Exercise 5(E)

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0;\ x\neq 3,\ x\neq -\frac{3}{2}$$

Solution:

Given equation,

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0$$

$$\Rightarrow \frac{2x(2x+3)+1(x-3)+3x+9}{(x-3)(2x+3)}=0$$

$4x^2 + 6x + x – 3 + 3x + 9 = 0$
$4x^2 + 10x + 6 = 0$
$4x^2 + 4x + 6x + 6 = 0$
$4x(x + 1) + 6(x + 1) = 0$
$(4x + 6) (x + 1) = 0$
So, $4x + 6 = 0$ or $x + 1 = 0$
$x = -1$ or $x = -6/4 = -3/2$ (rejected as this value is excluded in the domain)
Therefore,
$x = -1$ is the only solution

Solution:

Given, $(2x + 3)^2 = 81$
Taking square root on both sides we have,
$2x + 3 = \pm 9$
$2x = \pm 9 – 3$
$x = (\pm 9 – 3)/ 2$
So,
$x = (9 – 3)/ 2$ or $(-9 – 3)/ 2$
Therefore,
$x = 3$ or $x = -6$

Solution:

Given equation, $a^2x^2 – b^2 = 0$
$(ax)^2 – b^2 = 0$
$(ax + b)(ax - b) = 0$
So,
$ax + b = 0$ or $ax – b = 0$
Therefore,
$x = -b/a$ or b/a

Solution:

Given equation, $x^2 – 11/4\ x + 15/8 = 0$
Taking L.C.M we have,
$(8x^2 – 22x + 15)/ 8 = 0$
$8x^2 – 22x + 15 = 0$
$8x^2 – 12x – 10x + 15 = 0$
$4x(2x - 3) – 5(2x - 3) = 0$
$(4x - 5)(2x - 3) = 0$
So, $4x – 5 = 0$ or $2x – 3 = 0$
Therefore,
$x = 5/4$ or $x = 3/2$

1. $x + 4/x = -4; x \neq 0$
Solution:

Given equation, $x + 4/x = -4$
$(x^2 + 4)/ x = -4$
$x^2 + 4 = -4x$
$x^2 + 4x + 4 = 0$
$x^2 + 2x + 2x + 4 = 0$
$x(x + 2) + 2(x + 2) = 0$
$(x + 2)(x + 2) = 0$
$(x + 2)^2 = 0$
Taking square - root we have,
$x + 2 = 0$
Therefore, $x = -2$

2. $2x^4 – 5x^2 + 3 = 0$
Solution:

Given equation, $2x^4 – 5x^2 + 3 = 0$
Let's take $x^2 = y$
Then, the equation becomes
$2y^2 – 5y + 3 = 0$
$2y^2 – 2y – 3y + 3 = 0$
$2y(y - 1) - 3(y - 1) = 0$
$(2y - 3)\ (y - 1) = 0$
So, $2y – 3 = 0$ or $y – 1 = 0$
$y = 3/2$ or $y = 1$
And, we have taken $y = x^2$
Thus,
$x^2 = 3/2$ or $x^2 = 1$
$x = \pm \sqrt{(3/2)}$ or $x = \pm 1$

3. $x^4 – 2x^2 – 3 = 0$
Solution:

Given equation, $x^4 – 2x^2 – 3 = 0$
Let's take $x^2 = y$
Then, the equation becomes
$y^2 – 2y – 3 = 0$
$y^2 – 3y + y – 3 = 0$
$y(y - 3) + 1(y - 3) = 0$
$(y + 1)(y - 3) = 0$
So, $y + 1 = 0$ or $y – 3 = 0$
$y = -1$ or $y = 3$
And, we have taken $y = x^2$
Thus,
$x^2 = - 1$(impossible, no real solution)
$x^2 = 3$
$x = \pm \sqrt{3}$

4. $9\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) - 52 = 0$

Solution:

Let us take $(x + 1/x) = y$ …. (1)
Now, squaring it on both sides
$(x + 1/x)^2 = y^2$
$x^2 + 1/x^2 + 2 = y^2$
So,
$x^2 + 1/x^2 = y^2 – 2$ ….. (2)
Using (1) and (2) in the given equation, we have
$9(y^2 – 2) – 9(y) – 52 = 0$
$9y^2 – 18 – 9y – 52 = 0$
$9y^2 – 9y – 70 = 0$
$9y^2 – 30y + 21y – 70 = 0$
$3y(3y - 10) + 7(3y - 10) = 0$
$(3y + 7)(3y - 10) = 0$
So, $3y + 7 = 0$ or $3y – 10 = 0$
$y = -7/3$ or $y = 10/3$
Now,

