

Exercise 4.2

1. In a ΔABC , D and E are points on the sides AB and AC respectively such that $DE \parallel BC$.

i) If $AD = 6$ cm, $DB = 9$ cm and $AE = 8$ cm, Find AC.

Solution:

Given: ΔABC , $DE \parallel BC$, $AD = 6$ cm, $DB = 9$ cm and $AE = 8$ cm.

Required to find AC.

By using Thales Theorem, [As $DE \parallel BC$]

$$AD/BD = AE/CE$$

Let $CE = x$.

So then,

$$6/9 = 8/x$$

$$6x = 72 \text{ cm}$$

$$x = 72/6 \text{ cm}$$

$$x = 12 \text{ cm}$$

$$\therefore AC = AE + CE = 12 + 8 = 20.$$

ii) If $AD/DB = 3/4$ and $AC = 15$ cm, Find AE.

Solution:

Given: $AD/DB = 3/4$ and $AC = 15$ cm [As $DE \parallel BC$]

Required to find AE.

By using Thales Theorem, [As $DE \parallel BC$]

$$AD/DB = AE/CE$$

Let, $AE = x$, then $CE = 15 - x$.

$$\Rightarrow \frac{3}{4} = \frac{x}{(15-x)}$$

$$45 - 3x = 4x$$

$$-3x - 4x = -45$$

$$7x = 45$$

$$x = 45/7$$

$$x = 6.43 \text{ cm}$$

$$\therefore AE = 6.43 \text{ cm}$$

iii) If $AD/DB = 2/3$ and $AC = 18$ cm, Find AE.

Solution:

Given: $AD/DB = 2/3$ and $AC = 18$ cm

Required to find AE.

By using Thales Theorem, [As $DE \parallel BC$]

$$AD/DB = AE/CE$$

Let, $AE = x$ and $CE = 18 - x$

$$\Rightarrow 23 = x / (18-x)$$

$$\begin{aligned}3x &= 36 - 2x \\5x &= 36 \text{ cm} \\x &= 36/5 \text{ cm} \\x &= 7.2 \text{ cm} \\\therefore \text{AE} &= 7.2 \text{ cm}\end{aligned}$$

iv) If AD = 4 cm, AE = 8 cm, DB = x - 4 cm and EC = 3x - 19, find x.

Solution:

Given: AD = 4 cm, AE = 8 cm, DB = x - 4 and EC = 3x - 19
Required to find x.

By using Thales Theorem, [As DE \parallel BC]

$$\text{AD/BD} = \text{AE/CE}$$

Then, $4/(x - 4) = 8/(3x - 19)$

$$4(3x - 19) = 8(x - 4)$$

$$12x - 76 = 8(x - 4)$$

$$12x - 8x = -32 + 76$$

$$4x = 44 \text{ cm}$$

$$x = 11 \text{ cm}$$

v) If AD = 8 cm, AB = 12 cm and AE = 12 cm, find CE.

Solution:

Given: AD = 8 cm, AB = 12 cm, and AE = 12 cm.

Required to find CE,

By using Thales Theorem, [As DE \parallel BC]

$$\text{AD/BD} = \text{AE/CE}$$

$$8/4 = 12/\text{CE}$$

$$8 \times \text{CE} = 4 \times 12 \text{ cm}$$

$$\text{CE} = (4 \times 12)/8 \text{ cm}$$

$$\text{CE} = 48/8 \text{ cm}$$

$$\therefore \text{CE} = 6 \text{ cm}$$

vi) If AD = 4 cm, DB = 4.5 cm and AE = 8 cm, find AC.

Solution:

Given: AD = 4 cm, DB = 4.5 cm, AE = 8 cm

Required to find AC.

By using Thales Theorem, [As DE \parallel BC]

$$\text{AD/BD} = \text{AE/CE}$$

$$4/4.5 = 8/\text{AC}$$

$$\text{AC} = (4.5 \times 8)/4 \text{ cm}$$

$$\therefore \text{AC} = 9 \text{ cm}$$

vii) If AD = 2 cm, AB = 6 cm and AC = 9 cm, find AE.

