NCERT Solutions for Class-XII Math

Chapter-7
Exercise- Miscellaneous

NCERT Maths Class 12

1. $\frac{1}{x-x^3}$

1. Given: $\frac{1}{x-x^3}$

Let $I=\frac{1}{x-x^3}=\frac{1}{x\left(1-x^2\right)}=\frac{1}{x(1+x)(1-x)}$

Using partial differentiation:

let $\frac{1}{x(1+x)(1-x)}=\frac{A}{x}+\frac{B}{1+x}+\frac{C}{1-x}$ ...(1)

$\Rightarrow \frac{1}{x(1+x)(1-x)}=\frac{A(1+x)(1-x)+B(x)(1-x)+C(x)(1+x)}{x(1+x)(1-x)}$

$\Rightarrow \frac{1}{x(1+x)(1-x)}=\frac{A\left(1-x^2\right)+Bx(1-x)+Cx(1+x)}{x(1+x)(1-x)}$

$\Rightarrow 1 = A - Ax^2 + Bx - Bx^2 + Cx + Cx^2$

$\Rightarrow 1 = A + (B+C)x + (-A-B+C)x^2$

Equating the coefficients of x, x^2 and constant value. We get:

(a) A = 1

(b) B+C = 0 $\Rightarrow$ B = -C

(c) -A –B +C =0

$\Rightarrow$ -1 – (-C) +C = 0

$\Rightarrow$ 2C = 1 $\Rightarrow$ C = ½

So, B = -1/2

Put these values in equation (1)

$\Rightarrow \frac{1}{x(1+x)(1-x)}=\frac{1}{x}+\frac{-\left(\frac{1}{2}\right)}{1+x}+\frac{\left(\frac{1}{2}\right)}{1-x}$

$\Rightarrow \int\frac{1}{x(1+x)(1-x)}dx=\int\frac{1}{x}dx-\frac{1}{2}\int\frac{1}{1+x}dx+\frac{1}{2}\int\frac{1}{1-x}dx$

$$=\log|x|-\frac{1}{2}\log|1+x|+\frac{1}{2}\log|1-x|$$

$$=\log|x|-\log\left|(1+x)^{\frac{1}{2}}\right|+\log\left|(1-x)^{\frac{1}{2}}\right|$$

$$=\log\left|\frac{x}{(1+x)^{\frac{1}{2}}(1-x)^{\frac{1}{2}}}\right|+C$$

$$=\log\left|\frac{(x^2)^{\frac{1}{2}}}{(1+x)(1-x)^{\frac{1}{2}}}\right|+C$$

$$=\log\left|\frac{(x^2)^{\frac{1}{2}}}{(1-x^2)^{\frac{1}{2}}}\right|+C$$

$$=\log\left|\left(\frac{x^2}{1-x^2}\right)^{\frac{1}{2}}\right|+C$$

$$\Rightarrow I=\frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right|+C$$

2. $\frac{1}{\sqrt{x+a}+\sqrt{x+b}}$

2. $\frac{1}{\sqrt{x+a}+\sqrt{x+b}}=\frac{1}{\sqrt{x+a}+\sqrt{x+b}}\times\frac{\sqrt{x+a}-\sqrt{x+b}}{\sqrt{x+a}-\sqrt{x+b}}$

$$=\frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}$$

$$=\frac{\left(\sqrt{x+a}-\sqrt{x+b}\right)}{a-b}$$

$$\Rightarrow\int\frac{1}{\sqrt{x+a}-\sqrt{x+b}}dx=\frac{1}{a-b}\int\left(\sqrt{x+a}-\sqrt{x+b}\right)dx$$

$$=\frac{1}{(a-b)}\left[\frac{(x+a)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{(x+b)^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$=\frac{2}{3(a-b)}\left[(x+a)^{\frac{3}{2}}-(x+b)^{\frac{3}{2}}\right]+C$$

3. $\dfrac{1}{x\sqrt{ax-x^2}}[Hint: Put x = \dfrac{a}{t}]$

3. Given: $\dfrac{1}{x\sqrt{ax-x^2}}$

let $I = \dfrac{1}{x\sqrt{ax-x^2}}$

put $x = \dfrac{a}{t} \Rightarrow dx = -\dfrac{a}{t^2}dt$

$$\Rightarrow \int \frac{1}{x\sqrt{ax-x^2}}dx = \int \frac{1}{\frac{a}{t}\sqrt{\frac{a.a}{t}-\left(\frac{a}{t}\right)^2}}.-\frac{a}{t^2}dt$$

$$= \int \frac{-1}{at}.\frac{1}{\sqrt{\frac{1}{t}-\left(\frac{1}{t}\right)^2}}dt$$

$$= -\frac{1}{a}\int \frac{1}{\sqrt{\frac{t^2}{t}-\left(\frac{t}{t}\right)^2}}dt$$

$$= -\frac{1}{a}\int \frac{1}{\sqrt{t-1}}dt$$

$$= -\frac{1}{a}\int (t-1)^{-\frac{1}{2}}dt$$

$$= -\frac{1}{a}\left[\frac{\sqrt{(t-1)}}{\frac{1}{2}}\right]+C$$

$$= -\frac{2}{a}\left[\sqrt{\left(\frac{a}{x}-1\right)}\right]+C \qquad \text{because, } t = \frac{a}{x}$$

$$\Rightarrow I = -\frac{2}{a}\left[\sqrt{\left(\frac{a-x}{x}\right)}\right]+C$$

4. $\dfrac{1}{x^2(x^4+1)^{\frac{3}{4}}}$

4. $\dfrac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}$

Multiplying and dividing by x^{-3}, we obtain

$$\frac{x^{-3}}{x^2 \cdot x^{-3}\left(x^4+1\right)^{\frac{3}{4}}} = \frac{x^{-3}\left(x^4+1\right)^{\frac{-3}{4}}}{x^2 \cdot x^{-3}}$$

$$= \frac{\left(x^4+1\right)^{\frac{-3}{4}}}{x^5 \cdot \left(x^4\right)^{-\frac{3}{4}}}$$

$$= \frac{1}{x^5}\left(\frac{x^4+1}{x^4}\right)^{\frac{-3}{4}}$$

$$= \frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}$$

Let $\frac{1}{x^4} = t \Rightarrow -\frac{4}{x^5}dx = dt \Rightarrow \frac{1}{x^5}dx = -\frac{dt}{4}$

$$\therefore \int \frac{1}{x^2\left(x^4+1\right)^{\frac{3}{4}}}dx = \int \frac{1}{x^5}\left(1+\frac{1}{x^4}\right)^{-\frac{3}{4}}dx$$

$$= -\frac{1}{4}\int (1+t)^{-\frac{3}{4}}dt$$

$$= -\frac{1}{4}\left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}}\right] + C$$

$$= -\frac{1}{4}\frac{1}{x^4}\Bigg)^{\frac{1}{4}} + C$$

$$= -\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}} + C$$

5. $\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}$ [Hint $\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}$, $putx = t^6$]

5. Given: $\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}$

or we can write it as, $\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}$

Let $x = t^6 \Rightarrow dx = 6t^5 dt$

$$\Rightarrow \int \frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)}.dx = \int \frac{6t^5}{t^2(1+t)}.dt$$

$$= 6.\int \frac{t^3}{(1+t)}.dt$$

After division we get,

$$\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = 6.\int \left[\left(t^2 - t + 1\right) - \frac{1}{(1+t)}\right].dt$$

$$= 6.\left\{\int t^2.dt - \int t.dt + \int 1.dt - \int \left[\frac{1}{(1+t)}\right].dt\right\}$$

$$= 6\left[\left(\frac{t^3}{3}\right) - \left(\frac{t^2}{2}\right) + t - \log(1+t)\right]$$

$$= 6\left[\left(\frac{\left(x^{\frac{1}{6}}\right)^3}{3}\right) - \left(\frac{\left(x^{\frac{1}{6}}\right)^2}{2}\right) + \left(x^{\frac{1}{6}}\right) - \log\left(1+\left(x^{\frac{1}{6}}\right)\right)\right] + C$$

$$= \left[\left(2x^{\frac{1}{2}}\right) - \left(3x^{\frac{1}{3}}\right) + 6.x^{\frac{1}{6}} - 6.\log\left(1+x^{\frac{1}{6}}\right)\right] + C$$

$$= 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(1+x^{\frac{1}{6}}\right) + C$$

