

NCERT Solutions for Class 10 Maths Chapter 8 Introduction to Trigonometry EXERCISE 8.2

1. Evaluate the following:

$$(i) \quad \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

$$(ii) \quad 2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$$

$$(iii) \quad \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$$

$$(iv) \quad \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$(v) \quad \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

Solution: (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4}$$

$$= 1$$

(ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2 (1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4}$$

$$= 2$$

$$(i) \quad \frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2}$$

$$\begin{aligned}
&= \frac{1/\sqrt{2}}{\frac{2}{\sqrt{3}} + \frac{2\sqrt{3}}{\sqrt{3}}} \\
&= \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}} \\
&= \frac{\sqrt{3}(2\sqrt{2} - 2\sqrt{6})}{(2\sqrt{2} + 2\sqrt{6})(2\sqrt{2} - 2\sqrt{6})} \\
&= \frac{2\sqrt{3}(\sqrt{6} - \sqrt{2})}{16} \\
&= \frac{3\sqrt{2} - \sqrt{6}}{8}
\end{aligned}$$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{1}{\sqrt{3}}}$$

$$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4}$$

$$= \frac{(3\sqrt{3} - 4)(3\sqrt{3} - 4)}{(3\sqrt{3} + 4)(3\sqrt{3} - 4)}$$

$$= \frac{(3\sqrt{3} - 4)(3\sqrt{3} - 4)}{(3\sqrt{3})(3\sqrt{3}) - 4 \times 4}$$

$$= \frac{27 + 16 - 24\sqrt{3}}{27 - 16}$$

$$= \frac{43 - 24\sqrt{3}}{11}$$

(v) $\frac{5\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + 4\left(\frac{2}{\sqrt{3}}\right)\left(\frac{2}{\sqrt{3}}\right) - 1 \times 1}{\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}$

$$= \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1+\frac{3}{4}}{4}} = \frac{67}{12}$$

2. Choose the correct option and justify your choice:

(i) $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} =$

- A. $\sin 60^\circ$ B. $\cos 60^\circ$
C. $\tan 60^\circ$ D. $\sin 30^\circ$

Solution: $\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)}$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{6}{4\sqrt{3}}$$

$$= \frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Hence, (A) is correct

(ii) $\frac{1 - \tan^2 45^\circ}{1 + \tan^2 45^\circ} =$

- A. $\tan 90^\circ$ B. 1
C. $\sin 45^\circ$ D. 0

Solution: $\frac{1-1*1}{1+1*1}$

$$= \frac{1-1}{1+1}$$

$$= \frac{0}{2}$$

$$= 0$$

Hence, (D) is correct

(iii) $\sin 2A = 2 \sin A$ is true when $A =$

A. 0° B. 30°

C. 45° D. 60°

Solution: Out of the given alternatives, only $A = 0^\circ$ is correct

As

$$\sin 2A = \sin 0^\circ = 0$$

$$2 \sin A = 2 \sin 0^\circ = 2(0) = 0$$

Hence, (A) is correct

(iv) $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} =$

A. $\cos 60^\circ$

B. $\sin 60^\circ$

C. $\tan 60^\circ$

D. $\sin 30^\circ$

Solution: $\frac{2 \left(\frac{1}{\sqrt{3}}\right)}{1 - \frac{1}{\sqrt{3}} * \frac{1}{\sqrt{3}}} =$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \sqrt{3}$$

Out of the given alternatives, only $\tan 60^\circ = \sqrt{3}$

Hence, (C) is correct

3. If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0^\circ < A + B \leq 90^\circ$; $A > B$, find A and B

Solution: $\tan(A + B) = \sqrt{3}$

$$\tan(A + B) = \tan 60^\circ$$

$$A + B = 60^\circ \quad (i)$$

$$\tan(A - B) = \frac{1}{\sqrt{3}}$$

$$\tan(A - B) = \tan 30^\circ$$

$$A - B = 30^\circ \quad (ii)$$

On adding both equations, we obtain

$$2A = 90$$

$$\Rightarrow A = 45$$

From equation (i), we obtain

$$45 + B = 60$$

$$B = 15^\circ$$

Therefore, $\angle A = 45^\circ$ and $\angle B = 15^\circ$

4. State whether the following are true or false. Justify your answer

- (i) $\sin(A + B) = \sin A + \sin B$
- (ii) The value of $\sin \theta$ increases as θ increases
- (iii) The value of $\cos \theta$ increases as θ increases.
- (iv) $\sin \theta = \cos \theta$ for all values of θ .
- (v) $\cot A$ is not defined for $A = 0^\circ$

Solution: (i) $\sin (A + B) = \sin A + \sin B$

Let $A = 30^\circ$ and $B = 60^\circ$

$$\sin (A + B) = \sin (30^\circ + 60^\circ)$$

$$= \sin 90^\circ$$

$$= 1$$

$$\sin A + \sin B = \sin 30^\circ + \sin 60^\circ$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{1 + \sqrt{3}}{2}$$

Clearly, $\sin (A + B) \neq \sin A + \sin B$

Hence, the given statement is false

(ii) The value of $\sin \theta$ increases as θ increases in the interval of $0^\circ < \theta < 90^\circ$ as $\sin 0^\circ = 0$

$$\sin 30^\circ = \frac{1}{2} = 0.5$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2} = 0.886$$

$$\sin 90^\circ = 1$$

Hence, the given statement is true

(iii) $\cos 0^\circ = 1$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60^\circ = \frac{1}{2} = 0.5$$

$$\cos 90^\circ = 0$$

It can be observed that the value of $\cos \theta$ does not increase in the interval of $0^\circ < \theta < 90^\circ$

Hence, the given statement is false

(iv) $\sin \theta = \cos \theta$ for all values of θ .

This is true when $\theta = 45^\circ$

As,

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

It is not true for all other values of θ

$$\text{As } \sin 30^\circ = \frac{1}{2} \text{ and } \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Hence, the given statement is false

(v) $\cot A$ is not defined for $A = 0^\circ$

$$\cot A = \frac{\cos A}{\sin A}$$

$$\cot 0^\circ = \frac{\cos 0}{\sin 0}$$

$$= \frac{1}{0} = \text{undefined}$$

Hence, the given statement is true