10 Maths Chapter 7 Coordinate Geometry EXERCISE 7.4

1. Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A (2, – 2) and B (3, 7)

Solution:

Let the given line divide the line segment joining the points A(2, - 2) and B(3, 7) in a ratio k : 1

Coordinates of the point of division = $(\frac{3k+2}{k+1}, \frac{7k-2}{k+1})$

This point also lies on $2x + y - 4 = 0$

Therefore,

$2\,(\frac{3k+2}{k+1}) + (\frac{7k-2}{k+1}) - 4 = 0$

$\frac{6k+4+7k-2-4k-4}{k+1} = 0$

$9k - 2 = 0$

$k = \frac{2}{9}$

Hence, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, - 2) and B(3, 7) is 2:9

2. Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear

Solution: If the given points are collinear, then the area of triangle formed by these points will be 0

Area of the triangle = $\frac{1}{2}$ [$x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)$

= $\frac{1}{2}$ [x (2 – 0) + 1 (0 – y) + 7 (y – 2)

0 = $\frac{1}{2}$ (2x – y + 7y – 14)

2x + 6y – 14 = 0

$x + 3y - 7 = 0$

This is the required relation between x and y

3. Find the centre of a circle passing through the points (6, – 6), (3, – 7) and (3, 3)

Solution: Let O (x, y) be the centre of the circle. And let the points (6, - 6), (3, - 7), and (3, 3) be representing the points A, B,and C on the circumference of the circle

$OA = [(x- 6)^2 + (y+ 6)^2]^{1/2}$

$OB = [(x- 3)^2 + (y+ 7)^2]^{1/2}$

$OC = [(x- 3)^2 + (y- 3)^2]^{1/2}$

However, OA = OB

$[(x - 6)^2 + (y + 6)^2]^{1/2} = [(x - 3)^2 + (y + 7)^2]^{1/2}$

$-6x - 2y + 14 = 0$

$3x + y = 7$ (i)

Similarly, OA = OC

$[(x - 6)^2 + (y + 6)^2]^{1/2} = [(x - 3)^2 + (y - 3)^2]^{1/2}$

$-6x + 18y + 54 = 0$

$-3x + 9y = -27$ (ii)

On adding equation (i) and (ii), we obtain

$10y = - 20$

$y = - 2$

From equation (i), we obtain

$3x - 2 = 7$

$3x = 9$

$x = 3$

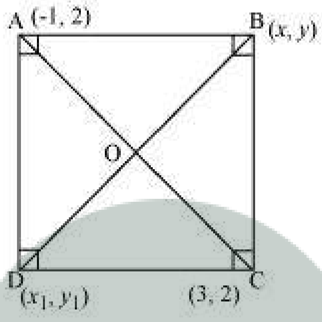
Therefore, the centre of the circle is (3, - 2)

4. The two opposite vertices of a square are (–1, 2) and (3, 2). Find the coordinates of the other two vertices

Solution: Let ABCD be a square having (- 1, 2) and (3, 2) as vertices A and C respectively. Let (x, y), (x1, y1) be the coordinate of vertex B and D respectively

We know that the sides of a square are equal to each other

AB = BC



$[(x + 1)^2 + (y - 2)^2]^{1/2} = [(x - 3)^2 + (y - 2)^2]^{1/2}$

$x^2 + 2x + 1 + y^2 – 4y + 4 = x^2 + 9 – 6x + y^2 + 4 – 4y$

$8x = 8$

$x = 1$

We know that in a square, all interior angles are of 90°.

In ΔABC,

$AB^2 + BC^2 = AC^2$

$[(x + 1)^2 + (y - 2)^2]^{1/2 * 2} + [(x - 3)^2 + (y - 2)^2]^{1/2 * 2} = [(3 + 1)^2 + (2 - 2)^2]^{1/2 * 2}$

$4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$

$2y^2 + 16 - 8y = 16$

$2y^2 - 8y = 0$

$y(y - 4) = 0$

$y = 0$ or 4

We know that in a square, the diagonals are equal inlength and bisect each other at 90°.

Let O be the mid-point ofAC. Therefore, it will also be the mid-point of BD

5. The Class X students of asecondary school inKrishinagarhave been allotteda rectangular plot of land fortheir gardening activity.Sapling of Gulmohar areplanted on the boundary at adistance of 1m from each other.There is a triangular grassylawn in the plot as shown inthe Fig. 7.14. The students areto sow seeds of floweringplants on the remaining area of the plot

(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?