6. $\dfrac{5x}{(x+1)(x^2+9)}$

6. Let $\dfrac{5x}{(x+1)(x^2+9)} = \dfrac{A}{(x+1)} + \dfrac{Bx+C}{(x^2+9)}$

$$\Rightarrow 5x = A(x^2+9) + (Bx+C)(x+1)$$

$$\Rightarrow 5x = Ax^2 + 9A + Bx^2 + Bx + Cx + C$$

Equating the coefficients of x^2, x, and constant term, we obtain

$A + B = 0B$

$+C = 5$

$9A + C = 0$

On solving these equations, we obtain

$A=-\frac{1}{2}, B=\frac{1}{2}$, and $C=\frac{9}{2}$

From equation (1), we obtain

$$\frac{5x}{(x+1)(x^2+9)}=\frac{-1}{2(x+1)}+\frac{\frac{x}{2}+\frac{9}{2}}{(x^2+9)}$$

$$\int\frac{5x}{(x+1)(x^2+9)}dx=\int\frac{-1}{2(x+1)}+\frac{(x+9)}{2(x^2+9)}\Bigg\}dx$$

$$=-\frac{1}{2}\log|x+1|+\frac{1}{2}\int\frac{x}{x^2+9}dx+\frac{9}{2}\int\frac{1}{x^2+9}dx$$

$$=-\frac{1}{2}\log|x+1|+\frac{1}{4}\int\frac{2x}{x^2+9}dx+\frac{9}{2}\int\frac{1}{x^2+9}dx$$

$$=-\frac{1}{2}\log|x+1|+\frac{1}{4}\log|x^2+9|+\frac{9}{2}\cdot\frac{1}{3}\tan^{-1}\frac{x}{3}$$

$$=-\frac{1}{2}\log|x+1|+\frac{1}{4}\log(x^2+9)+\frac{3}{2}\tan^{-1}\frac{x}{3}+C$$

7. $\frac{\sin x}{\sin(x-a)}$

7. Given: $\frac{\sin x}{\sin(x-a)}$

Let $I=\frac{\sin x}{\sin(x-a)}$

Let x – a = t ⇒ x = t + a ⇒ dx = dt

$$\Rightarrow\int\frac{\sin x}{\sin(x-a)}dx=\int\frac{\sin(t+a)}{\sin(t)}dt$$

As, { sin (A+B) = sin A cos B + cos A sin B}

$$\Rightarrow\int\frac{\sin x}{\sin(x-a)}dx=\int\frac{\sin t\cos a+\cos t\sin a}{\sin(t)}dt$$

$$=\int\frac{\sin t\cos a}{\sin t}+\frac{\cos t\sin a}{\sin t}dt$$

$$=\int(\cos a+\cot t\sin a)dt$$

$$=\int(\cos a)dt+\int(\cot t\sin a)dt$$

$$=(\cos a)\int 1.dt+\sin a.\int(\cot t)dt$$

$$=(\cos a).t+\sin a.\log|\sin t|+C$$

$$=(\cos a).(x-a)+\sin a.\log|\sin(x-a)|+C$$

$$= \sin a.\log\left|\sin(x-a)\right| + x.\cos a - a.\cos a + C$$

$$= \sin a.\log\left|\sin(x-a)\right| + x.\cos a + C_2$$

8. $\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}$

8. $\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} = \frac{e^{4\log x}\left(e^{\log x} - 1\right)}{e^{2\log x}\left(e^{\log x} - 1\right)}$

$= e^{2\log x}$

$= e^{\log x^2}$

$= x^2$

$\therefore \int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx = \int x^2 dx = \frac{x^3}{3} + C$

9. $\frac{\cos x}{\sqrt{4 - \sin^2 x}}$

9. Given: $\frac{\cos x}{\sqrt{4 - \sin^2 x}}$

let $I = \frac{\cos x}{\sqrt{4 - \sin^2 x}}$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$

$\Rightarrow \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx = \int \frac{1}{\sqrt{4 - t^2}} dt$

$= \int \frac{1}{\sqrt{\left(2^2 - t^2\right)}} dt$

$= \sin^{-1}\left(\frac{t}{2}\right) + C$

$\Rightarrow I = \sin^{-1}\left(\frac{\sin x}{2}\right) + C$

10. $\frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x.\cos^2 x}$

10. $\frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x\cos^2 x} = \frac{\left(\sin^4 x + \cos^4 x\right)\left(\sin^4 x - \cos^4 x\right)}{\sin^2 x + \cos^2 x - \sin^2 x\cos^2 x - \sin^2 x\cos^2 x}$

$$=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^2 x+\cos^2 x\right)\left(\sin^2 x-\cos^2 x\right)}{\left(\sin^2 x-\sin^2 x\cos^2 x\right)+\left(\cos^2 x-\sin^2 x\cos^2 x\right)}$$

$$=\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^2 x-\cos^2 x\right)}{\sin^2 x\left(1-\cos^2 x\right)+\cos^2 x\left(1-\sin^2 x\right)}$$

$$=\frac{-\left(\sin^4 x+\cos^4 x\right)\left(\cos^2 x-\sin^2 x\right)}{\left(\sin^4 x+\cos^4 x\right)}$$

$$=-\cos 2x$$

$$\therefore \int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx=\int -\cos 2x dx=-\frac{\sin 2x}{2}+C$$

11. $\frac{1}{\cos(x+a)\cos(x+b)}$

11. Given: $\frac{1}{\cos\left(x+a\right)\cos\left(x+b\right)}$

let, $I=\frac{1}{\cos\left(x+a\right)\cos\left(x+b\right)}$

Multiply and divide by sin (a-b), we get

$$I=\frac{1}{\sin\left(a-b\right)}.\left(\frac{\sin\left(a-b\right)}{\cos\left(x+a\right)\cos\left(x+b\right)}\right)$$

$$=\frac{1}{\sin\left(a-b\right)}.\left(\frac{\sin\left(a-b+x-x\right)}{\cos\left(x+a\right)\cos\left(x+b\right)}\right)$$

$$=\frac{1}{\sin\left(a-b\right)}.\left(\frac{\sin\left[\left(x+a\right)-\left(x+b\right)\right]}{\cos\left(x+a\right)\cos\left(x+b\right)}\right)$$

As,{ sin (A-B) = sin A cos B - cos A sin B}

$$\Rightarrow I=\frac{1}{\sin\left(a-b\right)}.\left(\frac{\sin\left(x+a\right).\cos\left(x+b\right)-\cos\left(x+a\right).\sin\left(x+b\right)}{\cos\left(x+a\right)\cos\left(x+b\right)}\right)$$

$$=\frac{1}{\sin\left(a-b\right)}.\left(\frac{\sin\left(x+a\right).\cos\left(x+b\right)}{\cos\left(x+a\right)\cos\left(x+b\right)}-\frac{\cos\left(x+a\right).\sin\left(x+b\right)}{\cos\left(x+a\right)\cos\left(x+b\right)}\right)$$

$$=\frac{1}{\sin\left(a-b\right)}.\left(\frac{\sin\left(x+a\right)}{\cos\left(x+a\right)}-\frac{\sin\left(x+b\right)}{\cos\left(x+b\right)}\right)$$

$$=\frac{1}{\sin\left(a-b\right)}.\left[\tan\left(x+a\right)-\tan\left(x+b\right)\right]$$

$$\Rightarrow \int \frac{1}{\cos(x+a)\cos(x+b)}dx = \int \frac{1}{\sin(a-b)}.\left[\tan(x+a)-\tan(x+b)\right]dx$$

$$= \frac{1}{\sin(a-b)}\left\{\int \tan(x+a)dx - \int \tan(x+b)dx\right\}$$

$$= \frac{1}{\sin(a-b)}\left[-\log\left|\cos(x+a)\right| - \left(-\log\left|\cos(x+a)\right|\right)\right]$$

$$= \frac{1}{\sin(a-b)}\left[-\log\left|\cos(x+a)\right| + \log\left|\cos(x+a)\right|\right]$$

$$\Rightarrow I = \frac{1}{\sin(a-b)}.\log\left|\frac{\cos(x+b)}{\cos(x+a)}\right| + C$$