Solution:

Given: AD = 2 cm, AB = 6 cm and AC = 9 cm

Required to find AE.

$$DB = AB - AD = 6 - 2 = 4 \text{ cm}$$

By using Thales Theorem, [As DE || BC]

$$AD/BD = AE/CE$$

$$2/4 = x/(9-x)$$

$$4x = 18 - 2x$$

$$6x = 18$$

$$x = 3 \text{ cm}$$

$$\therefore AE = 3 \text{ cm}$$

viii) If AD/BD = 4/5 and EC = 2.5 cm, Find AE.

Solution:

Given: AD/BD = 4/5 and EC = 2.5 cm

Required to find AE.

By using Thales Theorem, [As DE || BC]

$$AD/BD = AE/CE$$

$$\text{Then, } 4/5 = AE/2.5$$

$$\therefore AE = 4 \times 2.55 = 2 \text{ cm}$$

ix) If AD = x cm, DB = x - 2 cm, AE = x + 2 cm, and EC = x - 1 cm, find the value of x.

Solution:

Given: AD = x, DB = x - 2, AE = x + 2 and EC = x - 1

Required to find the value of x.

By using Thales Theorem, [As DE || BC]

$$AD/BD = AE/CE$$

$$\text{So, } x/(x-2) = (x+2)/(x-1)$$

$$x(x-1) = (x-2)(x+2)$$

$$x^2 - x - x^2 + 4 = 0$$

$$x = 4$$

x) If AD = 8x - 7 cm, DB = 5x - 3 cm, AE = 4x - 3 cm, and EC = (3x - 1) cm, Find the value of x.

Solution:

Given: AD = 8x - 7, DB = 5x - 3, AER = 4x - 3 and EC = 3x - 1

Required to find x.

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By using Thales Theorem, [As DE \parallel BC]

$$AD/BD = AE/CE$$

$$(8x-7)/(5x-3) = (4x-3)/(3x-1)$$

$$(8x-7)(3x-1) = (5x-3)(4x-3)$$

$$24x^2 - 29x + 7 = 20x^2 - 27x + 9$$

$$4x^2 - 2x - 2 = 0$$

$$2(2x^2 - x - 1) = 0$$

$$2x^2 - x - 1 = 0$$

$$2x^2 - 2x + x - 1 = 0$$

$$2x(x-1) + 1(x-1) = 0$$

$$(x-1)(2x+1) = 0$$

$$\Rightarrow x = 1 \text{ or } x = -1/2$$

We know that the side of triangle can never be negative. Therefore, we take the positive value.

$$\therefore x = 1.$$

xi) If AD = 4x - 3, AE = 8x - 7, BD = 3x - 1, and CE = 5x - 3, find the value of x.

Solution:

Given: AD = 4x - 3, BD = 3x - 1, AE = 8x - 7 and EC = 5x - 3

Required to find x.

By using Thales Theorem, [As DE \parallel BC]

$$AD/BD = AE/CE$$

$$\text{So, } (4x-3)/(3x-1) = (8x-7)/(5x-3)$$

$$(4x-3)(5x-3) = (3x-1)(8x-7)$$

$$4x(5x-3) - 3(5x-3) = 3x(8x-7) - 1(8x-7)$$

$$20x^2 - 12x - 15x + 9 = 24x^2 - 29x + 7$$

$$20x^2 - 27x + 9 = 24x^2 - 29x + 7$$

$$\Rightarrow -4x^2 + 2x + 2 = 0$$

$$4x^2 - 2x - 2 = 0$$

$$4x^2 - 4x + 2x - 2 = 0$$

$$4x(x-1) + 2(x-1) = 0$$

$$(4x+2)(x-1) = 0$$

$$\Rightarrow x = 1 \text{ or } x = -2/4$$

We know that the side of triangle can never be negative. Therefore, we take the positive value.

$$\therefore x = 1$$

xii) If AD = 2.5 cm, BD = 3.0 cm, and AE = 3.75 cm, find the length of AC.

