EXERCISE 2.3

If A = {1, 2, 3}, B = {4, 5, 6}, which of the following are relations from A to B? Give reasons in support of your answer.
(i) {(1, 6), (3, 4), (5, 2)}
(ii) {(1, 5), (2, 6), (3, 4), (3, 6)}
(iii) {(4, 2), (4, 3), (5, 1)}
(iv) A × B
Solution:
Given,
A = {1, 2, 3}, B = {4, 5, 6}
A relation from A to B can be defined as:
A × B = {1, 2, 3} × {4, 5, 6}
= {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

(i) {(1, 6), (3, 4), (5, 2)}
No, it is not a relation from A to B. The given set is not a subset of A × B as (5, 2) is not a part of the relation from A to B.

(ii) {(1, 5), (2, 6), (3, 4), (3, 6)}
Yes, it is a relation from A to B. The given set is a subset of A × B.

(iii) {(4, 2), (4, 3), (5, 1)}
No, it is not a relation from A to B. The given set is not a subset of A × B.

(iv) A × B
A × B is a relation from A to B and can be defined as:
{(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6),(3, 4),(3, 5),(3, 6)}

2. A relation R is defined from a set A = {2, 3, 4, 5} to a set B = {3, 6, 7, 10} as follows: (x, y) R x is relatively prime to y. Express R as a set of ordered pairs and determine its domain and range.
Solution:
Relatively prime numbers are also known as co-prime numbers. If there is no integer greater than one that divides both (that is, their greatest common divisor is one).

Given: (x, y) ∈ R = x is relatively prime to y
Here,
2 is co-prime to 3 and 7.

3 is co-prime to 7 and 10.
4 is co-prime to 3 and 7.
5 is co-prime to 3, 6 and 7.
∴ R = {(2, 3), (2, 7), (3, 7), (3, 10), (4, 3), (4, 7), (5, 3), (5, 6), (5, 7)}
Domain of relation R = {2, 3, 4, 5}
Range of relation R = {3, 6, 7, 10}

3. Let A be the set of first five natural and let R be a relation on A defined as follows: (x, y) R x ≤ y
Express R and R^{-1} as sets of ordered pairs. Determine also
(i) the domain of R^{-1}
(ii) The Range of R.
Solution:
A is set of first five natural numbers.
So, A= {1, 2, 3, 4, 5}
Given: (x, y) R x ≤ y
1 is less than 2, 3, 4 and 5.
2 is less than 3, 4 and 5.
3 is less than 4 and 5.
4 is less than 5.
5 is not less than any number A
∴ R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5)}

"An inverse relation is the set of ordered pairs obtained by interchanging the first and second elements of each pair in the original relation. If the graph of a function contains a point (a, b), then the graph of the inverse relation of this function contains the point (b, a)".

∴ R^{-1} = {(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (2, 2), (3, 2), (4, 2), (5, 2), (3, 3), (4, 3), (5, 3), (4, 4), (5, 4) (5, 5)}

(i) Domain of R^{-1} = {1, 2, 3, 4, 5}
(ii) Range of R = {1, 2, 3, 4, 5}

4. Find the inverse relation R^{-1} in each of the following cases:
(i) R= {(1, 2), (1, 3), (2, 3), (3, 2), (5, 6)}
(ii) R= {(x, y) : x, y ∈ N; x + 2y = 8}
(iii) R is a relation from {11, 12, 13} to (8, 10, 12} defined by y = x – 3

Solution:
(i) Given:
R= {(1, 2), (1, 3), (2, 3), (3, 2), (5, 6)}
So, R^{-1} = {(2, 1), (3, 1), (3, 2), (2, 3), (6, 5)}

(ii) Given,
R= {(x, y): x, y ∈ N; x + 2y = 8}
Here, x + 2y = 8
x = 8 – 2y
As y ∈ N, Put the values of y = 1, 2, 3,…… till x ∈ N
When, y = 1, x = 8 – 2(1) = 8 – 2 = 6
When, y = 2, x = 8 – 2(2) = 8 – 4 = 4
When, y = 3, x = 8 – 2(3) = 8 – 6 = 2
When, y = 4, x = 8 – 2(4) = 8 – 8 = 0
Now, y cannot hold value 4 because x = 0 for y = 4 which is not a natural number.
∴ R = {(2, 3), (4, 2), (6, 1)}
R^{-1} = {(3, 2), (2, 4), (1, 6)}