12. $\dfrac{x^3}{\sqrt{1-x^8}}$

12. $\dfrac{x^3}{\sqrt{1-x^8}}$

Let $x^4 = t \Rightarrow 4x^3dx = dt$

$$\Rightarrow \int \frac{x^3}{\sqrt{1-x^8}}dx = \frac{1}{4}\int \frac{dt}{\sqrt{1-t^2}}$$

$$= \frac{1}{4}\sin^{-1}t + C$$

$$= \frac{1}{4}\sin^{-1}\left(x^4\right) + C$$

13. $\dfrac{e^x}{(1+e^x)(2+e^x)}$

13. $\dfrac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}$

Let $e^x = t \Rightarrow e^x dx = dt$

$$\Rightarrow \int \frac{e^x}{\left(1+e^x\right)\left(2+e^x\right)}dx = \int \frac{dt}{(t+1)(t+2)}$$

$$= \int \left[\frac{1}{(t+1)} - \frac{1}{(t+2)}\right]dt$$

$$= \log\left|t+1\right| - \log\left|t+2\right| + C$$

$$= \log\left|\frac{t+1}{t+2}\right| + C$$

$$= \log\left|\frac{1+e^x}{2+e^x}\right| + C$$

14. $\dfrac{1}{(x^2+1)(x^2+4)}$

14. Given: $\dfrac{1}{\left(x^2+1\right)\left(x^2+4\right)}$

Let $I = \dfrac{1}{\left(x^2+1\right)\left(x^2+4\right)}$

Using partial differentiation:

let $\dfrac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \dfrac{Ax+B}{\left(x^2+1\right)} + \dfrac{Cx+D}{\left(x^2+4\right)}$ ...(1)

$$\Rightarrow \frac{1}{(x+1)\left(x^2+9\right)} = \frac{(Ax+B)\left(x^2+4\right)+(Cx+D)\left(x^2+1\right)}{(x+1)\left(x^2+9\right)}$$

$\Rightarrow 1 = (Ax + B)(x^2 + 4)+(Cx + D)(x^2 + 1)$

$\Rightarrow 1 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Cx + Dx^2 + D$

$\Rightarrow 1 = (A+C)x^3 + (B+D)x^2 + (4A+C)x + (4B+D)$

Equating the coefficients of x, x^2, x^3 and constant value. We get:

(a) $A + C = 0 \Rightarrow C = -A$

(b) $B + D = 0 \Rightarrow B = -D$

(c) $4A + C = 0 \Rightarrow 4A = -C \Rightarrow 4A = A \Rightarrow 3A = 0 \Rightarrow A = 0 \Rightarrow C = 0$

(d) $4B + D = 1 \Rightarrow 4B - B = 1 \Rightarrow B = 1/3 \Rightarrow D = -1/3$

Put these values in equation (1)

$$\Rightarrow \frac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \frac{Ax+B}{\left(x^2+1\right)} + \frac{Cx+D}{\left(x^2+4\right)}$$

$$\Rightarrow \frac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \frac{(0)x+\frac{1}{3}}{\left(x^2+1\right)} + \frac{(0)x+\left(-\frac{1}{3}\right)}{\left(x^2+4\right)}$$

$$\Rightarrow \frac{1}{\left(x^2+1\right)\left(x^2+4\right)} = \frac{\frac{1}{3}}{\left(x^2+1\right)} + \frac{\left(-\frac{1}{3}\right)}{\left(x^2+4\right)}$$

$$\Rightarrow \int \frac{1}{\left(x^2+1\right)\left(x^2+4\right)}dx = \frac{1}{3}.\int\frac{1}{\left(x^2+1\right)}dx - \frac{1}{3}.\int\frac{1}{\left(x^2+4\right)}dx$$

$$\Rightarrow \int \frac{1}{(x^2+1)(x^2+4)}dx = \frac{1}{3}.\int \frac{1}{(x^2+1^2)}dx - \frac{1}{3}.\int \frac{1}{(x^2+2^2)}dx$$

$$= \frac{1}{3}.\tan^{-1}x - \frac{1}{3}.\frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

$$\Rightarrow I = \frac{1}{3}.\tan^{-1}x - \frac{1}{6}\tan^{-1}\frac{x}{2} + C$$

15. $\cos^3 xe^{\log\sin x}$

15. $\cos^3 xe^{\log\sin x} = \cos^3 x \times \sin x$

Let cos x = t ⇒ – sin x dx = dt

$$\Rightarrow \int \cos^3 xe^{\log\sin x}dx = \int \cos^3 x\sin x dx$$

$$= -\int t \cdot dt$$

$$= -\frac{t^4}{4} + C$$

$$= -\frac{\cos^4 x}{4} + dt$$

16. $e^{3\log x}(x^4+1)^{-1}$

16. Given: $e^{3\log x}(x^4+1)^{-1}$

Let $I = e^{3\log x}(x^4+1)^{-1}$

$$= e^{\log x^3}(x^4+1)^{-1}$$

$$= \frac{x^3}{x^4+1}$$

Let $x^4 = t \Rightarrow 4x^3\,dx = dt \Rightarrow x^3\,dx = dt/4$

$$\Rightarrow \int e^{3\log x}(x^4+1)^{-1} = \int \frac{x^3}{x^4+1}dx$$

$$= \int \frac{1}{t+1}.\frac{dt}{4}$$

$$= \frac{1}{4}.\int \frac{1}{t+1}.dt$$

$$= \frac{1}{4}\log(t+1) + C$$

$$\Rightarrow I = \frac{1}{4}\log(x^4+1) + C$$

17. $f'(ax+b)[f(ax+b)]^n$

17. $f'(ax+b)[f(ax+b)]^n$

Let $f(ax+b)=t \Rightarrow af'(ax+b)dx=dt$

⇨ $\Rightarrow \int f'(ax+b)[f(ax+b)]^n dx=\frac{1}{a}\int t^n dt$

⇨ $=\frac{1}{a}\left[\frac{t^{n+1}}{n+1}\right]$

⇨ $=\frac{1}{a(n+1)}(f(ax+b))^{n+1}+\text{C}$

18. $\frac{1}{\sqrt{\sin^3 x\sin(x+\alpha)}}$

18. Given: $\frac{1}{\sqrt{\sin^3 x\ \sin(x+\alpha)}}$

$\text{let I}=\frac{1}{\sqrt{\sin^3 x\ \sin(x+\alpha)}}$

As,{ sin (A+B) = sin A cos B + cos A sin B}

$\Rightarrow \text{I}=\frac{1}{\sqrt{\sin^3 x(\sin x\cos\alpha+\cos x\sin\alpha)}}$

$\Rightarrow \text{I}=\frac{1}{\sqrt{\sin^3 x\left(\sin x\cos\alpha+\cos x.\frac{\sin x}{\sin x}\sin\alpha\right)}}$

$=\frac{1}{\sqrt{\sin^3 x\left(\sin x\cos\alpha+\sin x.\frac{\cos x}{\sin x}\sin\alpha\right)}}$

$=\frac{1}{\sqrt{\sin^4 x(\cos\alpha+\cot x\sin\alpha)}}$

$=\frac{1}{\sin^2 x\sqrt{(\cos\alpha+\cot x\sin\alpha)}}$

$=\frac{\text{cosec}^2 x}{\sqrt{(\cos\alpha+\cot x\sin\alpha)}}$

now, let $(\cos\alpha+\cot x\sin\alpha)=t \Rightarrow -\text{cosec}^2 x.\sin\alpha dx=dt$

$\Rightarrow \int\frac{1}{\sqrt{\sin^3 x\ \sin(x+\alpha)}}dx=\int\frac{\text{cosec}^2 x}{\sqrt{(\cos\alpha+\cot x\sin\alpha)}}dx$

$$= \int \frac{1}{\sqrt{t}} . -\frac{dt}{\sin\alpha}$$

$$= -\frac{1}{\sin\alpha}\int \frac{1}{\sqrt{t}}.dt$$

$$= -\frac{1}{\sin\alpha}\int t^{-\frac{1}{2}}.dt$$

$$= -\frac{1}{\sin\alpha}\left[\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right]+C$$

$$= -\frac{2}{\sin\alpha}\left[\sqrt{t}\right]+C$$

$$= -\frac{2}{\sin\alpha}\left[\sqrt{(\cos\alpha+\cot x\sin\alpha)}\right]+C$$

$$= -\frac{2}{\sin\alpha}\left[\sqrt{\left(\cos\alpha+\frac{\cos x}{\sin x}\sin\alpha\right)}\right]+C$$

$$= -\frac{2}{\sin\alpha}\left[\sqrt{\frac{(\cos\alpha\sin x+\cos x\sin\alpha)}{\sin x}}\right]+C$$

$$\Rightarrow I = -\frac{2}{\sin\alpha}\left[\sqrt{\frac{\sin(x+\alpha)}{\sin x}}\right]+C$$

