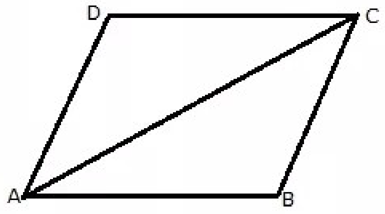
Exercise 16(B)

Solution 1:

(i) Suppose ABCD is a parallelogram (given)



Consider the triangles ABC and ADC:

$AB = CD$ [ABCD is a parallelogram]

$AD = BC$ [ABCD is a parallelogram]

$AD = AD$ [common]

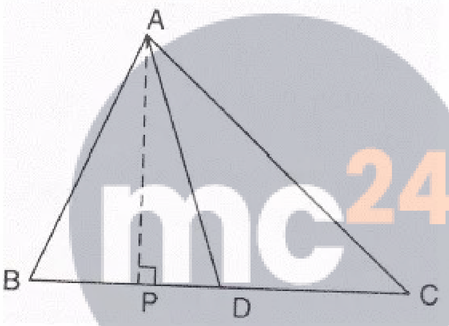
By Side – Side – Side criterion of congruence, we have,

$\triangle ABC \cong \triangle ADC$

Area of congruent triangles are equal.

Therefore, Area of ABC = Area of ADC

(ii) Consider the following figure:



Here $AP \perp BC$

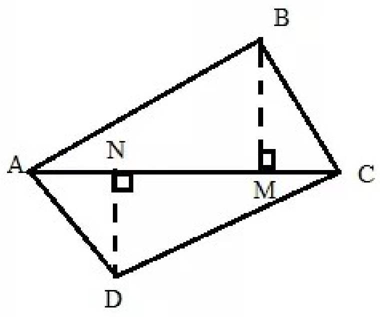
Since Ar.(ΔABD)= $\frac{1}{2}BD \times AP$

And, Ar.(ΔADC)= $\frac{1}{2}DC \times AP$

$$\therefore \frac{Area(\Delta ABD)}{Area(\Delta ADC)} = \frac{\frac{1}{2}BD \times AP}{\frac{1}{2}DC \times AP} = \frac{BD}{DC},$$

hence proved

(iii) Consider the following figure:



Here

Ar.(ΔABC)= $\frac{1}{2}BM \times AC$

And, Ar.(ΔADC)= $\frac{1}{2}DN \times AC$

$$\therefore \frac{Area(\Delta ABC)}{Area(\Delta ADC)} = \frac{\frac{1}{2}BM \times AC}{\frac{1}{2}DN \times AC} = \frac{BM}{DN},$$

hence proved

Solution 2:

AD is the median of Δ ABC. Therefore it will divide Δ ABC into two triangles of equal areas.

$\therefore$ Area(Δ ABD)= Area(Δ ACD) (i)

ED is the median of Δ EBC

$\therefore$ Area(Δ EBD)= Area(Δ ECD) (ii)

Subtracting equation (ii) from (i), we obtain

Area(Δ ABD)- Area(Δ EBD)= Area(Δ ACD)- Area(Δ ECD)

Area (Δ ABE) = Area (Δ ACE). Hence proved

Solution 3:

AD is the median of $\triangle$ ABC. Therefore it will divide $\triangle$ ABC into two triangles of equal areas.

$\therefore$ Area($\triangle$ ABD)= Area($\triangle$ ACD)

Area ($\triangle$ ABD)= $\frac{1}{2}$ Area($\triangle$ ABC) (i)

In $\triangle$ ABD, E is the mid-point of AD. Therefore BE is the median.

$\therefore$ Area($\triangle$ BED)= Area($\triangle$ ABE)

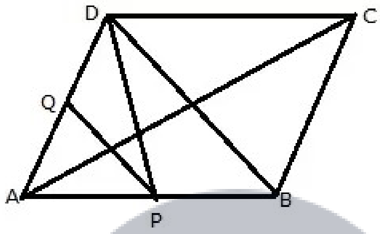
Area($\triangle$ BED)= $\frac{1}{2}$ Area($\triangle$ ABD)

Area($\triangle$ BED)= $\frac{1}{2} \times \frac{1}{2}$ Area($\triangle$ ABC)[from equation (i)]

Area($\triangle$ BED)= $\frac{1}{4}$ Area($\triangle$ ABC)

Solution 4:

We have to join PD and BD.



