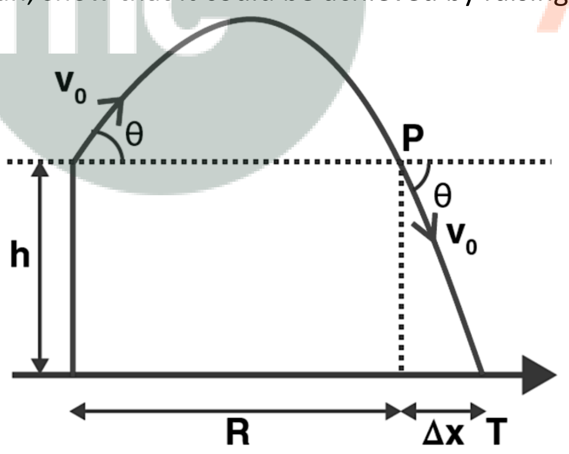
Exemplar Solutions for Class 11 Physics Chapter 3 - Motion In A Plane

Long Answers

29. A hill is 500 m high. Supplies are to be sent across the hill using a canon that can hurl packets at a speed of 125 m/s over the hill. The canon is located at a distance of 800 m from the foot of hill and can be moved on the ground at a speed of 2 m/s so that its distance from the hill can be adjusted. What is the shortest time in which a packet can reach on the ground across the hill? Take $g = 10\ m/s^2$.

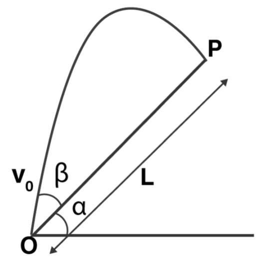
Answer: Given, Speed of packets = 125 m/s Height of the hill = 500 m Distance between the canon and the foot of the hill, d = 800 m The vertical component of the velocity should be minimum so that the time taken to cross the hill will be the shortest. $u_y = \sqrt{2gh} \geq \sqrt{(2)(10)(500)} \geq 100$ m/s But, $u^2 = u_x^2 + u_y^2$ Therefore, horizontal component of the initial velocity, $u_x = \sqrt{(125^2 - 100^2)} = 75$ m/s Time taken by the packet to reach the top of the hill, $t = u_y/g = 100/10 = 10$s Time taken to reach the ground from the top of the hill = t' = t = 10s Horizontal distance covered in 10s = (75)(10) = 750 m Therefore, time taken by canon = 50/2 = 25 s Therefore, total time taken by a packet to reach the ground = 25 + 10 + 10 = 45 s.

30. A gun can fire shells with maximum speed v_0 and the maximum horizontal range that can be achieved is $R = v_0^2/g$. If a target farther away by distance Δx has to be hit with the same gun, show that it could be achieved by raising the gun to a height at least $h = \Delta x[1+\Delta x/R]$



Answer: $R = v_0^2/g$ which is the maximum range of projectile Therefore, the angle of projection, θ = 45° The gun is placed at a height h so that it can hit the target The vertical downward direction is taken as negative Horizontal component of initial velocity = $v_0 \cos\theta$ Vertical component of initial velocity = $v_0 \sin\theta$ Displacement along y-axis, $-h = (v_0 \sin\theta)t + \frac{1}{2}(-g)t^2$ Displacement along x-axis, $t = (R + \Delta x)/v_0 \cos\theta$ Substituting all the equations, we get $h = \Delta x\ (1+ \Delta x/R)$

31. A particle is projected in air at an angle β to a surface which itself is inclined at an angle α to the horizontal. a) find an expression of range on the plane surface b) time of flight c) β at which range will be maximum

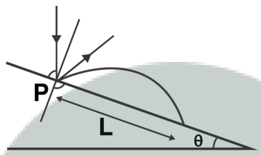


Answer: a) Time of flight, T = (2u sin(α + β))/(g cos β)

b) Range down an inclined plane, R = (2u² sin(α + β)cos α)/(g cos² β)

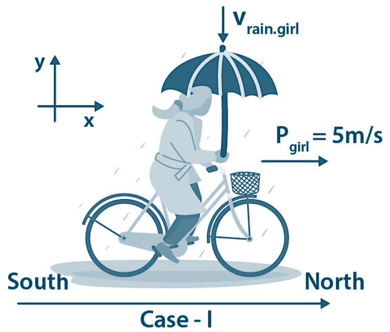
c) Maximum range down an inclined plane, R_{max} = u²/(g(1 - sin β))

32. A particle falling vertically from a height hits a plane surface inclined to horizontal at an angle θ with speed v_0 and rebounds elastically. Find the distance along the plane where it will hit second time.