19. $\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}, x\in[0,1]$

19. Let $I=\int\frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx$

It is known that, $\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}=\frac{\pi}{2}$

$$\Rightarrow I=\int\frac{\left(\frac{\pi}{2}-\cos^{-1}\sqrt{x}\right)-\cos^{-1}\sqrt{x}}{\frac{\pi}{2}}dx$$

$$=\frac{2\pi}{\pi}\int\left(\frac{2}{2}-2\cos^{-1}\sqrt{x}\right)dx$$

$$=\frac{2\pi}{\pi}\cdot\frac{2}{2}\int 1\cdot dx-\frac{4}{\pi}\int\cos^{-1}\sqrt{x}dx$$

$$=x-\frac{4}{\pi}\int\cos^{-1}\sqrt{x}dx$$

Let $I_1 = \int \cos^{-1}\sqrt{x}dx$

Also, let $\sqrt{x} = t \Rightarrow dx = 2tdt$

$\Rightarrow I_1 = 2\int \cos^{-1}t \cdot tdt$

$= 2\left[\cos^{-1}t \cdot \frac{t^2}{2} - \int \frac{-1}{\sqrt{1-t^2}} \cdot \frac{t^2}{2}dt\right]$

$= t^2\cos^{-1}t + \int \frac{t^2}{\sqrt{1-t^2}}dt$

$= t^2\cos^{-1}t - \int \frac{1-t^2-1}{\sqrt{1-t^2}}dt$

$= t^2\cos^{-1}t - \int \sqrt{1-t^2}dt + \int \frac{1}{\sqrt{1-t^2}}dt$

$= t^2\cos^{-1}t - \frac{t}{2}\sqrt{1-t^2} - \frac{1}{2}\sin^{-1}t + \sin^{-1}t$

$= t^2\cos^{-1}t - \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t$

From equation (1), we obtain

$I = x - \frac{4}{\pi}\left[t^2\cos t - \frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t\right]$

$= x - \frac{4}{\pi}\left[x\cos^{-1}\sqrt{x} - \frac{\sqrt{x}}{2}\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{x}\right]$

$= x - \frac{4\pi}{\pi}\left[x\left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right) - \frac{\sqrt{x-x^2}}{2} + \frac{\sin^{-1}\sqrt{x}}{}\right]$

$= x - 2x + \frac{4x}{\pi}\sin^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x-x^2} - \frac{2}{\pi}\sin^{-1}\sqrt{x}$

$= -x + \frac{2}{\pi}\left[(2x-1)\sin^{-1}\sqrt{x}\right] + \frac{2}{\pi}\sqrt{x-x^2} + C$

$= \frac{2(2x-1)}{\pi}\sin^{-1}\sqrt{x} + \frac{2}{\pi}\sqrt{x-x^2} - x + C$

20. $\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$

20. Given: $\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$

let $I=\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}$

Let x= $\cos^2\theta \Rightarrow dx = -2\sin\theta \cos\theta \, d\theta$

$\Rightarrow \sqrt{x}=\cos\theta \text{ or } \theta=\cos^{-1}\sqrt{x}$

$\Rightarrow I=\int\sqrt{\frac{1-\sqrt{\cos^2\theta}}{1+\sqrt{\cos^2\theta}}}(-2\sin\theta\cos\theta)d\theta$

$=\int\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-2\sin\theta\cos\theta)d\theta$

$=\int-\sqrt{\frac{2\sin^2\left(\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\theta}{2}\right)}}(2\sin\theta\cos\theta)d\theta$

$=\int-\sqrt{\frac{\sin^2\left(\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right)}}\left(2\sin2\frac{\theta}{2}\cos2\frac{\theta}{2}\right)d\theta$

$=\int-\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}.(2).\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right).\left(2\cos^2\left(\frac{\theta}{2}\right)-1\right)d\theta$

$\Rightarrow\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx=\int-4.\left[\sin^2\left(\frac{\theta}{2}\right)\right]\left(2\cos^2\left(\frac{\theta}{2}\right)-1\right)d\theta$

$=\int-4.\left\{\left[2.\sin^2\left(\frac{\theta}{2}\right)\cos^2\left(\frac{\theta}{2}\right)\right]-\sin^2\left(\frac{\theta}{2}\right)\right\}d\theta$

$=\int-2.\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^2d\theta+4\int\sin^2\left(\frac{\theta}{2}\right)d\theta$

$=-2.\int\sin^2\theta d\theta+4\int\sin^2\left(\frac{\theta}{2}\right)d\theta$

$=-2.\int\frac{1-\cos2\theta}{2}d\theta+4\int\frac{1-\cos\theta}{2}d\theta$

$=-2\left[\frac{\theta}{2}-\frac{\sin2\theta}{4}\right]+4\left[\frac{\theta}{2}-\frac{\sin\theta}{2}\right]+C$

$=-\theta+\frac{\sin2\theta}{2}+2\theta-2\sin\theta+C$

$=\theta+\frac{2.\sin\theta.\cos\theta}{2}-2\sin\theta+C$

$$=\theta+\frac{2.\sqrt{1-\cos^2\theta}.\cos\theta}{2}-2\sqrt{1-\cos^2\theta}+C$$

$$=\cos^{-1}\sqrt{x}+\sqrt{1-x}.\sqrt{x}-2\sqrt{1-x}+C$$

$$=\cos^{-1}\sqrt{x}+\sqrt{x(1-x)}-2\sqrt{1-x}+C$$

$$\Rightarrow I=\cos^{-1}\sqrt{x}+\sqrt{x-x^2}-2\sqrt{1-x}+C$$

21. $\frac{2+\sin 2x}{1+cos 2x}e^x$

21. $I=\int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x$

$$=\int\left(\frac{2+2\sin x\cos x}{2\cos^2 x}\right)e^x$$

$$=\int\left(\frac{1+\sin x\cos x}{\cos^2 x}\right)e^x$$

$$=\int\left(\sec^2 x+\tan x\right)e^x$$

Let $f(x)=\tan x\Rightarrow f'(x)=\sec^2 x$

$\therefore I=\int\left(f(x)+f'(x)\right]e^x dx$

$$=e^x f(x)+C$$

$$=e^x\tan x+C$$

22. $\frac{x^2+x+1}{(x+1)^2(x+2)}$

22. Given: $\frac{x^2+x+1}{(x+1)^2(x+2)}$

Let $I=\frac{x^2+x+1}{(x+1)^2(x+2)}$

Using partial differentiation:

let $\frac{x^2+x+1}{(x+1)^2(x+2)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{(x+2)}$ ...(1)

$$\Rightarrow\frac{x^2+x+1}{(x+1)^2(x+2)}=\frac{A(x+1)(x+2)+B(x+2)+C(x+1)^2}{(x+1)^2(x+2)}$$

$$\Rightarrow\frac{x^2+x+1}{(x+1)^2(x+2)}=\frac{A(x^2+3x+2)+B(x+2)+C(x^2+2x+1)}{(x+1)^2(x+2)}$$

$\Rightarrow x^2 + x + 1 = Ax^2 + 3Ax + 2A + Bx + 2B + Cx^2 + 2Cx + C$

$\Rightarrow x^2 + x + 1 = (2A+2B+C) + (3A+B+2C)x + (A+C)x^2$

Equating the coefficients of x, x^2 and constant value. We get:

(a) $A + C = 1$

(b) $3A + B + 2C = 1$

(c) $2A+2B+C =1$

After solving we get:

A=-2, B=1 and C=3

$$\Rightarrow \frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{(x+2)}$$

$$\Rightarrow \int \frac{x^2+x+1}{(x+1)^2(x+2)}dx = \int\left(\frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{(x+2)}\right)dx$$

$$= -2.\int\left(\frac{1}{x+1}\right)dx + \int\left(\frac{1}{(x+1)^2}\right)dx + 3.\int\left(\frac{1}{(x+2)}\right)dx$$

$$= -2.\int\left(\frac{1}{x+1}\right)dx + \int\left((x+1)^{-2}\right)dx + 3.\int\left(\frac{1}{(x+2)}\right)dx$$

$$= -2\log|x+1| + \left(\frac{(x+1)^{-1}}{(-1)}\right) + 3\log|x+1| + C$$

$$= -2\log|x+1| - \frac{1}{(x+1)} + 3\log|x+1| + C$$

23. $\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

23. $I = \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$

Let $x = \cos\theta \Rightarrow dx = -\sin\theta d\theta$

$$I = \int \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}(-\sin\theta d\theta) = -\int \tan^{-1}\tan\frac{\theta}{2}\cdot\sin\theta d\theta = -\frac{1}{2}\int\theta\cdot\sin\theta d\theta$$

$$= -\frac{1}{2}\left[\theta\cdot(-\cos\theta) - \int 1\cdot(-\cos\theta)d\theta\right] = -\frac{1}{2}[-\theta\cos\theta + \sin\theta] = +\frac{1}{2}\theta\cos\theta - \frac{1}{2}\sin\theta$$

$$= \frac{1}{2}\cos^{-1}x\cdot x - \frac{1}{2}\sqrt{1-x^2} + C = \frac{x}{2}\cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + C = \frac{1}{2}\left(x\cos^{-1}x - \sqrt{1-x^2}\right) + C$$