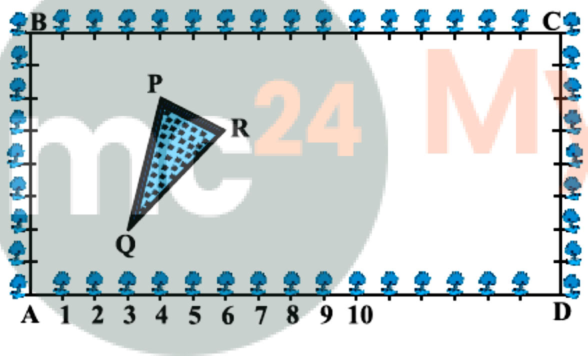


Fig. 7.14

Solution: (i) Taking A as origin,

We will take AD as *x*-axis and AB as *y*-axis.

It can be observed that the coordinates of point P,Q, and R are (4, 6), (3, 2), and (6, 5) respectively

Area of the triangle PQR = $\frac{1}{2}$ [x_1 ($y_2 - y_3$) + x_2 ($y_3 - y_1$) + x_3 ($y_1 - y_2$)

= $\frac{1}{2}$ [4 (2 – 5) + 3 (5 – 6) + 6 (6 – 2)

= $\frac{1}{2}$ (-12 – 3 + 24)

= $\frac{9}{2}$ Square units

(ii) Taking C as origin, CB as *x*-axis, and CD as *y*-axis, the coordinates of vertices P, Q, and R are (12, 2), (13, 6),and (10, 3) respectively

Area of the triangle PQR = $\frac{1}{2}$ [x_1 ($y_2 - y_3$) + x_2 ($y_3 - y_1$) + x_3 (y_1 - y_2)

= $\frac{1}{2}$ [12 (6 – 3) + 13 (3 – 2) + 10 (2 – 6)

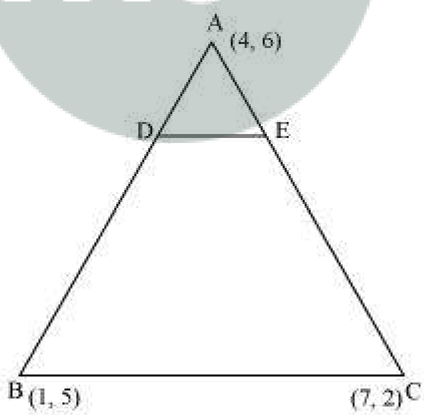
= $\frac{1}{2}$ (36 + 13 – 40)

= $\frac{9}{2}$ Square units

It can be observed that the area of the triangle is same in both the cases

6. The vertices of a Δ ABC are A(4, 6), B(1, 5) and C(7, 2). A line is drawn to intersect sidesAB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of theΔ ADE and compare it with the area of Δ ABC (Recall Theorem 6.2 and Theorem 6.6)

Solution: Given that,



$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$$

$\frac{AD}{AD+DB} = \frac{AE}{AE+EC} = \frac{1}{4}$

$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3

Coordinates of point D = $(\frac{1*1+3*4}{1+3}, \frac{1*5+3*6}{1+3})$

= $(\frac{13}{4}, \frac{23}{4})$

Coordinates of point E = $(\frac{1*7+3*4}{1+3}, \frac{1*2+3*6}{1+3})$

= $(\frac{19}{4}, \frac{20}{4})$

Area of the triangle = $\frac{1}{2}$ [x_1 ($y_2 - y_3$) + x_2 ($y_3 - y_1$) + x_3 (y_1 - y_2)]

= $\frac{1}{2}$ [4 $(\frac{23}{4} - \frac{20}{4})$ + $\frac{13}{4}$ $(\frac{20}{4}$ - 6) + $\frac{19}{4}$ (6 – $\frac{23}{4}$)]

= $\frac{1}{2}$ [3 – $\frac{13}{4}$ + $\frac{19}{16}$]

= $\frac{1}{2}$ [$\frac{48-52+19}{16}$]

= $\frac{15}{32}$ Square units

Area of triangle ABC = $\frac{1}{2}$ [4 (5 – 2) + 1 (2 - 6) + 7 (6 – 5)]

= $\frac{1}{2}$ (12 – 4 + 7)

= $\frac{15}{2}$ Square units

Clearly, the ratio between the areas of ΔADE and ΔABC is 1:16

We know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel tothe third side of the triangle.

These two triangles so formed (here ΔADE and ΔABC) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles

And, ratio between the areas of ΔADE and ΔABC = $(\frac{1}{4})^2$

= $\frac{1}{16}$

7. Let A (4, 2), B(6, 5) and C (1, 4) be the vertices of Δ ABC

(i) The median from A meets BC at D. Find the coordinates of the point D

(ii) Find the coordinates of the point P on AD such that AP: PD = 2: 1

(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ: QE = 2: 1 and CR: RF = 2: 1

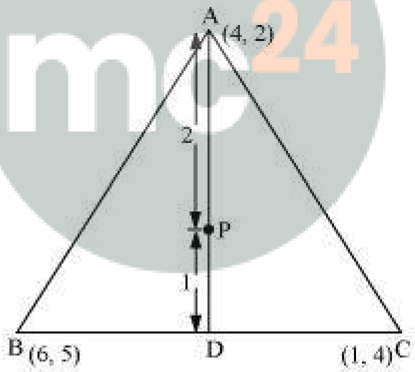
(iv) What do you observe?

