EXERCISE 10.4

1. If two equal chords of a circle intersect, prove that the parts of one chord are separately equal to the parts of the other chord.

Solution:

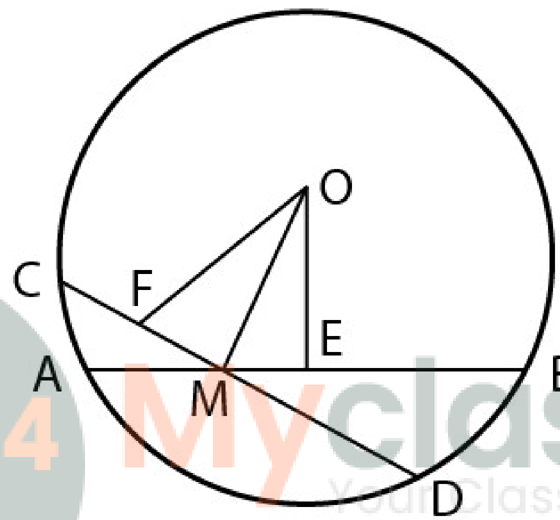
According to the question,

AB and CD are two equal chords of a circle with centre O, intersect each other at M.

To prove:

(i) MB = MC and

(ii) AM = MD



Proof:

AB is a chord and OE ⊥ to AB from the centre O,

Since, perpendicular from the centre to a chord bisect the chord

We get,

AE = ½ AB

Similarly,

FD = ½ CD

It is given that,

AB = CD⇒ ½ AB = ½ CD

So, AE = FD … (1)

Since equal chords are equidistance from the centre,

And AB = CD

So, OE = OF

Now, as proved, in right triangles MOE and MOF,

hyp. OE = hyp. OF [Common side]

OM = OM

ΔMOE ≅ ΔMOF

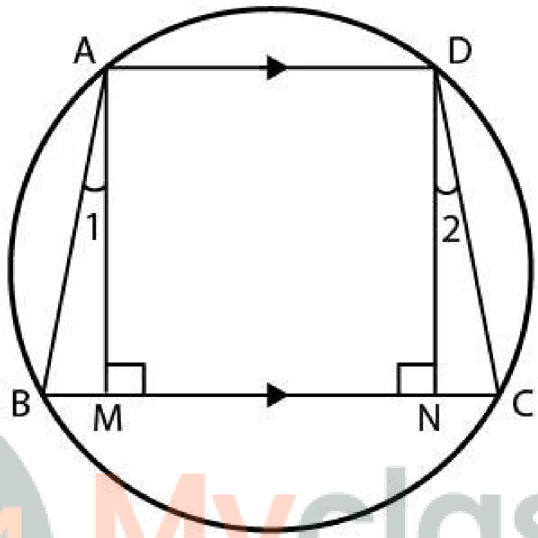
ME = MF … (2)

Subtracting equations (2) from (1), we get

AE – ME = FD – MF

⇒ AM = MD [Proved part (ii)]
Again, AB = CD [Given]
And AM = MD [Proved]
AB – AM = CD – MD [Equals subtracted from equal]
Hence, MB = MC [Proved part (i)]

2. If non-parallel sides of a trapezium are equal, prove that it is cyclic.
Solution:

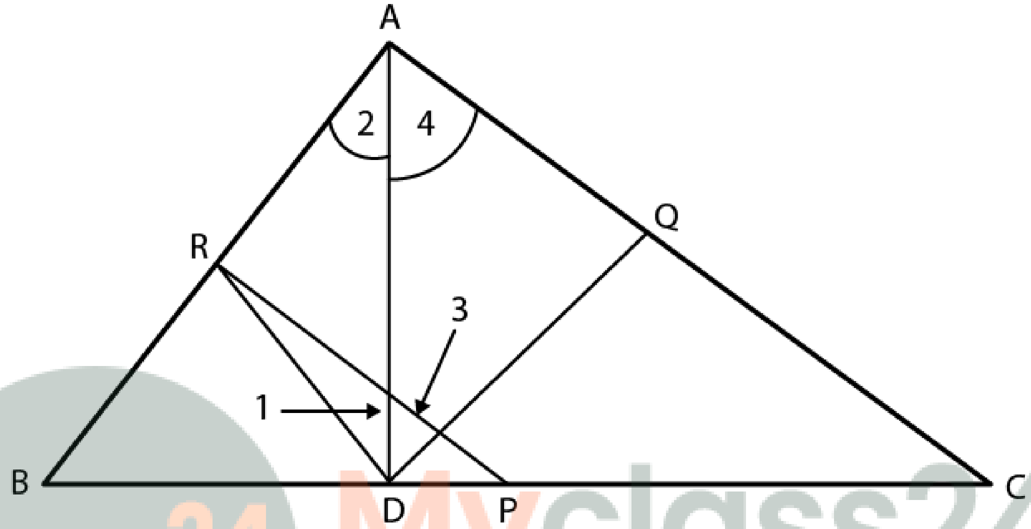


According to the question,
We have,
ABCD is a trapezium in which AD||BC
Non-parallel sides AB and DC of the trapezium ABCD are equal i.e.,
AB = DC.
To prove: Trapezium ABCD is cyclic.
Construction: Draw AM and DN such that they are perpendicular on BC.
Proof: In right triangles AMB and DNC,
$\angle AMB = \angle DNC = 90^{\circ}$
AB = DC [Given]
Since perpendicular distance between two parallel lines are same,
AM = DN
$\Delta AMB \cong \Delta DNC$ [By RHS congruence rule]
$\angle B = \angle C$ [CPCT]
And $\angle 1 = \angle 2$
$\angle BAD = \angle 1 + 90$
$= \angle 2 + 90$
$= \angle CDA$
Now, in quadrilateral ABCD
$\angle B + \angle C + \angle CDA + \angle BAD = 360$
$\angle B + \angle B + \angle CDA + \angle CDA = 360$
$2(\angle B + \angle CDA) = 360$
$\angle B + \angle CDA = 180$

We know that,
If any pair of opposite angles of a quadrilateral is 180º, then the quadrilateral is cyclic.
Hence, the trapezium ABCD is cyclic.