24. $\frac{\sqrt{x^2+1}\left[\log(x^2+1) - 2\log x\right]}{x^4}$

24. Given: $\dfrac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x^4}$

$$\text{let } I=\frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x^4}$$

$$=\frac{\sqrt{x^2+1}}{x^4}\left[\log\left(x^2+1\right)-\log x^2\right]$$

$$=\frac{1}{x^3}\sqrt{\frac{x^2+1}{x^2}}\left[\log\left(\frac{x^2+1}{x^2}\right)\right]$$

$$=\frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\left[\log\left(1+\frac{1}{x^2}\right)\right]$$

$$\text{now let} 1+\frac{1}{x^2}=t\Rightarrow -\frac{2}{x^3}dx=dt$$

$$\Rightarrow \int\frac{\sqrt{x^2+1}\left[\log\left(x^2+1\right)-2\log x\right]}{x^4}dx=\int\frac{1}{x^3}\sqrt{1+\frac{1}{x^2}}\left[\log\left(1+\frac{1}{x^2}\right)\right]dx$$

$$=\int-\frac{1}{2}.\sqrt{t}\left[\log(t)\right]dt$$

$$\text{because, } \int u.vdx=u.\int vdx-\int\frac{du}{dx}.\left\{\int vdx\right\}\,dx$$

$$=\int-\frac{1}{2}.\sqrt{t}\left[\log(t)\right]dt\ =-\frac{1}{2}\left[\log t.\int\sqrt{t}dt-\int\frac{d}{dt}\log t.\left\{\int\sqrt{t}dt\right\}\,dt\right]$$

$$=-\frac{1}{2}\left[\log t.\frac{t^{\frac{3}{2}}}{\frac{3}{2}}-\int\frac{1}{t}.\left\{\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right\}dt\right]$$

$$=-\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t-\int\left\{\frac{t^{\frac{3}{2}-1}}{\frac{3}{2}}\right\}dt\right]$$

$$=-\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t-\frac{2}{3}\int t^{\frac{1}{2}}dt\right]$$

$$=-\frac{1}{2}\left[\frac{2}{3}t^{\frac{3}{2}}\log t-\frac{2}{3}.\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right]$$

$$=\left[-\frac{1}{2}.\frac{2}{3}t^{\frac{3}{2}}\log t+\frac{1}{2}.\frac{2}{3}.\frac{2}{3}.t^{\frac{3}{2}}\right]$$

$$=-\frac{1}{3}t^{\frac{3}{2}}\left[\log t-\frac{2}{3}\right]$$

$$\Rightarrow I=-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{\frac{3}{2}}\left[\log\left(1+\frac{1}{x^2}\right)-\frac{2}{3}\right]+C$$

25. $\int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

25. $I=\int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

$$=\int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2})}{2\sin^2\frac{x}{2}}\right)dx=\int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{\operatorname{cosec}^2\frac{x}{2}}{2}-\cot\frac{x}{2}\right)dx \text{ Let } f(x)=-\cot\frac{x}{2}$$

$$\Rightarrow f'(x)=-\left(-\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}\right)=\frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$$

$$\therefore I=\int_{\frac{\pi}{2}}^{\pi} e^x\left(f(x)+f'(x)\right]dx=\left[e^x\cdot f(x)dx\right]_{\frac{\pi}{2}}^{x}=-\left[e^x\cdot\cot\frac{x}{2}\right]_{\frac{\pi}{2}}^{x}=-\left[e^{\pi}\times\cot\frac{\pi}{2}-e^{\frac{\pi}{2}}\times\cot\frac{\pi}{4}\right]$$

$$=-\left[e^{\pi}\times 0-e^{\frac{\pi}{2}}\times 1\right]=e^{\frac{\pi}{2}}$$

26. $\int_0^{\frac{\pi}{4}}\frac{\sin x\cos x}{\cos^4 x+\sin^4 x}dx$

26. Given: $\int_0^{\frac{\pi}{4}}\frac{\sin x\cos x}{\cos^4 x+\sin^4 x}\,dx$

let, $I=\int_0^{\frac{\pi}{4}}\frac{\sin x\cos x}{\cos^4 x+\sin^4 x}\,dx$

$$=\int_0^{\frac{\pi}{4}}\frac{\sin x\cos x}{\cos^4 x\left(1+\frac{\sin^4 x}{\cos^4 x}\right)}\,dx$$

$$=\int_0^{\frac{\pi}{4}}\frac{\tan x\sec^2 x}{\left(1+\tan^4 x\right)}\,dx$$

Now let $\tan^2 x = t \Rightarrow 2\tan x\sec^2 x\,dx = dt$

And when x=0 then t=0 and when x=π /4 then t=1

$$\Rightarrow I=\int_0^{\frac{\pi}{4}}\frac{\tan x\sec^2 x}{\left(1+\tan^4 x\right)}\ dx=\int_0^1\frac{1}{\left(1+t^2\right)}\left(\frac{dt}{2}\right)$$

$$\Rightarrow I=\frac{1}{2}\left[\tan^{-1}t\right]_0^1$$

$$=\frac{1}{2}\left[\tan^{-1}1-\tan^{-1}0\right]$$

$$\Rightarrow I=\frac{1}{2}.\frac{\pi}{4}$$

$$\Rightarrow I=\frac{\pi}{8}$$

27. $\int\limits_0^{\frac{\pi}{2}}\frac{\cos^2 xdx}{\cos^2 x+4\sin^2 x}$

27. Let $I=\int_0^{\frac{\pi}{2}}\frac{\cos^2 x}{\cos^2 x+4\sin^2 x}dx$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^2 x}{\cos^2 x+4\left(1-\cos^2 x\right)}dx\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{\cos^2 x}{\cos^2 x+4-4\cos^2 x}dx\Rightarrow I=\frac{-1}{3}\int_0^{\frac{\pi}{2}}\frac{4-3\cos^2 x-4}{4-3\cos^2 x}dx$$

$$\Rightarrow I=\frac{-1}{3}\int_0^{\frac{\pi}{2}}\frac{4-3\cos^2 x}{4-3\cos^2 x}dx+\frac{1}{3}\int_0^{\frac{\pi}{2}}\frac{4}{4-3\cos^2 x}dx\Rightarrow I=\frac{-1}{3}\int_0^{\frac{\pi}{2}}1dx+\frac{1}{3}\int_1^{\frac{\pi}{2}}\frac{4\sec^2 x}{4\sec^2 x-3}dx$$

$$\Rightarrow I=\frac{-1}{3}[x]_0^{\frac{\pi}{2}}+\frac{1}{3}\int_0^{\frac{\pi}{2}}\frac{4\sec^2 x}{4\left(1+\tan^2 x\right)-3}dx\Rightarrow I=-\frac{\pi}{6}+\frac{2}{3}\int_0^{\frac{\pi}{2}}\frac{2\sec^2 x}{1+4\tan^2 x}dx \qquad (1)$$

Consider, $\int_0^{\pi}\frac{2\sec^2 x}{1+4\tan^2 x}dx$

Let $2\tan x=t\Rightarrow 2\sec^2 xdx=dt$

When $x=0, t=0$ and when $x=\frac{\pi}{2}, t=\infty$

⇨ $\Rightarrow\int_0^{\frac{\pi}{2}}\frac{2\sec^2 x}{1+4\tan^2 x}dx=\int_0^{\infty}\frac{dt}{1+t^2}$

⇨ $=\left[\tan^{-1}t\right]_0^{\infty}$

⇨ $=\left[\tan^{-1}(\infty)-\tan^{-1}(0)\right]$

⇨ $=\frac{\pi}{2}$

Therefore, from (1), we obtain

$$I = -\frac{\pi}{6} + \frac{2}{3}\left[\frac{\pi}{2}\right] = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$$

28. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{s\text{in}\,2x}} dx$

28. Given: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

let, $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{-(-\sin 2x)}}\, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{-(-1+1-2\sin x\cos x)}}\, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{1-\left(\sin^2 x + \cos^2 x - 2\sin x\cos x\right)}}\, dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{(1-\left(\sin x - \cos x\right)^2)}}\, dx$

Now let sin x – cos x = t ⇒(cos x + sin x)dx = dt

when $x = \frac{\pi}{6} \Rightarrow t = \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$ and when $x = \frac{\pi}{3} \Rightarrow t = \frac{\sqrt{3}}{2} - \frac{1}{2} = \frac{\sqrt{3}-1}{2}$

$\Rightarrow \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{(1-\left(\sin x - \cos x\right)^2)}}\, dx = \int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{1}{\sqrt{(1-(t)^2)}}\, dt$

$= \int_{-\left(\frac{\sqrt{3}-1}{2}\right)}^{\frac{\sqrt{3}-1}{2}} \frac{1}{\sqrt{(1-(t)^2)}}\, dt$

let $f(x) = \frac{1}{\sqrt{(1-(t)^2)}}$ and $f(-x) = \frac{1}{\sqrt{(1-(-t)^2)}} = \frac{1}{\sqrt{(1-(t)^2)}} = f(x)$

i.e. f(x) = f(-x)

so, f(x) is an even function.

