Exercise 6(D)

Solution 1:

$$\frac{9}{x}-\frac{4}{y}=8 \qquad \ldots(1)$$

$$\frac{13}{x}+\frac{7}{y}=101 \qquad \ldots(2)$$

Multiplying equation no. (1) by 7 and (2) by 4.

$$\frac{63}{x}-\frac{28}{y}=56 \qquad \ldots(3)$$

$$\frac{52}{x}+\frac{28}{y}=404 \qquad \ldots(4)$$

$$\frac{115}{x}=460$$

$$x=\frac{115}{460} \Rightarrow x=\frac{1}{4}$$

From (1)

$$9\times\left(\frac{4}{1}\right)-\frac{4}{y}=8$$

$$-\frac{4}{y}=-28 \Rightarrow y=\frac{1}{7}$$

Solution 2:

$\frac{3}{x}+\frac{2}{y}=10$ ....(i)

$\frac{9}{x}-\frac{7}{y}=10.5$ ....(ii)

Multiplying equation (i) by 3, we get

$\frac{9}{x}+\frac{6}{y}=30$ ....(iii)

Subtracting (ii) from (iii), we get

$\frac{13}{y}=19.5$

$\Rightarrow y=\frac{13}{19.5}=\frac{2}{3}$

From (i),

$\frac{3}{x}+\frac{2\times3}{2}=10$

$\Rightarrow \frac{3}{x}+3=10$

$\Rightarrow \frac{3}{x}=7$

$\Rightarrow x=\frac{3}{7}$

Solution 3:

$5x+\frac{8}{y}=19$ ....(i)

$3x-\frac{4}{y}=7$ ....(ii)

Multiplying equation (ii) by 2, we get

$6x-\frac{8}{y}=14$ ....(iii)

Adding (i) and (iii), we get

$11x=33$

$\Rightarrow x=3$

Substituting $x=3$ in equation 91), we get

$5(3)+\frac{8}{y}=19$

$\Rightarrow \frac{8}{y}=19-15$

$\Rightarrow y=\frac{8}{4}=2$

Solution 4:

$4x + \frac{6}{y} = 15$ ....(i)

$3x - \frac{4}{y} = 7$ ....(ii)

Multiplying (i) by 4 and (ii) by 6

$16x + \frac{24}{y} = 60$ ....(iii)

$18x - \frac{24}{y} = 42$ ....(iv)

Adding (iii) and (iv), we get

$34x = 102$

$\Rightarrow x = 3$

Substituting $x = 3$ in (i), we get

$4(3) + \frac{6}{y} = 15$

$\Rightarrow \frac{6}{y} = 15 - 12$

$\Rightarrow y = \frac{6}{3} = 2$

Now, $y = ax - 2$

$\Rightarrow 2 = a(3) - 2$

$\Rightarrow 2 = 3a - 2$

$\Rightarrow 3a = 4$

$\Rightarrow a = \frac{4}{3} = 1\frac{1}{3}$

Solution 5:

$\frac{3}{x} - \frac{2}{y} = 0$ ...(1)

$\frac{2}{x} + \frac{5}{y} = 19$ ...(2)

Multiplying equation no. (1) by 5 and (2) by 2.

$\frac{15}{x} - \frac{10}{y} = 0$ ...(3)

$\frac{4}{x} + \frac{10}{y} = 38$ ....(4)

$\frac{19}{x} = 38 \quad \Rightarrow x = \frac{1}{2}$

From (1) $3\left(\frac{1}{2}\right) - \frac{2}{y} = 0 \quad \Rightarrow y = \frac{1}{3}$

$\therefore y = ax + 3$

$\frac{1}{3} = a\left(\frac{1}{2}\right) + 3$

$\frac{a}{2} = \frac{-8}{3} \Rightarrow a = \frac{-16}{3}$

Solution 6:

(i)

$$\frac{20}{x+y}+\frac{3}{x-y}=7 \quad ...(1)$$

$$\frac{8}{x+y}-\frac{15}{x+y}=5 \quad ...(2)$$

Multiplying equation no. (1) by 8 and (2) by 3.