(iii) Given,
R is a relation from {11, 12, 13} to (8, 10, 12} defined by y = x – 3
Here,
x = {11, 12, 13} and y = (8, 10, 12}
y = x – 3
When, x = 11, y = 11 – 3 = 8 ∈ (8, 10, 12}
When, x = 12, y = 12 – 3 = 9 ∉ (8, 10, 12}
When, x = 13, y = 13 – 3 = 10 ∈ (8, 10, 12}
∴ R = {(11, 8), (13, 10)}
R^{-1} = {(8, 11), (10, 13)}

5. Write the following relations as the sets of ordered pairs:
(i) A relation R from the set {2, 3, 4, 5, 6} to the set {1, 2, 3} defined by x = 2y.
(ii) A relation R on the set {1, 2, 3, 4, 5, 6, 7} defined by (x, y) ∈ R ⇔ x is relatively prime to y.
(iii) A relation R on the set {0, 1, 2,…,10} defined by 2x + 3y = 12.
(iv) A relation R form a set A = {5, 6, 7, 8} to the set B = {10, 12, 15, 16, 18} defined by (x, y) R x divides y.
Solution:
(i) A relation R from the set {2, 3, 4, 5, 6} to the set {1, 2, 3} defined by x = 2y.
Let A = {2, 3, 4, 5, 6} and B = {1, 2, 3}

Given, x = 2y where y = {1, 2, 3}
When, y = 1, x = 2(1) = 2
When, y = 2, x = 2(2) = 4
When, y = 3, x = 2(3) = 6
∴ R = {(2, 1), (4, 2), (6, 3)}

(ii) A relation R on the set {1, 2, 3, 4, 5, 6, 7} defined by (x, y) ∈ R ⇔ x is relatively prime to y.
Given:
(x, y) R x is relatively prime to y
Here,
2 is co-prime to 3, 5 and 7.
3 is co-prime to 2, 4, 5 and 7.
4 is co-prime to 3, 5 and 7.
5 is co-prime to 2, 3, 4, 6 and 7.
6 is co-prime to 5 and 7.
7 is co-prime to 2, 3, 4, 5 and 6.
∴ R ={(2, 3), (2, 5), (2, 7), (3, 2), (3, 4), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 4), (5, 6), (5, 7), (6, 5), (6, 7), (7, 2), (7, 3), (7, 4), (7, 5), (7, 6), (7, 7)}

(iii) A relation R on the set {0, 1, 2,…, 10} defined by 2x + 3y = 12.
Given,
(x, y) R 2x + 3y = 12
Where x and y = {0, 1, 2,…, 10}
2x + 3y = 12
2x = 12 – 3y
x = (12-3y)/2
When, y = 0, x = (12-3(0))/2 = 12/2 = 6
When, y = 2, x = (12-3(2))/2 = (12-6)/2 = 6/2 = 3
When, y = 4, x = (12-3(4))/2 = (12-12)/2 = 0/2 = 0
∴ R = {(0, 4), (3, 2), (6, 0)}

(iv) A relation R form a set A = {5, 6, 7, 8} to the set B = {10, 12, 15, 16, 18} defined by (x, y) ∈ R ⇔ x divides y.
Given,
(x, y) R x divides y
Where, x = {5, 6, 7, 8} and y = {10, 12, 15, 16, 18}
Here,
5 divides 10 and 15.

6 divides 12 and 18.
7 divides none of the value of set B.
8 divides 16.
∴ R = {(5, 10), (5, 15), (6, 12), (6, 18), (8, 16)}

6. Let R be a relation in N defined by (x, y) ∈ R ⇔ x + 2y = 8. Express R and R^{-1} as sets of ordered pairs.
Solution:
Given,
(x, y) R x + 2y = 8 where x ∈ N and y ∈ N
x + 2y= 8
x = 8 – 2y
Putting the values y = 1, 2, 3,…… till x ∈ N
When, y = 1, x = 8 – 2(1) = 8 – 2 = 6
When, y = 2, x = 8 – 2(2) = 8 – 4 = 4
When, y = 3, x = 8 – 2(3) = 8 – 6 = 2
When, y = 4, x = 8 – 2(4) = 8 – 8 = 0
Now, y cannot hold value 4 because x = 0 for y = 4 which is not a natural number.
∴ R = {(2, 3), (4, 2), (6, 1)}
R^{-1} = {(3, 2), (2, 4), (1, 6)}

7. Let A = {3, 5} and B = {7, 11}. Let R = {(a, b): a ∈ A, b ∈ B, a-b is odd}. Show that R is an empty relation from A into B.
Solution:
Given,
A = {3, 5} and B = {7, 11}
R = {(a, b): a ∈ A, b ∈ B, a-b is odd}
On putting a = 3 and b = 7,
a – b = 3 – 7 = -4 which is not odd
On putting a = 3 and b = 11,
a – b = 3 – 11 = -8 which is not odd
On putting a = 5 and b = 7:
a – b = 5 – 7 = -2 which is not odd
On putting a = 5 and b = 11:
a – b = 5 – 11 = -6 which is not odd
∴ R = { } = Φ
R is an empty relation from A into B.
Hence proved.