It is also known that if f(x) is an even function then,

$\left\{\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx\right\}$

$$\Rightarrow I = 2.\int_0^{\frac{\sqrt{3}-1}{2}} \frac{1}{\sqrt{(1-(t)^2)}}\, dt$$

$$\Rightarrow I = \left[2.\sin^{-1} t\right]_0^{\frac{\sqrt{3}-1}{2}}$$

$$\Rightarrow I = 2.\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

29. $\int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$

29. Let $I = \int_0^1 \frac{dx}{\sqrt{1+x}-\sqrt{x}}$

$$I = \int_0^1 \frac{1}{\left(\sqrt{1+x}-\sqrt{x}\right)} \times \frac{\left(\sqrt{1+x}+\sqrt{x}\right)}{\left(\sqrt{1+x}+\sqrt{x}\right)} dx = \int_0^1 \frac{\sqrt{1+x}+\sqrt{x}}{1+x-x} dx = \int_0^1 \sqrt{1+x}dx + \int_0^1 \sqrt{x}dx$$

$$= \left[\frac{2}{3}(1+x)^{\frac{3}{2}}\right]_0^1 + \left[\frac{2}{3}(x)^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}\left[(2)^{\frac{3}{2}} - 1\right]^2 + \frac{2}{3}[1] = \frac{2}{3}(2)^{\frac{3}{2}} = \frac{2 \cdot 2\sqrt{2}}{3} = \frac{4\sqrt{2}}{3}$$

30. $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9+16\sin 2x} dx$

30. Given: $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9+16\sin 2x}\, dx$

let, $I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9+16\sin 2x}\, dx$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9+16(2\sin x \cos x)}\, dx$$

Now let sin x – cos x = t ⇒(cos x + sin x)dx = dt

⇒ $(\sin x - \cos x)^2 = t^2$

⇒ $\sin^2 x + \cos^2 x - 2 \sin x \cos x = t^2$

⇒ $1 - 2 \sin x \cos x = t^2$

⇒ $1 - t^2 = 2 \sin x \cos x$

And when x=0 then t=-1 and when x=π /4 then t=0

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \frac{\tan x \sec^2 x}{\left(1+\tan^4 x\right)}\, dx = \int_{-1}^{0} \frac{1}{\left(9+16\left(1-t^2\right)\right)}\, (dt)$$

$$=\int_{-1}^{0}\frac{1}{\left(9+16-16t^2\right)}(dt)$$

$$=\int_{-1}^{0}\frac{1}{\left(25-16t^2\right)}(dt)$$

$$=\int_{-1}^{0}\frac{1}{\left((5)^2-(4t)^2\right)}(dt)$$

$$=\frac{1}{4}\left[\frac{1}{2(5)}\log\left|\frac{5+4t}{5-4t}\right|\right]_{-1}^{0}+C$$

$$=\frac{1}{40}\left[\log\left|\frac{5+0}{5-0}\right|-\log\left|\frac{5+4(-1)}{5-4(-1)}\right|\right]+C$$

$$=\frac{1}{40}\left[\log|1|-\log\left|\frac{1}{9}\right|\right]+C$$

$$=\frac{1}{40}\left[\log\left|\frac{1}{\frac{1}{9}}\right|\right]+C$$

$$=\frac{1}{40}\left[\log|9|\right]+C$$

$$=\frac{1}{40}\log 9$$

31. $\int_0^{\frac{\pi}{2}}\sin 2x\tan^{-1}(\sin x)dx$

31. Let $I=\int_0^{\frac{\pi}{2}}\sin 2x\tan^{-1}(\sin x)dx=\int_0^{\frac{\pi}{2}}2\sin x\cos x\tan^{-1}(\sin x)dx$

Also, let $\sin x=t\Rightarrow\cos x dx=dt$

When $x=0,t=0$ and when $x=\frac{\pi}{2},t=1$

$\Rightarrow I=2\int_0^1 t\tan^{-1}(t)dt$ (1)

Consider $\int t\cdot\tan^{-1}t dt=\tan^{-1}t\cdot\int t\,dt-\int\left\{\frac{d}{dt}\left(\tan^{-1}t\right)\int t\,dt\right\}dt$

$$=\tan^{-1}t\cdot\frac{t^2}{2}-\int\frac{1}{1+t^2}\cdot\frac{t^2}{2}dt=\frac{t^2\tan^{-1}t}{2}-\frac{1}{2}\int\frac{t^2+1-1}{1+t^2}dt=\frac{t^2\tan^{-1}t}{2}-\frac{1}{2}\int 1dt+\frac{1}{2}\int\frac{1}{1+t^2}dt$$

$$=\frac{t^2\tan^{-1}t}{2}-\frac{1}{2}\cdot t+\frac{1}{2}\tan^{-1}t \Rightarrow \int_0^1 t\cdot\tan^{-1}t\,dt=\left[\frac{t^2\cdot\tan^{-1}t}{2}-\frac{t}{2}+\frac{1}{2}\tan^{-1}t\right]_0^1=\frac{1}{2}\left[\frac{\pi}{4}-1+\frac{\pi}{4}\right]$$

$=\frac{1}{2}\left[\frac{\pi}{2}-1\right]=\frac{\pi}{4}-\frac{1}{2}$ From equation (1), we obtain

$$I=2\left[\frac{\pi}{4}-\frac{1}{2}\right]=\frac{\pi}{2}-1$$

32. $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}dx$

32. Given: $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}\,dx$

let, $I=\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}\,dx \ldots.(1)$

as, $\left\{\int_0^a f(x)dx=\int_0^a f(a-x)dx\right\}$

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}\,dx$$

$$=\int_0^{\pi}\frac{(\pi-x)(-\tan(x))}{(-\sec x)+(-\tan x)}\,dx$$

$$=\int_0^{\pi}\frac{-(\pi-x)(\tan(x))}{-[(\sec x)+(\tan x)]}\,dx$$

$$=\int_0^{\pi}\frac{(\pi-x)(\tan(x))}{\sec x+\tan x}\,dx \quad \ldots.(2)$$

Adding (1) and (2), we get

$$2I=\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}+\frac{(\pi-x)(\tan(x))}{\sec x+\tan x}\,dx$$

$$2I=\int_0^{\pi}\frac{\pi\tan x}{\sec x+\tan x}\,dx$$

$$=\int_0^{\pi}\frac{\pi.\frac{\sin x}{\cos x}}{\frac{1}{\cos x}+\frac{\sin x}{\cos x}}\,dx$$

$$2I=\pi.\int_0^{\pi}\frac{(\sin x)}{(1+\sin x)}\,dx$$

$$= \pi.\int_0^{\pi} \frac{(-1+1+\sin x)}{(1+\sin x)}\ dx$$

$$= \pi.\int_0^{\pi} \frac{(-1)}{(1+\sin x)}\ dx + \pi.\int_0^{\pi} \frac{(1+\sin x)}{(1+\sin x)}\ dx$$

$$= \pi.\int_0^{\pi} \frac{(-1)}{(1+\sin x)} \times \frac{(1-\sin x)}{(1-\sin x)} dx + \pi.\int_0^{\pi} 1.\ dx$$

$$= -\pi.\int_0^{\pi} \frac{(1-\sin x)}{(1-\sin^2 x} dx + \pi.\int_0^{\pi} 1.\ dx$$

$$2I = -\pi.\int_0^{\pi} \frac{(1-\sin x)}{\cos^2 x}\ dx + \pi.\int_0^{\pi} 1.\ dx$$

$$2I = -\pi.\int_0^{\pi} \left\{\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right\}\ dx + \pi.\int_0^{\pi} 1.\ dx$$

$$2I = -\pi.\int_0^{\pi} \{\sec^2 x - \tan x \sec x\}\ dx + \pi.\int_0^{\pi} 1.\ dx$$

$$\Rightarrow 2I = -\pi.[\tan x - \sec x]_0^{\pi} + [x]_0^{\pi}$$

$$\Rightarrow 2I = -\pi.[\tan \pi - \sec \pi - \tan 0 + \sec 0] + \pi.[\pi - 0]$$