3. If P, Q and R are the mid-points of the sides BC, CA and AB of a triangle and AD is the perpendicular from A on BC, prove that P, Q, R and D are concyclic.
Solution:

To prove: R, D, P and Q are concyclic.

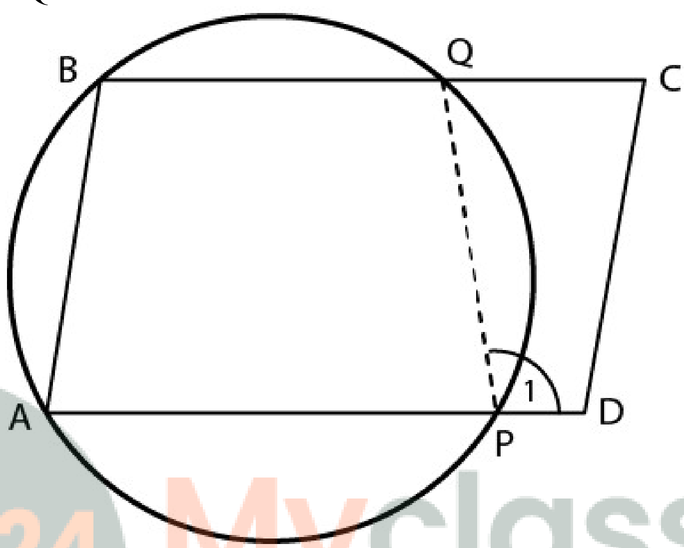


Construction: Join RD, QD, PR and PQ.
RP joins the mid-point of AB, i.e., R, and the mid-point of BC, i.e., P.
Using midpoint theorem,
RP||AC
Similarly,
PQ||AB.
So, we get,
ARPQ is a parallelogram.
So, ∠RAQ = ∠RPQ [Opposite angles of a ||gm]…(1)
ABD is a right angled triangle and DR is a median,
RA = DR and ∠1 = ∠2 …(2)
Similarly ∠3 = ∠4 …(3)
Adding equations (2) and (3),
We get,
∠1+ ∠3 = ∠2 +∠4
⇒ ∠RDQ = ∠RAQ
∠RPQ [Proved above]
Since ∠D and ∠P are subtended by RQ on the same side of it, we get the points R, D, P and Q concyclic.
Hence, R, D, P and Q are concyclic.

4. ABCD is a parallelogram. A circle through A, B is so drawn that it intersects AD at P and BC at Q. Prove that P, Q, C and D are concyclic.

Solution:

According to the question,
ABCD is a parallelogram.
A circle through A, B is so drawn that it intersects AD at P and BC at Q.
To prove: P, Q, C and D are concyclic.
Construction: Join PQ.

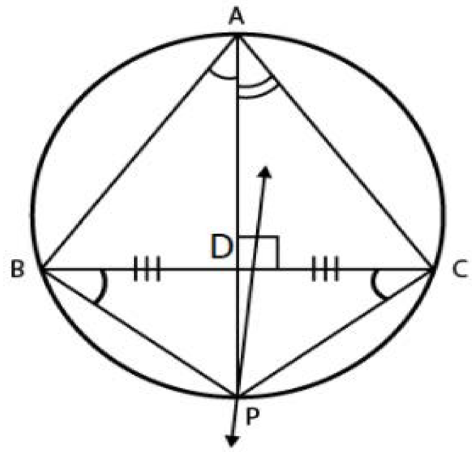


Extend side AP of the cyclic quadrilateral APQB to D.
External angle, $\angle 1$= interior opposite angle, $\angle B$
Since, BA||CD and BC cuts them
$\angle B + \angle C = 180^o$
Since, Sum of interior angles on the same side of the transversal = 180^o
Or $\angle 1 + \angle C = 180^o$
So, PDCQ is cyclic quadrilateral.
Hence, the points P, Q, C and D are concyclic.

5. Prove that angle bisector of any angle of a triangle and perpendicular bisector of the opposite side if intersect, they will intersect on the circumcircle of the triangle.

Solution:

According to the question,
Triangle ABC and *l* is perpendicular bisector of BC.



To prove:
Angles bisector of ∠A and perpendicular bisector of BC intersect on the circumcircle of ΔABC.
Proof:
Let the angle bisector of ∠A intersect circumcircle of ΔABC at D.
Construction: Join BP and CP.
Since, angles in the same segment are equal
We have, ∠BAP = ∠BCP
We know that,
AP is bisector of ∠A.
Then,
∠BAP = ∠BCP = ½ ∠A …(1)
Similarly,
We have,
∠PAC = ∠PBC = ½ ∠A …(2)
From equations (1) and (2),
We have
∠BCP = ∠PBC
We know that,
If the angles subtended by two Chords of a circle at the centre are equal, the chords are equal.
So,
BP = CP
Here, P is on perpendicular bisector of BC.
Hence, angle bisector of ∠A and perpendicular bisector of BC intersect on the circumcircle of ΔABC.