EXERCISE 29.8

Evaluate the following limits:

1. $\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x$

Solution:

Given:

The limit $\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x$

Let us assume, $y = \frac{\pi}{2} - x$

So,

$x \to \frac{\pi}{2}, y \to 0$

$$\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x = \lim_{y \to 0} y \tan\left(\frac{\pi}{2} - y\right)$$

$$= \lim_{y \to 0} y \frac{\sin\left(\frac{\pi}{2} - y\right)}{\cos\left(\frac{\pi}{2} - y\right)}$$ [We know that, tan = sin/cos]

$$= \lim_{y \to 0} y \frac{\cos y}{\sin y}$$

Upon simplification, we get

$$= \lim_{y \to 0} \cos y - \lim_{y \to 0} \frac{y}{\sin y}$$

Substituting the value of y = 0, then

$$= \cos 0 - \frac{0}{\sin 0}$$

$$= 1 - 0$$

$$= 1$$

∴ The value of $\lim_{x \to \frac{\pi}{2}} \left(\frac{\pi}{2} - x\right) = 1$

2. $\lim_{x \to \pi/2} \frac{\sin 2x}{\cos x}$

Solution:

Given:

The limit $\lim_{x \to \pi/2} \frac{\sin 2x}{\cos x}$

We know, $\sin 2x = 2\sin x.\cos x$
So,
$$\lim_{x\to\pi/2} \frac{\sin 2x}{\cos x} = \lim_{x\to\pi/2} \frac{2\sin x \cos x}{\cos x}$$
Upon simplification, we get
$$= \lim_{x\to\pi/2} 2\sin x$$
Substitute the value of x, we get
$$= 2\sin\frac{\pi}{2}$$
$$= 2 \times 1$$
$$= 2$$

$\therefore$ The value of $\lim_{x\to\pi/2} \frac{\sin 2x}{\cos x} = 2$

3. $\lim_{x\to\pi/2} \frac{\cos^2 x}{1-\sin x}$

Solution:
Given:
The limit $\lim_{x\to\pi/2} \frac{\cos^2 x}{1-\sin x}$
We know that, $\cos^2 x = 1-\sin^2 x$
So, by substituting this value we get,
$$\lim_{x\to\pi/2} \frac{\cos^2 x}{1-\sin x} = \lim_{x\to\pi/2} \frac{1-\sin^2 x}{1-\sin x}$$
Upon expansion,
$$= \lim_{x\to\pi/2} \frac{(1-\sin x)(1+\sin x)}{1-\sin x}$$
When simplified, we get
$$= \lim_{x\to\pi/2} 1+\sin x$$
Now, substitute the value of x, we get
$$= 1+\sin\frac{\pi}{2}$$
$$= 1+1$$
$$= 2$$

$\therefore$ The value of $\lim_{x\to\pi/2} \frac{\cos^2 x}{1-\sin x} = 2$

4. $\lim_{x\to\pi/3} \frac{\sqrt{1-\cos 6x}}{\sqrt{2}(\pi/3 - x)}$

Solution:

Given:

The limit $\lim_{x\to\pi/3} \frac{\sqrt{1-\cos 6x}}{\sqrt{2}(\pi/3 - x)}$

We know that, $1 - \cos 2x = 2\sin^2 x$

So,

$$\lim_{x\to\frac{\pi}{3}} \frac{\sqrt{1-\cos 6x}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)} = \lim_{x\to\frac{\pi}{3}} \frac{\sqrt{2\sin^2 3x}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)}$$

$$= \lim_{x\to\frac{\pi}{3}} \frac{\sqrt{2}\sin 3x}{\sqrt{2}\left(\frac{\pi}{3}-x\right)}$$

$$= \lim_{x\to\frac{\pi}{3}} \frac{\sin 3x}{\left(\frac{\pi}{3}-x\right)}$$

$$= \lim_{x\to\frac{\pi}{3}} \frac{3\sin 3x}{\pi - 3x}$$

We know that, $\sin x = \sin(\pi - x)$

So,

$$\lim_{x\to\frac{\pi}{3}} \frac{\sqrt{1-\cos 6x}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)} = \lim_{x\to\frac{\pi}{3}} \frac{3\sin(\pi-3x)}{\pi-3x} \quad \left[\text{We know that, } \lim_{x\to 0}\frac{\sin x}{x} = 1\right]$$

$$= 3$$

∴ The value of $\lim_{x\to\frac{\pi}{3}} \frac{\sqrt{1-\cos 6x}}{\sqrt{2}\left(\frac{\pi}{3}-x\right)} = 3$

5. $\lim_{x\to a} \frac{\cos x - \cos a}{x - a}$

Solution:

Given:

The limit $\lim_{x\to a} \frac{\cos x - \cos a}{x - a}$

We know that,

$$\left[\ \cos A - \cos B = 2\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{A+B}{2}\right)\right]$$

By substituting in the formula, we get

$$\lim_{x\to a}\frac{\cos x-\cos a}{x-a}=\lim_{x\to a}\frac{\left(-2\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)\right)}{x-a}$$

$$=-2\lim_{x\to a}\sin\left(\frac{x+a}{2}\right)\lim_{x\to a}\sin\frac{\left(\frac{x-a}{2}\right)}{x-a}$$

Upon simplification, we get

$$=-2\sin\left(\frac{a+a}{2}\right)\left(\lim_{x\to a}\sin\frac{\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}}\right)\times\frac{1}{2}$$

$$=-2\sin a\times 1\times\frac{1}{2}$$

$$=-\sin a$$

∴ The value of $\lim_{x\to a}\frac{\cos x-\cos a}{x-a}=-\sin a$