Solution:

Given: AD = 2.5 cm, AE = 3.75 cm and BD = 3 cm

Required to find AC.

By using Thales Theorem, [As DE \parallel BC]

$$AD/BD = AE/CE$$

$$\begin{aligned}2.5/3 &= 3.75/CE \\2.5 \times CE &= 3.75 \times 3 \\CE &= 3.75 \times 32.5 \\CE &= 11.252.5 \\CE &= 4.5\end{aligned}$$

$$\begin{aligned}\text{Now, } AC &= 3.75 + 4.5 \\ \therefore AC &= 8.25 \text{ cm.}\end{aligned}$$

2. In a $\triangle ABC$, D and E are points on the sides AB and AC respectively. For each of the following cases show that $DE \parallel BC$:

i) $AB = 12$ cm, $AD = 8$ cm, $AE = 12$ cm, and $AC = 18$ cm.

Solution:

Required to prove $DE \parallel BC$.

We have,

$AB = 12$ cm, $AD = 8$ cm, $AE = 12$ cm, and $AC = 18$ cm. (Given)

So,

$$BD = AB - AD = 12 - 8 = 4 \text{ cm}$$

And,

$$CE = AC - AE = 18 - 12 = 6 \text{ cm}$$

It's seen that,

$$AD/BD = 8/4 = 1/2$$

$$AE/CE = 12/6 = 1/2$$

Thus,

$$AD/BD = AE/CE$$

So, by the converse of Thale's Theorem

We have,

$$DE \parallel BC.$$

Hence Proved.

ii) $AB = 5.6$ cm, $AD = 1.4$ cm, $AC = 7.2$ cm, and $AE = 1.8$ cm.

Solution:

Required to prove $DE \parallel BC$.

We have,

$AB = 5.6$ cm, $AD = 1.4$ cm, $AC = 7.2$ cm, and $AE = 1.8$ cm. (Given)

So,

$$BD = AB - AD = 5.6 - 1.4 = 4.2 \text{ cm}$$

And,

$$CE = AC - AE = 7.2 - 1.8 = 5.4 \text{ cm}$$

It's seen that,

$$AD/BD = 1.4/4.2 = 1/3$$

$$AE/CE = 1.8/5.4 = 1/3$$

Thus,

$$AD/BD = AE/CE$$

So, by the converse of Thale's Theorem

We have,

$$DE \parallel BC.$$

Hence Proved.

iii) AB = 10.8 cm, BD = 4.5 cm, AC = 4.8 cm, and AE = 2.8 cm.

Solution:

Required to prove $DE \parallel BC$.

We have

$$AB = 10.8 \text{ cm, } BD = 4.5 \text{ cm, } AC = 4.8 \text{ cm, and } AE = 2.8 \text{ cm.}$$

So,

$$AD = AB - DB = 10.8 - 4.5 = 6.3$$

And,

$$CE = AC - AE = 4.8 - 2.8 = 2$$

It's seen that,

$$AD/BD = 6.3/4.5 = 2.8/2.0 = AE/CE = 7/5$$

So, by the converse of Thale's Theorem

We have,

$$DE \parallel BC.$$

Hence Proved.

iv) AD = 5.7 cm, BD = 9.5 cm, AE = 3.3 cm, and EC = 5.5 cm.

Solution:

Required to prove $DE \parallel BC$.

We have

$$AD = 5.7 \text{ cm, } BD = 9.5 \text{ cm, } AE = 3.3 \text{ cm, and } EC = 5.5 \text{ cm}$$

Now,

$$AD/BD = 5.7/9.5 = 3/5$$

And,

$$AE/CE = 3.3/5.5 = 3/5$$

Thus,

$$AD/BD = AE/CE$$

So, by the converse of Thale's Theorem

We have,

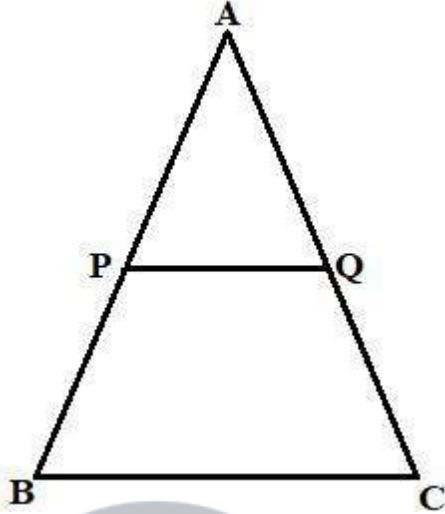
$$DE \parallel BC.$$

Hence Proved.