BD is the diagonal of the parallelogram ABCD. Therefore it divides the parallelogram into two equal parts.

$\therefore$ Area($\triangle$ ABD)= Area($\triangle$ DBC)

= $\frac{1}{2}$ Area (parallelogram ABCD) (i)

DP is the median of $\triangle$ ABD. Therefore it will divide $\triangle$ ABD into two triangles of equal areas.

$\therefore$ Area($\triangle$ APD)= Area($\triangle$ DPB)

= $\frac{1}{2}$ Area ($\triangle$ ABD)

= $\frac{1}{2} \times \frac{1}{2}$ Area(parallelogram ABCD)[from equation (i)]

= $\frac{1}{4}$ Area (parallelogram ABCD) (ii)

In $\triangle$ APD, Q is the mid-point of AD. Therefore PQ is the median.

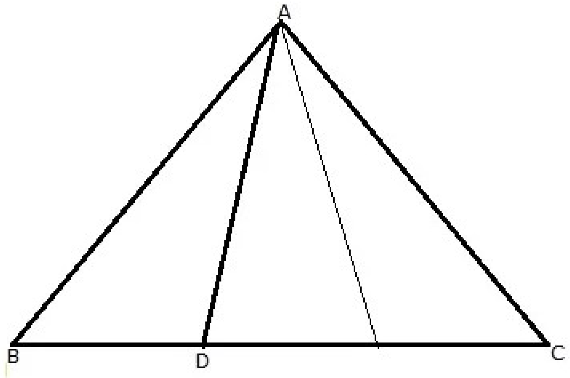
$\therefore$ Area($\triangle$ APQ)= Area($\triangle$ DPQ)

= $\frac{1}{2}$ Area($\triangle$ APD)

= $\frac{1}{2} \times \frac{1}{4}$ Area (parallelogram ABCD) [from equation (ii)]

Area ($\triangle$ APQ)= $\frac{1}{8}$ Area (parallelogram ABCD),hence proved

Solution 5:



In $\triangle$ ABC, $\because$ BD = $\frac{1}{2}$ DC $\Rightarrow \frac{BD}{DC} = \frac{1}{2}$

$\therefore$ Ar.($\triangle$ ABD):Ar.($\triangle$ ADC)=1:2

But Ar.($\triangle$ ABD)+Ar.($\triangle$ ADC)=Ar.($\triangle$ ABC)

Ar.($\triangle$ ABD)+2Ar.($\triangle$ ABD)=Ar.($\triangle$ ABC)

3 Ar.($\triangle$ ABD)= Ar.($\triangle$ ABC)

Ar.($\triangle$ ABD)= $\frac{1}{3}$ Ar.($\triangle$ ABC)

Solution 6:

Ratio of area of triangles with same vertex and bases along the same line is equal to ratio of their respective bases. So, we have

$$\frac{\text{Area of DPB}}{\text{Area of PCB}} = \frac{\text{DP}}{\text{PC}} = \frac{3}{2}$$

Given: Area of ΔDPB = 30 sq. cm

Let 'x' bet the area of the triangle PCB

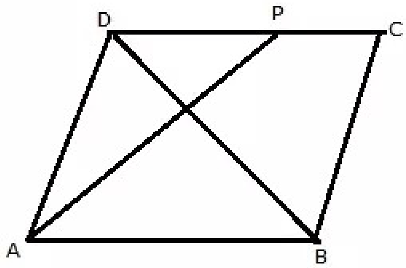
Therefore, we have,

$$\frac{30}{x} = \frac{3}{2}$$

$$\Rightarrow x = \frac{30}{3} \times 2 = 20 \ sq.\ cm.$$

So area of ΔPCB = 20 sq. cm

Consider the following figure.