$x + 1/x = -7/3$	or	$x + 1/x = 10/3$
$(x^2 + 1)/x = -7/3$	or	$(x^2 + 1)/x = 10/3$
$3x^2 – 10x + 3 = 0$	or	$3x^2 + 7x + 3 = 0$

$3x^2 – 9x – x + 3 = 0$ or
$3x(x - 3) – 1(x - 3) = 0$
$(3x - 1)(x - 3) = 0$
So, $x = 1/3$ or 3

$$x=\frac{-7\pm\sqrt{(-7)^2-4(3)(3)}}{2(3)}$$
$$x=\frac{-7\pm\sqrt{13}}{6}$$

5. $2\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)=11$

Solution:

Let us take $(x + 1/x) = y$ …. (1)
Now, squaring it on both sides
$(x + 1/x)^2 = y^2$
$x^2 + 1/x^2 + 2 = y^2$
So,
$x^2 + 1/x^2 = y^2 – 2$ ….. (2)
Using (1) and (2) in the given equation, we have
$2(y^2 – 2) – (y) = 11$
$2y^2 – 4 – y = 11$
$2y^2 – y – 15 = 0$
$2y^2 – 6y + 5y – 15 = 0$
$2y(y - 3) + 5(y - 3) = 0$
$(2y + 5) (y - 3) = 0$
So,
$2y + 5 = 0$ or $y - 3 = 0$
$y = -5/2$ or $y = 3$
Now,

$x + 1/x = -5/2$	or	$x + 1/x = 3$
$(x^2 + 1)/x = -5/2$	or	$(x^2 + 1)/x = 3$
$2(x^2 + 1) = -5x$	or	$x^2 + 1 = 3x$
$2x^2 + 5x + 2 = 0$	or	$x^2 - 3x + 1 = 0$
$2x^2 + 4x + x + 2 = 0$	or	
$2x(x + 2) + 1(x + 2) = 0$		$x=\frac{-3\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}$
$(2x + 1)(x + 2) = 0$		
Hence, $x = -1/2$ or -2		$x=\frac{-3\pm\sqrt{5}}{2}$

6. $\left(x^2+\frac{1}{x^2}\right)-3\left(x-\frac{1}{x}\right)-2=0$

Solution:

Let us take $(x - 1/x) = y$ …. (1)
Now, squaring it on both sides
$(x - 1/x)^2 = y^2$

$x^2 + 1/x^2 - 2 = y^2$
So,
$x^2 + 1/x^2 = y^2 + 2$ ….. (2)
Using (1) and (2) in the given equation, we have
$(y^2 + 2) – 3(y) – 2 = 0$
$y^2 - 3y = 0$
$y(y - 3) = 0$
So, $y = 0$ or $y – 3 = 0$
Now,

$(x - 1/x) = 0$	or	$(x - 1/x) = 3$
$x^2 – 1 = 0$	or	$x^2 – 1 = 3x$
$x^2 = 1$	or	$x^2 – 3x – 1 = 0$

Therefore,

$x = \pm 1$	or	$x=\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$

$$x=\frac{3 \pm \sqrt{13}}{2}$$

Exercise 5(F)

(i) (x + 5) (x - 5) = 24
Solution:

Given equation, (x + 5) (x - 5) = 24
$x^2 – 25 = 24$
$x^2 = 49$
Thus,
$x = \pm 7$

(ii) $3x^2 - 2\sqrt{6}x + 2 = 0$
Solution:

Given equation, $3x^2 - 2\sqrt{6}x + 2 = 0$
$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$
$\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$
$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$
So, $\sqrt{3}x - \sqrt{2} = 0$ or $\sqrt{3}x - \sqrt{2} = 0$
Therefore,
$x = \sqrt{(2/3)}, \sqrt{(2/3)}$ (equal roots)

(iii) $3\sqrt{2}x^2 – 5x - \sqrt{2} = 0$
Solution:

Given equation, $3\sqrt{2}x^2 – 5x - \sqrt{2} = 0$
$3\sqrt{2}x^2 – 6x + x - \sqrt{2} = 0$
$3\sqrt{2}x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$
$(3\sqrt{2}x + 1)(x - \sqrt{2}) = 0$
So, $3\sqrt{2}x + 1 = 0$ or $x - \sqrt{2} = 0$
Therefore,
$x = -1/ 3\sqrt{2}$ or $x = \sqrt{2}$

(iv) $2x – 3 = \sqrt{(2x^2 – 2x + 21)}$
Solution:

Given equation, $2x – 3 = \sqrt{(2x^2 – 2x + 21)}$
On squaring on both sides, we have
$(2x – 3)^2 = 2x^2 – 2x + 21$
$4x^2 + 9 – 12x = 2x^2 – 2x + 21$
$2x^2 – 10x – 12 = 0$
Dividing by 2, we get
$x^2 – 5x – 6 = 0$
$x^2 – 6x + x – 6 = 0$
$x(x - 6) + 1(x - 6) = 0$

(x + 1) (x - 6) = 0
So, x + 1 = 0 or x – 6 = 0
Thus, we get
x = -1 or x = 6
But, putting x = -1 the L.H.S become negative. And we know that the square root function always gives a positive value.
Therefore,
x = 6 is the only solution.