$$\Rightarrow 2I = -\pi.[0-(-1)-0+1] + \pi.[\pi]$$

$$\Rightarrow 2I = \pi.[-2+\pi]$$

$$\Rightarrow I = \frac{\pi}{2}.[\pi - 2]$$

33. $\int_1^4 [|x-1| + |x-2| + |x-3|]\ dx$

33. Let $I = \int_1^4 \left[|x-1| + |x-2| + |x-3|\right] dx$

$$\Rightarrow I = \int^4 ||x-1| dx + \int_1^4 |x-2| dx + \int^1 ||x-3| dx$$

$I = I_1 + I_2 + I_3$ (1)

where, $I_1 = \int_1^4 |x-1| dx, I_2 = \int_1^4 |x-2| dx$, and $I_3 = \int_1^4 |x-3| dx$

$$I_1 = \int_1^4 |x-1| dx$$

$(x-1) \geq 0$ for $1 \leq x \leq 4$

$$\therefore I_1 = \int^4 ((x-1) dx$$

$$\Rightarrow I_1 = \left[\frac{x^2}{x} - x\right]_1^4$$

$$\Rightarrow I_1 = \left[8 - 4 - \frac{1}{2} + 1\right] = \frac{9}{2} \qquad (2)$$

$$I_2 = \int_1^4 |x-2|\,dx$$

$x - 2 \geq 0$ for $2 \leq x \leq 4$ and $x - 2 \leq 0$ for $1 \leq x \leq 2$

$$\therefore I_2 = \int^2 ((2-x)\,dx + \int_2^4 (x-2)\,dx$$

$$\Rightarrow I_2 = \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$$

$$\Rightarrow I_2 = \left[4 - 2 - 2 + \frac{1}{2}\right] + [8 - 8 - 2 + 4]$$

$$\Rightarrow I_2 = \frac{1}{2} + 2 = \frac{5}{2} \qquad (3)$$

$$I_3 = \int^1 ||x-3|\,dx$$

$x - 3 \geq 0$ for $3 \leq x \leq 4$ and $x - 3 \leq 0$ for $1 \leq x \leq 3$

$$\therefore I_3 = \int^3 ((3-x)\,dx + \int_3^4 (x-3)\,dx$$

$$\Rightarrow I_3 = \left[3x - \frac{x^2}{2}\right]_1^3 + \left[\frac{x^2}{2} - 3x\right]_3^4$$

$$\Rightarrow I_3 = \left[9 - \frac{9}{2} - 3 + \frac{1}{2}\right] + \left[8 - 12 - \frac{9}{2} + 9\right]$$

$$\Rightarrow I_3 = [6-4] + \left[\frac{1}{2}\right] = \frac{5}{2}$$

From equations (1), (2), (3), and (4), we obtain

$$I = \frac{9}{2} + \frac{5}{2} + \frac{5}{2} = \frac{19}{2}$$

34. $\int_1^3 \frac{dx}{x^2(x+1)} = \frac{2}{3} + \log\frac{2}{3}$

34. Given: $\int_1^3 \frac{dx}{(x^2)(x+1)}$

To Prove : $\int_1^3 \frac{dx}{(x^2)(x+1)} = \frac{2}{3} + \log\frac{2}{3}$

Let $I = \frac{dx}{(x^2)(x+1)}$

Using partial differentiation:

let $\frac{1}{\left(x^2\right)(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$ ...(1)

$$\Rightarrow \frac{1}{\left(x^2\right)(x+1)}=\frac{A(x)(x+1)+B(x+1)+C\left(x^2\right)}{(x+1)\left(x^2\right)}$$

$$\Rightarrow 1=A\left(x^2+x\right)+(Bx+B)+Cx^2$$

⇒ 1 = Ax^2 +Ax+ B+Bx+ Cx^2

⇒ 1 = B + (A+B)x + (A+C)x^2

Equating the coefficients of x, x^2 and constant value. We get:

(a) B = 1

(b) A + B = 0 ⇒ A = -B ⇒ A = -1

(c) A + C =0 ⇒ C = -A ⇒ C = 1

Put these values in equation (1)

$$\Rightarrow \frac{1}{\left(x^2\right)(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$$

$$\Rightarrow \frac{1}{\left(x^2\right)(x+1)}=\frac{-1}{x}+\frac{1}{x^2}+\frac{1}{x+1}$$

$$\Rightarrow \int \frac{1}{\left(x^2\right)(x+1)}dx=\int -\frac{1}{x}dx+\int \frac{1}{\left(x^2\right)}dx+\int \frac{1}{(x+1)}dx$$

$$\Rightarrow \int_1^3 \frac{1}{\left(x^2\right)(x+1)}dx=\left[-\log|x|-x^{-1}+\log|x+1|\right]_1^3$$

$$\Rightarrow \int_1^3 \frac{1}{\left(x^2\right)(x+1)}dx=\left[-\frac{1}{x}+\log\left|\frac{x+1}{x}\right|\right]_1^3$$

$$=\left[-\frac{1}{3}+\log\left|\frac{3+1}{3}\right|-\left(-\frac{1}{1}+\log\left|\frac{1+1}{1}\right|\right)\right]$$

$$=\left[-\frac{1}{3}+\log\left|\frac{4}{3}\right|+\left(1-\log\left|\frac{2}{1}\right|\right)\right]$$

$$=\left[-\frac{1}{3}+1+\log\left|\frac{4}{3}\times\frac{1}{2}\right|\right]$$

$$\Rightarrow I=\left[\frac{2}{3}+\log\left|\frac{2}{3}\right|\right]$$

⇒ L.H.S = R.H.S

Hence proved.

35. $\int_{0}^{1} xe^{x} dx = 1$

35. Let $I = \int_{0}^{1} xe^{x} dx$ Integrating by parts, we obtain

$$I = x\int_{0}^{1} e^{x} dx - \int_{0}^{1}\left\{\left(\frac{d}{dx}(x)\right)\int e^{x} dx\right\}dx$$

$$= \left[xe^{x}\right]_{0}^{1} - \int_{0}^{1} e^{x} dx$$

$$= \left[xe^{x}\right]_{0}^{1} - \left[e^{x}\right]_{0}^{1}$$

$$= e - e + 1$$

$$= 1$$

Hence, the given result is proved.

36. $\int_{-1}^{1} x^{17} \cos^{4} x dx = 0$

36. Given: $\int_{-1}^{1} x^{17}.\cos^{4} xdx$

To Prove : $\int_{-1}^{1} x^{17}.\cos^{4} xdx = 0$

Let $I = \int_{-1}^{1} x^{17}.\cos^{4} xdx$

As we can see $f(x) = x^{17}.\cos^{4}x$ and $f(-x) = (-x)^{17}.\cos^{4}(-x) = -x^{17}.\cos^{4}x$

i.e. $f(x) = -f(-x)$

so, it is an odd function.

It is also known that if f(x) is an odd function then,

$$\left\{\int_{-a}^{a} f(x)dx = 0\right\}$$

$$\Rightarrow I = \int_{-1}^{1} x^{17}.\cos^{4} xdx = 0$$

Hence proved.

37. $\int_{0}^{\frac{\pi}{2}} \sin^{3} xdx = \frac{2}{3}$

37. Let $I = \int_{0}^{2} \sin^{3} xdx$

$$I = \int_{0}^{\frac{\pi}{2}} \sin^{2} x \cdot \sin xdx$$

$$= \int_0^{\frac{\pi}{2}} \left(1 - \cos^2 x\right) \sin x dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x dx - \int_0^{\frac{\pi}{2}} \cos^2 x \cdot \sin x dx$$

$$= [-\cos x]_0^{\frac{\pi}{2}} + \left[\frac{\cos^3 x}{3}\right]_0^{\frac{\pi}{2}}$$

$$= 1 + \frac{1}{3}[-1] = 1 - \frac{1}{3} = \frac{2}{3}$$

Hence, the given result is proved.

