NCERT Solutions for Class XII Maths

Chapter-7.9

NCERT Maths Class 12

1. $\int_{-1}^{1}(x+1)dx$

1. Let $I=\int_{-1}^{1}(x+1)dx$

$\int(x+1)dx=\frac{x^2}{2}+x=F(x)$

By second fundamental theorem of calculus, we obtain

$I=F(1)-F(-1)$

$=\left(\frac{1}{2}+1\right)-\left(\frac{1}{2}-1\right)$

$=\frac{1}{2}+1-\frac{1}{2}+1$

$=2$

2. Evaluate $\int_{2}^{3}\frac{1}{x}dx$

2. Let $I=\int_{2}^{3}\frac{1}{x}dx$

$\Rightarrow I=\int_{2}^{3}\frac{1}{x}dx$ $\quad [\int\frac{1}{x}dx=\log x]$

$\Rightarrow I=\left[\log|x|\right]_2^3$

$\Rightarrow I = \log|3| - \log|2|$

$\Rightarrow I = \log 3/2$

$\therefore \int_{2}^{3}\frac{1}{x}dx=\log\frac{3}{2}$

3. $\int_{1}^{2}\left(4x^3-5x^2+6x+9\right)dx$

3. $\int\left(4x^3-5x^2+6x+9\right)dx=4\left(\frac{x^4}{4}\right)-5\left(\frac{x^3}{3}\right)+6\left(\frac{x^2}{2}\right)+9(x)$

$=x^4-\frac{5x^3}{3}+3x^2+9x=F(x)$

By second fundamental theorem of calculus, we obtain

$$I = F(2) - F(1)$$

$$I = \left\{2^4 - \frac{5\cdot(2)^3}{3} + 3(2^2 + 9(2)\right\} - \left\{(1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1)\right\}$$

$$= \left(16 - \frac{40}{3} + 12 + 18\right) - \left(1 - \frac{5}{3} + 3 + 9\right)$$

$$= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9$$

$$= 33 - \frac{35}{3}$$

$$= \frac{99 - 35}{3}$$

$$= \frac{64}{3}$$

4. Evaluate $\int_0^{\frac{\pi}{4}} \sin 2x\,dx$

4. Let $I = \int_0^{\frac{\pi}{4}} \sin 2x\,dx$

$$\Rightarrow I = \int_0^{\frac{\pi}{4}} \sin 2x\,dx$$

$$\Rightarrow I = \left[-\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{4}} \qquad \left[\int \sin x\ dx = -\cos x\right]$$

$$\Rightarrow I = -\left(\cos 2\times\pi/4 - \cos 0\right)/2$$

$$\Rightarrow I = -\left(\cos \pi/2 - \cos 0\right)/2 = -\left(0 - 1\right)/2$$

$$\Rightarrow I = ½$$

$$\therefore \int_0^{\frac{\pi}{4}} \sin 2x\,dx = ½$$

5. $\int_0^{\frac{x}{2}} \cos 2x\,dx$

5. Let $I = \int_0^{\frac{\pi}{2}} \cos 2x\,dx$

$$\int \cos 2x dx = \left(\frac{\sin 2x}{2}\right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{2}\right) - F(0)$$

$$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right]$$

$$= \frac{1}{2}[\sin \pi - \sin 0]$$

$$= \frac{1}{2}[0 - 0] = 0$$

6. $\int_4^5 e^x dx$

6. Let $I = \int_4^5 e^x dx$

$$\int e^x dx = e^x = F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(5) - F(4)$

$= e^5 - e^4$

$= e^4 (e - 1)$

7. $\int_0^{\frac{x}{4}} \tan x \, dx$

7. Let $I = \int_0^{\frac{\pi}{4}} \tan x \, dx$

$$\int \tan x \, dx = -\log|\cos x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\log\left|\cos\frac{\pi}{4}\right| + \log|\cos 0|$$

$$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$$

$$= -\log(2)^{-\frac{1}{2}}$$

$$= \frac{1}{2}\log 2$$

8. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

8. Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$

$$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x \, dx$$

$$\Rightarrow I = \left[\log|\operatorname{cosec} x - \cot x|\right]_{\pi/6}^{\pi/4}$$

$[\int \text{cosec } x dx = \log|\text{cosec } x - \cot x| + c]$

$\Rightarrow I = \log|\text{cosec } \pi/4 - \cot \pi/4| - \log|\text{cosec } \pi/6 - \cot \pi/6|$

$\Rightarrow I = \log|\sqrt{2} - 1| - \log|2 - \sqrt{3}|$

$\Rightarrow I = \log\left|\frac{\sqrt{2}-1}{2-\sqrt{3}}\right|$

$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \text{cosec } x \, dx = \log\left(\frac{\sqrt{2}-1}{2-\sqrt{3}}\right)$

9. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

9. Let $I = \int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x = F(x)$

By second fundament theorem of calculus, we obtain

$I = F(1) - (0)$

$= \sin^{-1}(1) - \sin^{-1}(0)$

$= \frac{\pi}{2} - 0$

$= \frac{\pi}{2}$

10. Evaluate $\int_0^1 \frac{dx}{1+x^2}$

10. Let $I = \int_0^1 \frac{dx}{1+x^2}$

$\Rightarrow I = \int_0^1 \frac{dx}{1+x^2}$ $\left[\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\right]$