From the diagram, it is clear that,

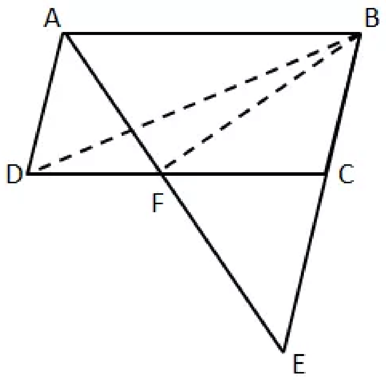
$$Area(\triangle CDB) = Area(\triangle DPB) + Area(\triangle CPB)$$
$$= 30 + 20$$
$$= 50 \ sq.\ cm$$

Diagonal of the parallelogram divides it into two triangles $\triangle ADB$ and $\triangle CDB$ of equal area.

Therefore,

$$Area(||gm\ ABCD) = 2 \times \triangle CDB$$
$$= 2 \times 50 = 100 \ sq.\ cm$$

Solution 7:



$BC = CE$ (given)
Also, in parallelogram ABCD, $BC = AD$
$\Rightarrow AD = CE$
Now, in ΔADF and ΔECF, we have
$AD = CE$
$\angle ADF = \angle ECF$ (Alternate angles)
$\angle DAF = \angle CEF$ (Alternate angles)
$\therefore \Delta ADF \cong \Delta ECF$ (ASA Criterion)
$\Rightarrow Area(\Delta ADF) = Area(\Delta ECF)$ ....(1)
Also, in ΔFBE, FC is the median (Since $BC = CE$)
$\Rightarrow Area(\Delta BCF) = Area(\Delta ECF)$ ....(2)
From (1) and (2),
$Area(\Delta ADF) = Area(\Delta BCF)$ ....(3)
Again, ΔADF and ΔBDF are on the base DF and between parallels DF and AB.
$\Rightarrow Area(\Delta BDF) = Area(\Delta ADF)$ ....(4)
From (3) and (4),
$Area(\Delta BDF) = Area(\Delta BCF) = 30 \text{ cm}^2$
Þ$Area(\Delta BCD) = Area(\Delta BDF) + Area(\Delta BCF) = 30 + 30 = 60 \text{ cm}^2$
Hence, Area of parallelogram $ABCD = 2 \times Area(\Delta BCD) = 2 \times 60 = 120 \text{ cm}^2$

Solution 8:

In ΔABC,

R and Q are the mid - points of AC and BC respectively.

$\Rightarrow RQ \parallel AB$

that is $RQ \parallel PB$

So, area(ΔPBQ) = area(ΔAPR)...(i)..(Since AP = PB and triangles on the same base and between the same parallels are equal in area)

Since P and R are the mid - points of AB and AC respectively.

$\Rightarrow PR \parallel BC$

that is $PR \parallel BQ$

So, quadrilateral PMQR is a parallelogram.

Also, area(ΔPBQ) = area(ΔPQR)...(ii)...(diagonal of a parallelogram divide the parallelogram in two triangles with equal area)

from (i) and (ii),

area(ΔPQR) = area(ΔPBQ) = area(ΔAPR)...(iii)

Similarly, P and Q are the mid - points of AB and BC respectively.

$\Rightarrow PQ \parallel AC$

that is $PQ \parallel RC$

So, quadrilateral PQCR is a parallelogram.

Also, area(ΔRQC) = area(ΔPQR)...(iv)...(diagonal of a parallelogram divide the parallelogram in two triangles with equal area)

From (iii) and (iv),

area(ΔPQR) = area(ΔPBQ) = area(ΔRQC) = area(ΔAPR)

So, area(ΔPBQ) = $\frac{1}{4}$ area(ΔABC)...(v)

Also, since S is the mid - point of PQ,

BS is the median of ΔPBQ

So, area(ΔQSB) = $\frac{1}{2}$ area(ΔPBQ)

from (v),

area(ΔQSB) = $\frac{1}{2} \times \frac{1}{4}$ area(ΔABC)

$\Rightarrow$ area(ΔABC) = 8 area(ΔQSB)