2. One root of the quadratic equation $8x^2 + mx + 15 = 0$ is ¾. Find the value of m. Also, find the other root of the equation.
Solution:

Given equation, $8x^2 + mx + 15 = 0$
One of the roots is ¾, and hence it satisfies the given equation
So,
$8(3/4)^2 + m(3/4) + 15 = 0$
$8(9/16) + m(3/4) + 15 = 0$
$18/4 + 3m/4 + 15 = 0$
Taking L.C.M, we have
$(18 + 3m + 60)/4 = 0$
$18 + 3m + 60 = 0$
$3m = - 78$
$m = -26$
Now, putting the value of m in the given equation, we get
$8x^2 + (-26)x + 15 = 0$
$8x^2 - 26x + 15 = 0$
$8x^2 – 20x – 6x + 15 = 0$
$4x(2x - 5) – 3(2x - 5) = 0$
$(4x – 3) (2x - 5) = 0$
So, $4x – 3 = 0$ or $2x – 5 = 0$
Therefore.
x = ¾ or x = 5/2

3. Show that one root of the quadratic equation $x^2 + (3 – 2a)x – 6a = 0$ is -3. Hence, find its other root.
Solution:

Given quadratic equation, $x^2 + (3 – 2a)x – 6a = 0$
Now, putting x = -3 we have
$(-3)^2 + (3 – 2a)(-3) – 6a = 0$
$9 – 9 + 6a – 6a = 0$
$0 = 0$
Since, x = -3 satisfies the given equation -3 is one of the root of the quadratic equation.
$x^2 + (3 – 2a)x – 6a = 0$
$x^2 + 3x – 2ax – 6a = 0$

$x(x + 3) – 2a(x + 3) = 0$
$(x – 2a) (x + 3) = 0$
So, $x – 2a = 0$ or $x + 3 = 0$
$x = 2a$ or $x = -3$
Hence, the other root is 2a.

4. If $p – 15 = 0$ and $2x^2 + px + 25 = 0$: find the values of x.
Solution:

Given equations, $p – 15 = 0$ and $2x^2 + px + 25 = 0$
Thus, $p = 15$
Now, using p in the quadratic equation, we get
$2x^2 + (15)x + 25 = 0$
$2x^2 + 10x + 5x + 25 = 0$
$2x(x + 5) + 5(x + 5) = 0$
$(2x + 5) (x + 5) = 0$
So, $2x + 5 = 0$ or $x + 5 = 0$
Hence,
$x = -5/2$ or $x = -5$

5. Find the solution of the quadratic equation $2x^2 – mx – 25n = 0$; if $m + 5 = 0$ and $n – 1 = 0$.
Solution:

Given,
$m + 5 = 0$ and $n – 1 = 0$
so,
$m = -5$ and $n = 1$
Now, putting these values in the given quadratic equation $2x^2 – mx – 25n = 0$, we get
$2x^2 – (-5)x – 25(1) = 0$
$2x^2 + 5x – 25 = 0$
$2x^2 + 10x – 5x – 25 = 0$
$2x(x + 5) -5(x + 5) = 0$
$(2x - 5) (x + 5) = 0$
So, $2x – 5 = 0$ or $x + 5 = 0$
Hence,
$x = 5/2$ or $x = -5$

6. If m and n are roots of the equation: $1/x – 1/(x-2) = 3$: where $x \neq 0$ and $x \neq 2$; find m x n.
Solution:

Given equation, $1/x – 1/(x-2) = 3$
$(x - 2 - x)/ (x(x - 2)) = 3$
$-2 = 3(x^2 – 2x)$
$3x^2 – 6x + 2 = 0$
Solving by using quadratic formula, we get

$$\Rightarrow x = \frac{6 \pm \sqrt{6^2 - 4(3)(2)}}{2 \times 3}$$

$$\Rightarrow x = \frac{6 \pm \sqrt{12}}{2 \times 3}$$

$$\Rightarrow x = \frac{\sqrt{3} \pm 1}{\sqrt{3}}$$

And, since m and n are roots of the equation, we have

m = (√3 + 1)/ √3 n = (√3 - 1)/ √3

So,

m x n = (√3 + 1)/ √3 x (√3 - 1)/ √3 = [$(\sqrt{3})^2$ – 1]/ $(\sqrt{3})^2$

Thus,

m x n = 2/3