38. $\int_0^{\frac{\pi}{4}} 2\tan^3 x dx = 1 - \log 2$

38. Given: $\int_0^{\frac{\pi}{4}} 2\tan^3 x dx$

To Prove : $\int_0^{\frac{\pi}{4}} 2\tan^3 x dx = 1 - \log 2$

Let $I = \int_0^{\frac{\pi}{4}} 2\tan^3 x dx \ \ldots(1)$

$$= \int_0^{\frac{\pi}{4}} 2.\tan x.\tan^2 x dx$$

$$= 2.\int_0^{\frac{\pi}{4}} \tan x.(\sec^2 x - 1) dx$$

$$\Rightarrow I = 2\left\{-\int_0^{\frac{\pi}{4}} \tan x dx + \int_0^{\frac{\pi}{4}} \tan x.\sec^2 x dx\right\}$$

$$\Rightarrow I = -[2\log \sec x]_0^{\pi/4} + 2.I_1 \ \ldots(2)$$

First solve for I1:

$$\Rightarrow I_1 = \int_0^{\frac{\pi}{4}} \tan x.\sec^2 x dx$$

Let tan x = t ⇒ $\sec^2$ x dx = dt

When x=0 then t= 0 and when x = π /2 then t = 1

$$\Rightarrow I_1 = \int_0^1 t.dt \quad = \left[\frac{t^2}{2}\right]_0^1$$

$$\Rightarrow I_1 = \frac{1}{2}$$

Put in equ. (2)

$\Rightarrow \mathrm{I} = [2\log\cos x]_0^{\pi/4} + 2.\frac{1}{2}$

$\Rightarrow \mathrm{I} = 2\left\{\log\cos\frac{\pi}{4} - \log\cos 0\right\} + 1$

$\Rightarrow \mathrm{I} = 2\left\{\log\frac{1}{\sqrt{2}} - \log 1\right\} + 1$

$\Rightarrow \mathrm{I} = \left\{\log\left(\frac{1}{\sqrt{2}}\right)^2 - \log(1)^2\right\} + 1$

$\Rightarrow \mathrm{I} = 1 - \log 2 + \log 1$

$\Rightarrow \mathrm{I} = 1 - \log 2$

L.H.S = R.H.S

Hence Proved.

39. $\int_0^1 \sin^{-1} x dx = \frac{\pi}{2} - 1$

39. Let $I = \int_0^1 \sin^{-1} x dx$

$\Rightarrow I = \int_1^1 \sin^{-1} x \cdot 1 \cdot dx$

Integrating by parts, we obtain

$I = \left[\sin^{-1} x \cdot x\right]_0^1 - \int_0^1 \frac{1}{\sqrt{1-x^2}} \cdot x dx$

$= \left[x \sin^{-1} x\right]_0^1 + \frac{1}{2}\int_0^1 \frac{(-2x)}{\sqrt{1-x^2}} dx$ Let $1 - x^2 = t \Rightarrow -2xdx = dt$

When $x = 0, t = 1$ and when $x = 1, t = 0$

$I = \left[x \sin^{-1} x\right]_0^1 + \frac{1}{2}\int^0 \frac{dt}{\sqrt{t}} \frac{dt}{\sqrt{t}}$

$= \left[x \sin^{-1} x\right]_0^1 + \frac{1}{2}[2\sqrt{t}]_1^0$

$= \sin^{-1}(1) + \left[-\sqrt{1}\right]$

$= \frac{\pi}{2} - 1$

Hence, the given result is proved.

40. Evaluate $\int_0^1 e^{2-3x} dx$ as a limit of a sum.

40. Given: $\int_0^1 e^{2-3x}dx$

Let $I = \int_0^1 e^{2-3x}dx$

because, $\int_a^b f(x)dx = (b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right]$

where, $h = \frac{b-a}{n}$

Here, a=0, b=1, and f(x)=e^{2-3x} and h = 1/n

$$\Rightarrow \int_0^1 e^{2-3x}dx = (1-0)\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f(0+h)+f(0+2h)+\ldots+f(0+(n-1)h)\right]$$

$$=(1)\lim_{n\to\infty}\frac{1}{n}\left[e^2+e^{2-3h}+e^{2-3(2h)}+\ldots+e^{2-3(n-1)h}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[e^2+e^2.e^{3h}++e^2.e^{-6h}\ldots+e^2.e^{-3(n-1)h}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{1+e^{3h}+e^{-6h}+\ldots+.e^{-3(n-1)h}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{\frac{1-\left(e^{-3h}\right)^n}{1-\left(e^{-3h}\right)}\right\}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{\frac{1-\left(e^{-\frac{3}{n}}\right)^n}{1-\left(e^{-\frac{3}{n}}\right)}\right\}\right] \qquad \text{as, } h=\frac{1}{n}$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[e^2\left\{\frac{\left(e^{-3}\right)-1}{\left(e^{-\frac{3}{n}}\right)-1}\right\}\right]$$

$$=e^2.\left(e^{-3}-1\right)\lim_{n\to\infty}\frac{1}{n}.\left(-\frac{n}{3}\right)\left[\left\{\frac{-\frac{3}{n}}{\left(e^{-\frac{3}{n}}\right)-1}\right\}\right]$$

$$=-\frac{\left(e^2.\left(e^{-3}-1\right)\right)}{3}\lim_{n\to\infty}\left[\left\{\frac{-\frac{3}{n}}{\left(e^{-\frac{3}{n}}\right)-1}\right\}\right]$$

as, $\lim_{n\to\infty}\left[\frac{x}{(e^x)-1}\right]=1$

$=\frac{-e^{-1}+e^2}{3}(1)$

$\Rightarrow I=\frac{1}{3}\left(e^2-\frac{1}{e}\right)$

41. $\int\frac{dx}{e^x+e^{-x}}$ is equal to

(A) $\log(e^x-e^{-x})+C$ (B) $\tan^{-1}(e^{-x})+C$

(C) $\log(e^x-e^{-x})+C$ (D) $\log(e^x+e^{-x})+C$

41. Let $I=\int\frac{dx}{e^x+e^{-x}}dx=\int\frac{e^x}{e^{2x}+1}dx$

Also, let $e^x=t\Rightarrow e^x dx=dt$

⇨ $\therefore I=\int\frac{dt}{1+t^2}$

⇨ $=\tan^{-1}t+C$

⇨ $=\tan^{-1}(e^x)+C$

Hence, the correct Answer is A .

42. $\int\frac{\cos 2x}{(\sin x+\cos x)^2}dx$ is equal to

(A) $\frac{-1}{\sin x+\cos x}+C$ (B) $\log|\sin x+\cos x|+C$

(C) $\log|\sin x-\cos x|+C$ (D) $\frac{1}{(\sin x+\cos x)^2}$

42. Given: $\int\frac{\cos 2x}{(\sin x+\cos x)^2}dx$

let $I=\int\frac{\cos 2x}{(\sin x+\cos x)^2}dx$

$=\int\frac{\cos^2 x-\sin^2 x}{(\sin x+\cos x)^2}dx$

$=\int\frac{(\cos x-\sin x)(\cos x+\sin x)}{(\sin x+\cos x)^2}dx$

$= \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx$

Put sin x + cos x= t ⇒ cos x – sin x = dt

$\Rightarrow \int \frac{(\cos x - \sin x)}{(\sin x + \cos x)} dx = \int \frac{dt}{t}$

⇨ $= \log|t| + C$

$= \log|\sin x + \cos x| + C$

Hence, correct option is (B).

43. If f (a + b – x) = f (x), then $\int_a^b x f(x)$ is equal to

43. $I = \int_a^b x f(x) dx$ (1)

$I = \int_a^b (a+b-x) f(a+b-x) dx \quad \left(\int_a^b f(x) dx = \int_a^b f(a+b-x) dx\right)$

$\Rightarrow I = \int_a^b (a+b-x) f(x) dx$

$\Rightarrow I = (a+b)\int_a^b f(x) dx - I$ [Using (1)]

$\Rightarrow I + I = (a+b)\int_a^b f(x) dx$

$\Rightarrow 2I = (a+b)\int_a^b f(x) dx$

$\Rightarrow I = \left(\frac{a+b}{2}\right)\int_a^b f(x) dx$

Hence, the correct Answer is D.

44. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$

(A) 1

(B) 0

(C) –1

(D) $\frac{\pi}{4}$

44. Given: $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$

Let $I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$

$= \int_0^1 \tan^{-1}\left(\frac{x+x-1}{1+x(1-x)}\right) dx$

$$= \int_0^1 \tan^{-1}\left(\frac{x-(1-x)}{1+x(1-x)}\right)dx$$

as, $\tan^{-1}\left(\frac{A-B}{1+AB)}\right) = \tan^{-1}(A)\tan^{-1}(B)$

$$= \int_0^1 [\tan^{-1}(x) - \tan^{-1}(1-x)]dx \quad \ldots(1)$$

as, $\left\{\int_0^a f(x)dx = \int_0^a f(a-x)dx\right\}$

$$= \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(1-(1-x))]dx$$

$$= \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(x)]dx \quad \ldots(2)$$

Adding (1) and (2), we get

$$2I = \int_0^1 [\tan^{-1}(x) - \tan^{-1}(1-x)]dx + \int_0^1 [\tan^{-1}(1-x) - \tan^{-1}(x)]dx$$

$$2I = \int_0^1 [\tan^{-1}(x) - \tan^{-1}(1-x) + \tan^{-1}(1-x) - \tan^{-1}(x)]dx$$

$\Rightarrow 2I = 0$

$\Rightarrow I = 0$

Hence, correct option is (B).