3. In a ΔABC , P and Q are the points on sides AB and AC respectively, such that $PQ \parallel BC$. If $AP = 2.4 \text{ cm}$, $AQ = 2 \text{ cm}$, $QC = 3 \text{ cm}$ and $BC = 6 \text{ cm}$. Find AB and PQ.

Solution:

Given: ΔABC , $AP = 2.4$ cm, $AQ = 2$ cm, $QC = 3$ cm, and $BC = 6$ cm. Also, $PQ \parallel BC$.
Required to find: AB and PQ .



By using Thales Theorem, we have [As it's given that $PQ \parallel BC$]

$$\begin{aligned} AP/PB &= AQ/QC \\ 2.4/PB &= 2/3 \\ 2 \times PB &= 2.4 \times 3 \\ PB &= (2.4 \times 3)/2 \text{ cm} \\ \Rightarrow PB &= 3.6 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Now finding, } AB &= AP + PB \\ AB &= 2.4 + 3.6 \\ \Rightarrow AB &= 6 \text{ cm} \end{aligned}$$

Now, considering ΔAPQ and ΔABC

We have,

$$\angle A = \angle A$$

$$\angle APQ = \angle ABC \text{ (Corresponding angles are equal, } PQ \parallel BC \text{ and } AB \text{ being a transversal)}$$

Thus, ΔAPQ and ΔABC are similar to each other by AA criteria.

Now, we know that

Corresponding parts of similar triangles are proportional.

$$\begin{aligned} \Rightarrow AP/AB &= PQ/BC \\ \Rightarrow PQ &= (AP/AB) \times BC \\ &= (2.4/6) \times 6 = 2.4 \end{aligned}$$

$$\therefore PQ = 2.4 \text{ cm.}$$

4. In a ΔABC , D and E are points on AB and AC respectively, such that $DE \parallel BC$. If $AD = 2.4$ cm, $AE = 3.2$ cm, $DE = 2$ cm and $BC = 5$ cm. Find BD and CE .

Solution:

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Given: ΔABC such that $AD = 2.4$ cm, $AE = 3.2$ cm, $DE = 2$ cm and $BE = 5$ cm. Also $DE \parallel BC$.
Required to find: BD and CE .

As $DE \parallel BC$, AB is transversal,

$$\angle APQ = \angle ABC \text{ (corresponding angles)}$$

As $DE \parallel BC$, AC is transversal,

$$\angle AED = \angle ACB \text{ (corresponding angles)}$$

In ΔADE and ΔABC ,

$$\angle ADE = \angle ABC$$

$$\angle AED = \angle ACB$$

$$\therefore \Delta ADE = \Delta ABC \text{ (AA similarity criteria)}$$

Now, we know that

Corresponding parts of similar triangles are proportional.

$$\Rightarrow \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$$

$$\frac{AD}{AB} = \frac{DE}{BC}$$

$$\frac{2.4}{2.4 + DB} = \frac{2}{5} \text{ [Since, } AB = AD + DB]$$

$$2.4 + DB = 6$$

$$DB = 6 - 2.4$$

$$DB = 3.6 \text{ cm}$$

In the same way,

$$\Rightarrow \frac{AE}{AC} = \frac{DE}{BC}$$

$$\frac{3.2}{3.2 + EC} = \frac{2}{5} \text{ [Since } AC = AE + EC]$$

$$3.2 + EC = 8$$

$$EC = 8 - 3.2$$

$$EC = 4.8 \text{ cm}$$

$\therefore BD = 3.6$ cm and $CE = 4.8$ cm.