[Note: The point which is common to all the three medians is called the centroidand this point divides each median in the ratio 2: 1]

(v) If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of Δ ABC, find the coordinates of the centroid of the triangle

Solution:(i) Median AD of the triangle will divide the side BC in two equal parts

Therefore, D is the mid-point of side BC



Coordinates of D = $(\frac{6+1}{2}, \frac{5+4}{2})$

= $(\frac{7}{2}, \frac{9}{2})$

(ii) Point P divides the side AD in a ratio 2:1

Coordinates of P = $(\frac{2*\frac{7}{2}+1*4}{2+1}, \frac{2*\frac{9}{2}+1*2}{2+1})$

= $(\frac{11}{3}, \frac{11}{3})$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC

Coordinates of E = $(\frac{4+1}{2}, \frac{2+4}{2})$

= $(\frac{5}{2}, 3)$

Point Q divides the side BE in a ratio 2:1

Coordinates of Q = $(\frac{2*\frac{5}{2}+ 1*6}{2+1}, \frac{2*3+1*5}{2+1})$

= $(\frac{11}{3}, \frac{11}{3})$

Median CF of the triangle will divide the side AB in two equal parts. Therefore, F is the mid-point of side AB

Coordinates of F = $(\frac{4+6}{2}, \frac{2+5}{2})$

= $(5, \frac{7}{2})$

Point R divides the side CF in a ratio 2:1

Coordinates of R = $(\frac{2*5+1*1}{2+1}, \frac{2*\frac{7}{2}+ 1*4}{2+1})$

= $(\frac{11}{3}, \frac{11}{3})$

(iv) It can be observed that the coordinates of point P, Q, R are the same.Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle

(v) Consider a triangle, ΔABC, having its vertices as A($x1$, $y1$), B($x2$, $y2$), and C($x3$,$y3$)

Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC

Coordinates of D = $(\frac{x2+x3}{2}, \frac{y2+y3}{2})$

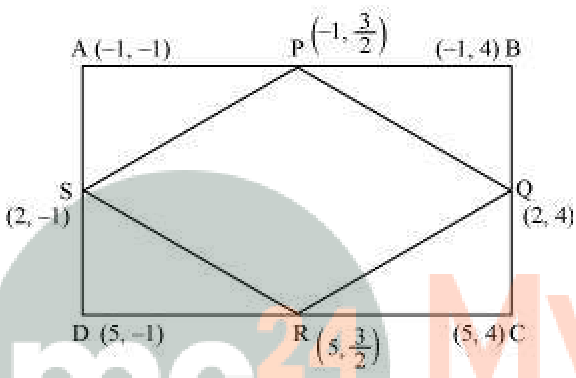
Let the centroid of this triangle be O.Point O divides the side AD in a ratio 2:1

Coordinates of O = $\left(\frac{2*\frac{x2}{3}+\frac{x3}{2}+1*x1}{2+1}, \frac{2*\frac{y2}{2}+\frac{y3}{2}+1*y1}{2+1}\right)$

= $\left(\frac{x1+x2+x3}{3}, \frac{y1+y2+y3}{3}\right)$

8. ABCD is a rectangle formed by the points A(–1, –1), B(– 1, 4), C(5, 4) and D(5, – 1). P, Q,R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateralPQRS a square? a rectangle? or a rhombus? Justify your answer

Solution: P is the mid-point of AB



Coordinates of P = $(-1, \frac{3}{2})$

Similarly coordinates of Q, R and S are (2, 4), $(5, \frac{3}{2})$ and (2, -1) respectively

Length of PQ = $[(-1-2)^2 + (\frac{3}{2} - 4)^2]^{1/2}$

= $\sqrt{\frac{61}{4}}$

Length of QR = $[(2-5)^2 + (4 - \frac{3}{2})^2]^{1/2}$

= $\sqrt{\frac{61}{4}}$

Length of RS = $[(5-2)^2 + (\frac{3}{2} + 1)^2]^{1/2}$

= $\sqrt{\frac{61}{4}}$

Length of SP = $[(2+1)^2 + (-1 - \frac{3}{2})^2]^{1/2}$

$= \sqrt{\frac{61}{4}}$

Length of PR = $[(-1- 5)^2 + (\frac{3}{2} - \frac{3}{2})^2]^{1/2}$

= 6

Length of QS = $[(2 - 2)^2 + (4 + 1)^2]^{1/2}$

= 5

It can be observed that all sides of the given quadrilateral are of the same measure. However, the diagonals are ofdifferent lengths. Therefore, PQRS is a rhombus