8. Let A = {1, 2} and B = {3, 4}. Find the total number of relations from A into B.
Solution:
Given,
A= {1, 2}, B= {3, 4}
n (A) = 2 (Number of elements in set A).
n (B) = 2 (Number of elements in set B).
We know,
n (A × B) = n (A) × n (B)
= 2 × 2
= 4 [since, n(x) = a, n(y) = b. total number of relations = 2^{ab}]
∴ Number of relations from A to B are 2^4 = 16.

9. Determine the domain and range of the relation R defined by
(i) R = {(x, x+5): x ∈ {0, 1, 2, 3, 4, 5}
(ii) R= {(x, x^3): x is a prime number less than 10}
Solution:
(i) R = {(x, x+5): x ∈ {0, 1, 2, 3, 4, 5}
Given,
R = {(x, x+5): x ∈ {0, 1, 2, 3, 4, 5}
∴ R = {(0, 0+5), (1, 1+5), (2, 2+5), (3, 3+5), (4, 4+5), (5, 5+5)}
R = {(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)}
So,
Domain of relation R = {0, 1, 2, 3, 4, 5}
Range of relation R = {5, 6, 7, 8, 9, 10}

(ii) R= {(x, x^3): x is a prime number less than 10}
Given,
R = {(x, x^3): x is a prime number less than 10}
Prime numbers less than 10 are 2, 3, 5 and 7
∴ R = {(2, 2^3), (3, 3^3), (5, 5^3), (7, 7^3)}
R = {(2, 8), (3, 27), (5, 125), (7, 343)}
So,
Domain of relation R = {2, 3, 5, 7}
Range of relation R = {8, 27, 125, 343}

10. Determine the domain and range of the following relations:
(i) R= {a, b): a ∈ N, a < 5, b = 4}
(ii) S= {a, b): b = |a-1|, a ∈ Z and |a| ≤ 3}
Solution:

(i) R= {a, b): a ∈ N, a < 5, b = 4}
Given,
R= {a, b): a ∈ N, a < 5, b = 4}
Natural numbers less than 5 are 1, 2, 3 and 4
a = {1, 2, 3, 4} and b = {4}
R = {(1, 4), (2, 4), (3, 4), (4, 4)}
So,
Domain of relation R = {1, 2, 3, 4}
Range of relation R = {4}

(ii) S= {a, b): b = |a-1|, a ∈ Z and |a| ≤ 3}
Given,
S= {a, b): b = |a-1|, a ∈ Z and |a| ≤ 3}
Z denotes integer which can be positive as well as negative
Now, |a| ≤ 3 and b = |a-1|
∴ a = {-3, -2, -1, 0, 1, 2, 3}
For, a = -3, -2, -1, 0, 1, 2, 3 we get,
S = {(-3, |-3 – 1|), (-2, |-2 – 1|), (-1, |-1 – 1|), (0, |0 – 1|), (1, |1 – 1|), (2, |2 – 1|), (3, |3 – 1|)}
S = {(-3, |-4|), (-2, |-3|), (-1, |-2|), (0, |-1|), (1, |0|), (2, |1|), (3, |2|)}
S = {(-3, 4), (-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1), (3, 2)}
b = 4, 3, 2, 1, 0, 1, 2
So,
Domain of relation S = {0, -1, -2, -3, 1, 2, 3}
Range of relation S = {0, 1, 2, 3, 4}

11. Let A = {a, b}. List all relations on A and find their number.
Solution:
The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B. If n (A) = p and n (B) = q, then n (A × B) = pq.
So, the total number of relations is 2^{pq}.
Now,
A × A = {(a, a), (a, b), (b, a), (b, b)}
Total number of relations are all possible subsets of A × A:
[{(a, a), (a, b), (b, a), (b, b)}, {(a, a), (a, b)}, {(a, a), (b, a)},{(a, a), (b, b)}, {(a, b), (b, a)}, {(a, b), (b, b)}, {(b, a), (b, b)}, {(a, a), (a, b), (b, a)}, {(a, b), (b, a), (b, b)}, {(a, a), (b, a), (b, b)}, {(a, a), (a, b), (b, b)}, {(a, a), (a, b), (b, a), (b, b)}]
n (A) = 2 ⇒ n (A × A) = 2 × 2 = 4
∴ Total number of relations = 2^4 = 16