$$\frac{160}{x+y}+\frac{24}{x-y}=56 \quad ...(3)$$

$$\frac{-45}{x+y}+\frac{24}{x-y}=15 \quad ...(4)$$

$$\frac{205}{x+y}=41$$

$x+y=5 \quad ...(5)$

From (1)

$$\frac{20}{5}+\frac{3}{x-y}=7$$

$$\frac{3}{x-y}=3$$

$x-y=1 \quad ...(6)$

$x+y=5 \quad ...(5)$

$x-y=1 \quad ...(6)$

$2x=6$

$x=3$

from (5)

$3+y=5 \Rightarrow y=2$

(ii)

Let $a = 3x + 4y$ and $b = 3x - 2y$

$$\therefore \frac{34}{3x+4y} + \frac{15}{3x-2y} = 5$$

$$\Rightarrow \frac{34}{a} + \frac{15}{b} = 5 \text{.....(i)}$$

$$\frac{25}{3x-2y} - \frac{8.50}{3x+4y} = 4.5$$

$$\Rightarrow -\frac{8.50}{a} + \frac{25}{b} = 4.5 \text{.....(ii)}$$

Multiply equation (ii)by 4, we get :

$$-\frac{34}{a} + \frac{100}{b} = 18$$

$$\frac{34}{a} + \frac{15}{b} = 5 \quad \text{[Equation (i)]}$$

$$+ \quad + \quad + \quad \text{[Adding]}$$

$$\frac{115}{b} = 23$$

$$\Rightarrow b = 5$$

$$\Rightarrow 3x - 2y = 5 \text{.....(iii)}$$

Substituting $b = 5$ in equation (i), we get

$$\frac{34}{a} + \frac{15}{5} = 5$$

$$\Rightarrow 2a = 34$$

$$\Rightarrow a = 17$$

$$\Rightarrow 3x + 4y = 17 \text{.....(iv)}$$

Subtracting equation (iv)from equation (iii), we get::

$$3x - 2y = 5$$

$$3x + 4y = 17$$

$$- \quad - \quad -$$

$$-6y = -12$$

$$\Rightarrow y = 2$$

SubStituting $y = 2$ in equation (iii), we get

$$3x - 2(2) = 5$$

$$\Rightarrow 3x = 9$$

$$\Rightarrow x = 3$$

$\therefore$ Solution is $x = 3$ and $y = 2$.

Solution 7:

(i)

$$x + y = 2xy \quad \ldots(1)$$

$$x - y = 6xy \quad \ldots(2)$$

$$2x = 8xy$$

$$2 = 8y$$

$$y = \frac{1}{4}$$

From (1)

$$x + \frac{1}{4} = 2x\left(\frac{1}{4}\right)$$

$$\frac{1}{2}x = \frac{-1}{4}$$

$$x = \frac{-1}{2}$$

(ii)

$x + y = 7xy$ ...(1)

$2x - 3 = -xy$ ...(2)

Multiplying equation no. (1) by 3.

$$3x + 3y = 21xy \quad \ldots(3)$$

$$2x - 3y = -xy \quad \ldots(4)$$

$$5x = 20xy$$

$$y = \frac{1}{4}$$

From (1)

$$x + \frac{1}{4} = 7x\left(\frac{1}{4}\right)$$

$$\frac{1}{4} = \frac{3}{4}x$$

$$x = \frac{1}{3}$$

Solution 8:

Given equations are $\frac{a}{x}-\frac{b}{y}=0$ and $\frac{ab^2}{x}+\frac{a^2b}{y}=a^2+b^2$

