

EXERCISE 19.6

1. $\int \sin^2(2x + 5) dx$

Solution:

We know that

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

By substituting the above formula

∴ The given equation becomes,

$$\Rightarrow \int \frac{1 - \cos 2(2x+5)}{2} dx$$

We know $\int \cos ax dx = \frac{1}{a} \sin ax + c$

$$\Rightarrow \frac{1}{2} \int dx - \frac{1}{2} \int \cos(4x + 10) dx$$

On integrating we get

$$\Rightarrow \frac{x}{2} - \frac{1}{8} \sin(4x + 10) + c$$

2. $\int \sin^3(2x + 1) dx$

Solution:

We know that $\sin 3x = -4\sin^3 x + 3\sin x$

The above formula can be written as

$$\Rightarrow 4\sin^3 x = 3\sin x - \sin 3x$$

The above equation becomes

$$\Rightarrow \sin^3 x = \frac{3 \sin x - \sin 3x}{4}$$

Now applying above formula to the given question we get

$$\Rightarrow \int \sin^3(2x + 1) dx = \int \frac{3 \sin(2x+1) - \sin 3(2x+1)}{4} dx$$

We know $\int \sin ax dx = \frac{-1}{a} \cos ax + c$

By substituting the above formula we get

$$\Rightarrow \frac{3}{8} \int \sin(2x + 1) dx - \frac{1}{4} \int \sin(6x + 3) dx$$

On integrating we get

$$\Rightarrow \frac{-3}{8} \cos(2x + 1) + \frac{1}{24} \cos(6x + 3) + c$$

3. $\int \cos^4 2x dx$

Solution:

Consider,

$$\cos^4 2x = (\cos^2 2x)^2$$

We know that

$$\Rightarrow \cos^2 x = \frac{1 + \cos 2x}{2}$$

The above equation

$$\Rightarrow (\cos^2 2x)^2 = \left(\frac{1 + \cos 4x}{2} \right)^2$$

$$\Rightarrow \left(\frac{1 + \cos 4x}{2} \right)^2 = \left(\frac{1 + 2 \cos 4x + \cos^2 4x}{4} \right)$$

$$\Rightarrow \cos^2 4x = \frac{1 + \cos 8x}{2}$$

$$\Rightarrow \left(\frac{1 + 2 \cos 4x + \cos^2 4x}{4} \right) = \frac{1}{4} + \frac{\cos 4x}{2} + \frac{1 + \cos 8x}{8}$$

Now the question becomes,



$$\Rightarrow \frac{1}{4} \int dx + \frac{1}{2} \int \cos 4x dx + \frac{1}{8} \int dx + \frac{1}{8} \int \cos 8x dx$$

We know $\int \cos ax dx = \frac{1}{a} \sin ax + c$

$$\Rightarrow \frac{x}{4} + \frac{1}{8} \sin 4x + \frac{x}{8} + \frac{\sin 8x}{64} + c$$

$$\Rightarrow \frac{24x + 8\sin 4x + \sin 8x}{64} + c$$

4. $\int \sin^2 bx dx$

Solution:

We know that

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

By substituting this formula,

\therefore The given equation becomes,

$$\Rightarrow \int \frac{1 - \cos 2bx}{2} dx$$

We know $\int \cos ax dx = \frac{1}{a} \sin ax + c$

$$\Rightarrow \frac{1}{2} \int dx - \frac{1}{2} \int \cos(2bx) dx$$

On integration

$$\Rightarrow \frac{x}{2} - \frac{1}{4b} \sin(2bx) + c$$