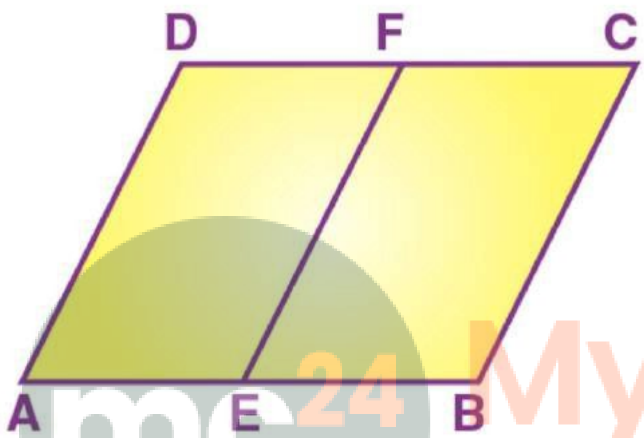
# EXERCISE 14C

**1. E is the mid-point of side AB and F is the midpoint of side DC of parallelogram ABCD. Prove that AEFD is a parallelogram.**

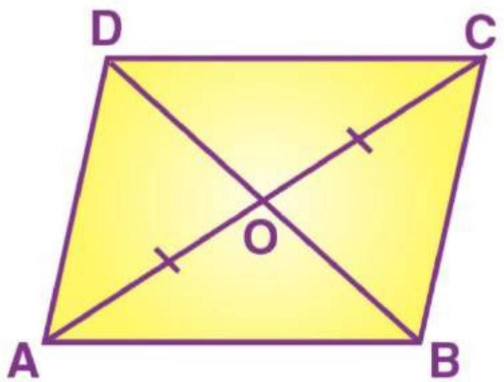
**Solution:**
Let us draw a parallelogram ABCD Where F is the midpoint
Of side DC of parallelogram ABCD
To prove: AEFD is a parallelogram




Proof:
In quadrilateral ABCD
AB parallel to DC
BC parallel to AD
AB = DC
Multiply both side by ½
½ AB = ½ DC
AE = DF
Also, AD parallel to EF
Therefore, AEFC is a parallelogram.

**2. The diagonal BD of a parallelogram ABCD bisects angles B and D. Prove that ABCD is a rhombus.**

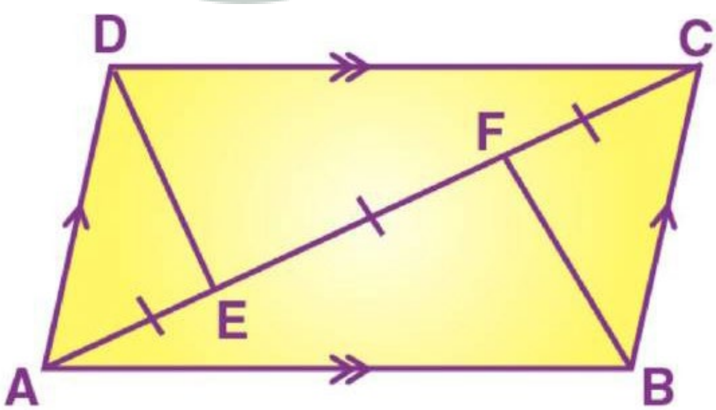
**Solution:**
Given ABCD is a parallelogram where the diagonal BD bisects parallelogram ABCD at angle B and D
To prove: ABCD is a rhombus
Proof: Let us draw a parallelogram ABCD where the diagonal BD bisects the parallelogram at angle B and D.
Construction: Let us join AC as a diagonal of the parallelogram ABCD



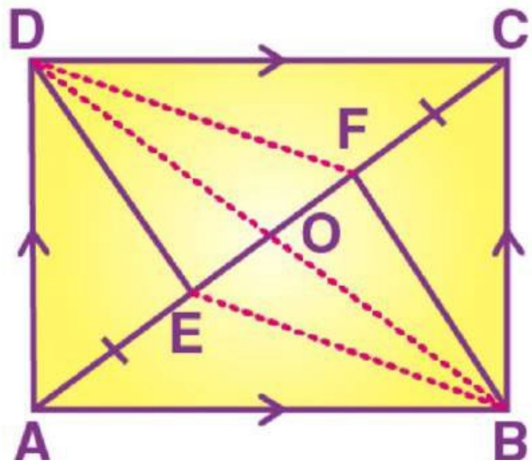

Since ABCD is a parallelogram
Therefore, AB = DC
AD = BC
Diagonal BD bisects angle B and D
So ∠COD = ∠DOA
Again, AC also bisects at A and C
Therefore ∠AOB = ∠BOC
Thus, ABCD is a rhombus.
Hence proved



**3. The alongside figure shows a parallelogram ABCD in which AE = EF = FC. Prove that:**
**(i) DE is parallel to FB**
**(ii) DE = FB**
**(iii) DEBF is a parallelogram.**



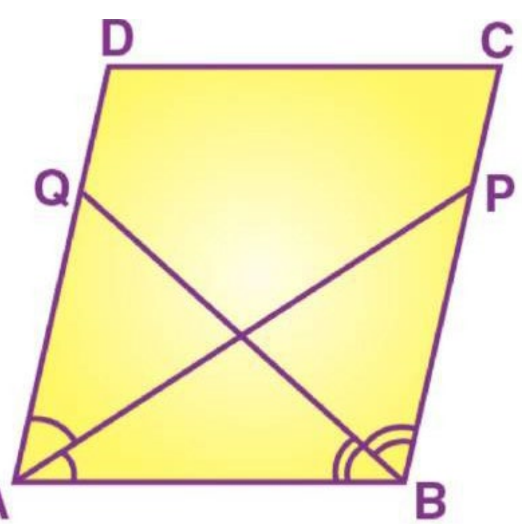

**Solution:**
Construction:
Join DF and EB
Join diagonal BD




Since the diagonals of a parallelogram bisect each other
Therefore, OA = OC and OB = OD
Also, AE = EF = FC
Now, OA = OC and AE = FC
OA - OC = OC – FC
OE = OF
Thus, in quadrilateral DEFB, we have
OB = OD and OE = OF
Diagonals of a quadrilateral DEFB bisect each other.
DEFB is a parallelogram
DE is parallel to FB
DE = FB (opposite sides are equal)