Taking $\frac{1}{x}=u$ and $\frac{1}{y}=v$, the above system of equations become

$au-bv+0=0$

$ab^2u+a^2bv-(a^2+b^2)=0$

By cross-multiplication, we have

$$\frac{u}{-b\times[-(a^2+b^2)]-a^2b\times 0}=\frac{-v}{a\times[-(a^2+b^2)]-ab^2\times 0}=\frac{1}{a\times a^2b-ab^2\times(-b)}$$

$$\Rightarrow \frac{u}{b(a^2+b^2)}=\frac{-v}{-a(a^2+b^2)}=\frac{1}{a^3b+ab^3}$$

$$\Rightarrow \frac{u}{b(a^2+b^2)}=\frac{v}{a(a^2+b^2)}=\frac{1}{ab(a^2+b^2)}$$

$$\Rightarrow u=\frac{b(a^2+b^2)}{ab(a^2+b^2)} \text{ and } v=\frac{a(a^2+b^2)}{ab(a^2+b^2)}$$

$$\Rightarrow u=\frac{1}{a} \text{ and } v=\frac{1}{b}$$

$$\Rightarrow \frac{1}{x}=\frac{1}{a} \text{ and } \frac{1}{y}=\frac{1}{b}$$

$\Rightarrow x=a$ and $y=b$

Solution 9:

$$\frac{2xy}{x+y}=\frac{3}{2}$$

$$\Rightarrow \frac{x+y}{xy}=\frac{4}{3}$$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}=\frac{4}{3} \quad ....(1)$$

$$\frac{xy}{2x-y}=-\frac{3}{10}$$

$$\Rightarrow \frac{2x-y}{xy}=-\frac{10}{3}$$

$$\Rightarrow -\frac{1}{x}+\frac{2}{y}=-\frac{10}{3} \quad ....(2)$$

Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$

Then, equations (1) and (2) become

$u+v=\frac{4}{3}$ and $-u+2v=-\frac{10}{3}$

$\Rightarrow 3u+3v=4$ and $-3u+6v=-10$

Adding, we have

$9v=-6$

$$\Rightarrow v=-\frac{6}{9}=-\frac{2}{3}$$

$$\Rightarrow \frac{1}{y}=-\frac{2}{3} \Rightarrow y=-\frac{3}{2}$$

Substituting $y=-\frac{3}{2}$ in (1), we have

$$\frac{1}{x}-\frac{2}{3}=\frac{4}{3}$$

$$\Rightarrow \frac{1}{x}=\frac{6}{3}=2$$

$$\Rightarrow x=\frac{1}{2}$$

Hence, $x=\frac{1}{2}$ and $y=-\frac{3}{2}$

Solution 10:

Given equations are $\frac{3}{2x}+\frac{2}{3y}=-\frac{1}{3}$ and $\frac{3}{4x}+\frac{1}{2y}=-\frac{1}{8}$

Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$

Then, the system of equations become

$\frac{3}{2}u+\frac{2}{3}v=-\frac{1}{3}$ and $\frac{3}{4}u+\frac{1}{2}v=-\frac{1}{8}$

$\Rightarrow \frac{9u+4v}{6}=-\frac{1}{3}$ and $\frac{3u+2v}{4}=-\frac{1}{8}$

$\Rightarrow 27u+12v=-6$ and $24u+16v=-4$

$\Rightarrow 27u+12v+6=0$ and $24u+16v+4=0$

$\Rightarrow \frac{u}{12\times 4-16\times 6}=\frac{-v}{27\times 4-24\times 6}=\frac{1}{27\times 16-24\times 12}$

$\Rightarrow \frac{u}{48-96}=\frac{-v}{108-144}=\frac{1}{432-288}$

$\Rightarrow \frac{u}{-48}=\frac{-v}{-36}=\frac{1}{144}$

$\Rightarrow \frac{u}{-48}=\frac{v}{36}=\frac{1}{144}$

$\Rightarrow u=\frac{-48}{144}=-\frac{1}{3}$ and $v=\frac{36}{144}=\frac{1}{4}$

$\Rightarrow \frac{1}{x}=-\frac{1}{3}$ and $\frac{1}{y}=\frac{1}{4}$

$\Rightarrow x=-3$ and $y=4$