Answer: When x and y axes are selected as shown in the diagram, the motion of projectile from O to A is y = 0 $u_y = v_0 \cos\theta$ $a_y = -g \cos\theta$ t = T With the help of kinematics for y-axis $y = u_y t + \frac{1}{2} a_y t^2$ $T = 2v_0 \cos\theta / g \cos\theta$ With the help of kinematics for x-axis $x = u_x t + \frac{1}{2} a_x t^2$ $L = (4v_0^2 \sin\theta)/g$

33: A girl riding a bicycle with a speed of 5 m/s towards north direction, observes rain falling vertically down. If she increases her speed to 10 m/s, rain appears to meet her at 45° to the vertical. What is the speed of the rain? In what direction does rain fall as observed by a ground based observer?



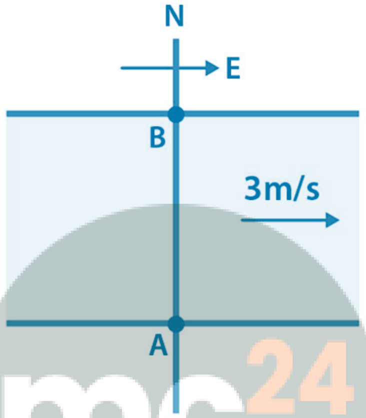
Answer: Let V_rg be the velocity of the rain drop that appears to the girl. All the vectors are drawn in reference to the frame from the ground.

Let the velocity of rain be: $v_r = a\hat{i} - b\hat{j}$

Case I According to the problem, the velocity of girl = vg Let $v_r g$ be the velocity of rain with respect to the girl The equation which is used for determining a is: v_r - vg = (aî + bĵ) - 5ĵ = (a - 5)î + bĵ Therefore, a = 5

Case II Now the velocity of the girl has increased = vg Following is the equation that is used for determining the value of b at tan 45°. v_r - vg = (aî + bĵ) - 10î = (a - 10)î + bĵ Therefore, the velocity of rain is: v_r = 5î - 5ĵ Speed of rain is: $|v_r| = \sqrt{(5^2 + (-5)^2)} = \sqrt{50} = 5\sqrt{2}$ m/s

34. A river is flowing due east with a speed 3 m/s. A swimmer can swim in still water at a speed of 4 m/s. a) if swimmer starts swimming due north, what will be his resultant velocity? b) if he wants to start from point A on south bank and reach opposite point B on north bank, i) which direction should he swim? ii) what will be his resultant speed? c) from two different cases as mentioned in a) and b) above, in which case will he reach opposite bank in shorter time?



Answer: Given, Speed of the river, v_r = 3 m/s Velocity of swimmer, v_s = 4 m/s

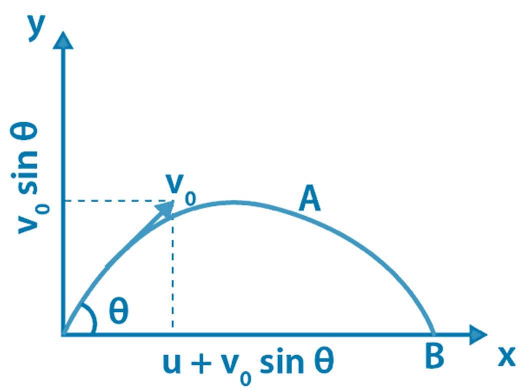
a) When the swimmer swims due north, the y-component has the velocity 4 m/s and x-component will have 3 m/s. following is the resultant velocity with respect to tan θ $v = \sqrt{(v_r^2 + v_s^2)} = \sqrt{((3)^2 + (4)^2)}$ = 5 m/s θ = 37°N

b) Following is the resultant speed of the swimmer when he wants to reach the point which is directly opposite to him and which is with respect to tan θ $v = \sqrt{(v_r^2 - v_s^2)} = \sqrt{((4)^2 - (3)^2)} = \sqrt{7}$ m/s $\theta = \tan^{-1}(3/\sqrt{7})$ of north

c) Time taken in case a) is d/4 sec Time taken in case b) is $d/\sqrt{7}$ As $d/4 < d/\sqrt{7}$, time taken in case a) is shorter than case b).

35. A cricket fielder can throw the cricket ball with a speed v_0. If he throws the ball while running with speed u at an angle θ to the horizontal, find a) the effective angle to the horizontal at which the ball is projected in air as seen by a spectator b) what will be time of flight? c) what is the distance from the point of projection at which the ball will land? d) find θ at which he should throw the ball that would maximise the horizontal range as found c) e) how does θ for maximum range change if $u > v_0$, $u = v_0$ and $u < v_0$ f) how does θ in e) compare with that for u = 0?