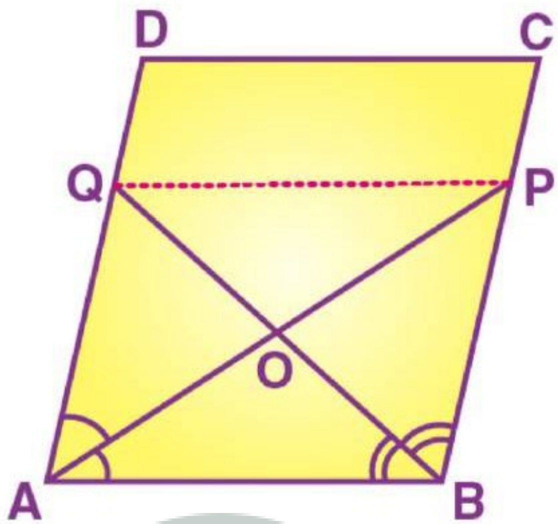
**4. In the alongside diagram, ABCD is a parallelogram in which AP bisects angle AP bisects angle A and BQ bisects angle B. Prove that:**




**(i) AQ = BP**
**(ii) PQ = CD.**

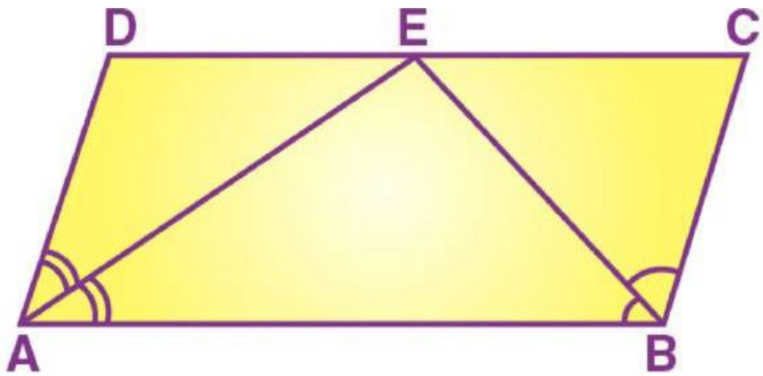
**(iii) ABPQ is a parallelogram.**

**Solution:**

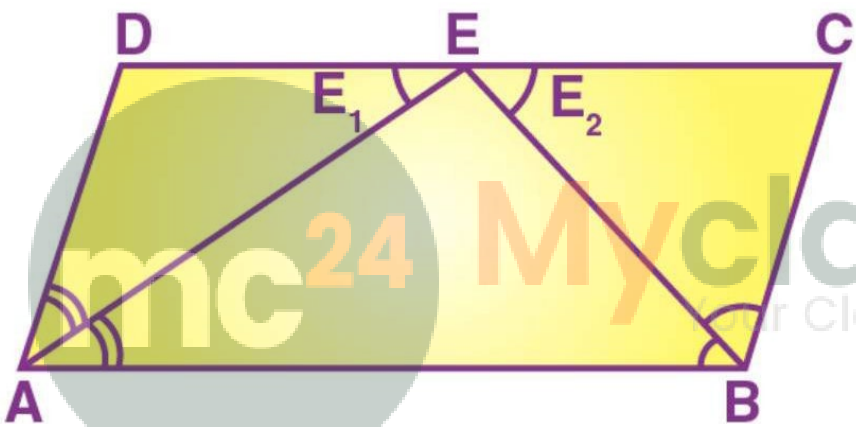



Consider the triangle AOQ and triangle BOP
∠AOQ = ∠BOP [opposite angles]
∠OAQ = ∠BPO [alternate angles]
△ AOQ ≅ △ BOP
Hence AQ = BP
Consider the triangle QOP and triangle AOB
∠AOB = ∠QOP [opposite angles]
∠OAB = ∠APQ [alternate angles]
△ QOP ≅ △ AOB
Hence PQ = AB = CD
Consider the quadrilateral QPCD
DQ = CP and DQ parallel to CP
Also, QP = DC and AB parallel to QP parallel to DC
Hence quadrilateral QPCD is a parallelogram.

**5. In the given figure, ABCD is a parallelogram. Prove that: AB = 2 BC.**




**Solution:**
Given: ABCD is a parallelogram
To prove: AB = 2BC






Proof: ABCD is a parallelogram
$\angle A + \angle D = \angle B + \angle C = 180^o$
From the triangle AEB we have
$\angle A/2 + \angle B/2 + \angle E = 180^o$
$\angle A - \angle A/2 + \angle D + \angle E_1 = 180^o$ [taking $E_1$ as a new angle]
$\angle A + \angle D + \angle E_1 = 180^o + \angle A/2$
$\angle E_1 = \angle A/2$ [since $\angle A + \angle D = 180^o$]
Again, similarly
$\angle E_2 = \angle B/2$
Now,
AB = DE + EC
= AD + BC
= 2 BC [since AD = BC]