Answer: Initial velocity in x-direction, $u_x = u + v_0 \cos\theta$ Initial velocity in y-direction, $u_y = v_0 \sin\theta$ where θ is the angle of projection



a) The angle of projection with the horizontal with respect to spectator is: $\tan\theta = u_y/u_x = (v_0\sin\theta)/(u + v_0\cos\theta)$

b) When the displacement along the y-axis is zero over the time period T $y = 0$, $u_y = v_0 \sin\theta$, $a_y = -g$, $t = T$ We know that, $y = u_yT + ½a_yt^2$ Solving for T, we get $T = 2u_0 \sin\theta/g$

c) Horizontal range is $R = (v_0/g)[2u \sin\theta + v_0 \sin 2\theta]$

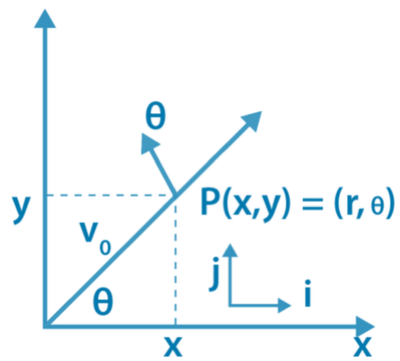
d) For horizontal range, $dR/d\theta = 0$ Then θ_{max} is: $\theta_{max} = \cos^{-1}((-u + \sqrt{(u^2 + 8v_0^2)})/(4v_0))$

e) If $u = v_0$, $\theta_{max} = \pi/2$

f) If $u = 0$, $\theta_{max} = 45°$ Then, $u = 0$ and $\theta \geq 45°$

36. Motion in two dimensions, in a plane can be studied by expressing position, velocity, and acceleration as vector in Cartesian coordinates $\mathbf{A} = A_x\hat{i} + A_y\hat{j}$ where $\hat{i}$ and $\hat{j}$ are unit vectors along x and y directions, respectively and A_x and A_y are corresponding components of **A**. Motion can also be studied by expressing vectors in circular polar coordinates as $\mathbf{A} = A_r\hat{r} + A\theta\hat{\theta}$ where $\hat{r} = (r/r) = \cos\theta\hat{i} + \sin\theta\hat{j}$ and $\hat{\theta} = -\sin\theta\hat{i} + \cos\theta\hat{j}$ are unit vectors along direction in which r and θ are increasing.

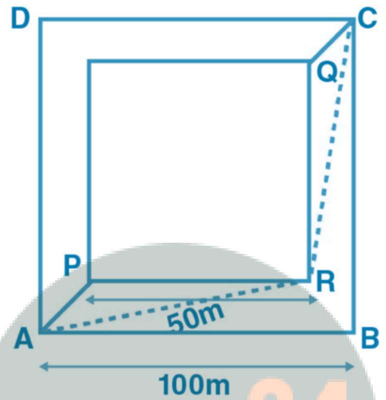
a) express $\hat{i}$ and $\hat{j}$ in terms of $\hat{r}$ and $\hat{\theta}$ b) show that both $\hat{r}$ and $\hat{\theta}$ are unit vectors and are perpendicular to each other c) show that $d/dt(\hat{r}) = \omega\hat{\theta}$ where $\omega = d\theta/dt$ and $d/dt(\hat{\theta}) = -\omega\hat{r}$ d) for a particle moving along a spiral given by $r = a\theta\hat{r}$ where a = 1 find dimensions of 'a' e) find velocity and acceleration in polar vector representation for particle moving along spiral described in d) above



Answer: a) The unit vector is given as: $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$ and $\hat{\theta} = -\sin\theta\hat{i} + \cos\theta\hat{j}$ Solving the above equations we get unit vector as: $\hat{i} = \hat{r}(\hat{r}\cos\theta - \hat{\theta}\sin\theta)$

b) r̂.î = (cosθî + sinθĵ).(-sinθî + cosθĵ) θ = 90°, angle between r̂ and θ̂
c) Given, r̂ = cosθî + sinθĵ Solving the above gives: r̂ = ω[-sinθî + cosθĵ]
d) Given, r = aθr̂ [a] = L = $[M^0L^1T^0]$
e) Given, a = 1 unit, r = θr̂ = θ[cosθî + sinθĵ] Differentiating the above equation, we get velocity as: v = dθ/dt r̂ + θθ̂ = ωr̂ + ωθr̂ By differentiating the above equation we can find acceleration as: a = $(d^2\theta/dt^2 r - \omega^2)\hat{r} + (2\omega^2 + d^2\theta/dt^2)\hat{\theta}$

37. A man wants to reach from A to the opposite corner of the square C. The sides of the square are 100 m. A central square of 50 m × 50 m is filled with sand. Outside this square, he can walk at a speed 1 m/s. In the central square, he walk only at a speed of v m/s. What is smallest value of v for which he can reach faster via a straight path through the sand than any path in the square outside the sand?



Answer: As shown in the diagram, APQC is the path taken by the man through the sand, time taken by him to go from A to C, T_{sand} = (AP + QC)/1 + PQ/v = 50√2(1/v + 1) The shortest path will be ARC from the diagram. $T_{outside}$ = AR+RC/1 s AR = $\sqrt{75^2 + 25^2}$ RC = AR = 25√10 s $T_{sand} < T_{outside}$ V > 0.81 m/s