$\Rightarrow I = \left[\tan^{-1} x\right]_0^1$

$\Rightarrow I = \tan^{-1}(1) - \tan^{-1}(0) = \pi/4 - 0$

$\Rightarrow I = \pi/4$

$\therefore \int_0^1 \frac{dx}{1+x^2} = \pi/4$

11. $\int_2^3 \frac{dx}{x^2-1}$

11. Let $I = \int_2^3 \frac{dx}{x^2-1}$

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| = F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(3) - F(2)$

$$= \frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right| - \log\left|\frac{2-1}{2+1}\right|\right]$$

$$= \frac{1}{2}\left[\log\left|\frac{2}{4}\right| - \log\left|\frac{1}{3}\right|\right]$$

$$= \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{1}{3}\right]$$

$$= \frac{1}{2}\left[\log\frac{3}{2}\right]$$

12. Evaluate $\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$

12. Let $I = \int_0^{\frac{\pi}{2}} \cos^2 x\, dx$

$\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \dfrac{1+\cos 2x}{2}$

putting the value $\cos^2 x$ in I

$$\Rightarrow I = \int_0^{\pi/2} \frac{1+\cos 2x}{2} dx = \frac{1}{2}\int_0^{\pi/2} dx + \frac{1}{2}\int_0^{\pi/2} \cos 2x\, dx \quad \left[\int \cos x dx = \sin x + c\right]$$

$$\Rightarrow I = \frac{1}{2}[x]_0^{\pi/2} + \frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{1}{2}\left(\frac{\pi}{2} - 0\right) + \frac{1}{4}\left(\sin 2\times\frac{\pi}{2} - \sin 2\times 0\right)$$

$$\Rightarrow I = \frac{\pi}{4} + \frac{1}{4}(0-0) = \pi/4$$

$$\therefore \int_0^{\frac{\pi}{2}} \cos^2 x\, dx = \pi/4$$

13. $\int_2^3 \frac{x dx}{x^2+1}$

13. Let $I = \int_2^3 \frac{x}{x^2+1} dx$

$$\int \frac{x}{x^2+1} dx = \frac{1}{2}\int \frac{2x}{x^2+1} dx = \frac{1}{2}\log(1+x^2) = F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(3) - F(2)$

$$= \frac{1}{2}\left[\log\left(1+(3)^2\right) - \log\left(1+(2)^2\right)\right]$$

$= \frac{1}{2}[\log(10) - \log(5)]$

$= \frac{1}{2} \text{lpg}\left(\frac{10}{5}\right) = \frac{1}{2}\log 2$

14. Evaluate $\int_0^1 \frac{2x+3}{5x^2+1} dx$

14. Let $I = \int_0^1 \frac{2x+3}{5x^2+1}$

Multiplying by 5 in numerator and denominator

$\Rightarrow I = \frac{1}{5}\int_0^1 \frac{5(2x+3)}{5x^2+1} dx = \frac{1}{5}\int_0^1 \frac{10x+15}{5x^2+1} dx$

$\Rightarrow I = \frac{1}{5}\int_0^1 \frac{10x}{5x^2+1} dx + 3\int_0^1 \frac{1}{5x^2+1}$

$\Rightarrow I = I_1 + I_2$

$I1 = \frac{1}{5}\int_0^1 \frac{10x}{5x^2+1} dx$

Let $5x^2 + 1 = t$ ……………………….(i)

$d(5x^2 + 1) = dt$

$10x \, dx = dt$ ……………………...(ii)

When $x = 0;\ t = 5 \times 0^2 + 1 = 1$

When $x = 1;\ t = 5 \times 1^2 + 1 = 6$

Substituting (i) and (ii) in I_1

$I_1 = \frac{1}{5}\int_1^6 \frac{dt}{t} = \frac{1}{5}\left[\log|t|\right]_1^6$ $\left[\int \frac{1}{x} dx = \log x\right]$

$I_1 = \frac{1}{5}(\log|6| - \log|1|) = \frac{1}{5}(\log 6 - 0)$

$I_1 = \frac{1}{5}\int_0^1 \frac{10x}{5x^2+1} dx = \frac{\log 6}{5}$

$I_2 = 3\int_0^1 \frac{1}{5x^2+1} dx = \frac{3}{5}\int_0^1 \frac{1}{x^2 + \frac{1}{5}} dx$

$\left[\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c\right]$

$I_2 = \frac{3}{5} \times \frac{1}{\frac{1}{\sqrt{5}}}\left[\tan^{-1}\sqrt{5}x\right]_0^1 = \frac{3}{5} \times \sqrt{5}(\tan^{-1}\sqrt{5} - \tan^{-1}0)$

$I_2 = 3/\sqrt{5}\tan^{-1}5$

$\because I = I_1 + I_2$

$\therefore I = 1/5 \log 6 + 3/\sqrt{5} \tan^{-1} 5$

$\therefore \int_0^1 \frac{2x+3}{5x^2+1} dx = 1/5 \log 6 + 3/\sqrt{5} \tan^{-1} 5$

15. $\int_0^1 xe^{x^2} dx$

15. Let $I = \int_0^1 xe^{x^3} dx$

Put $x^2 = t \Rightarrow 2x\, dx = dt$

As $x \to 0, t \to 0$ and as $x \to 0$ and as $x \to 1, t \to 1$,

$\therefore I = \frac{1}{2}\int_0^1 e^t dt$

$\frac{1}{2}\int e^t dt = \frac{1}{2}e^t = F(t)$

By second fundamental theorem of calculus, we obtain

$I = F(1) - F(0)$

$= \frac{1}{2}e - \frac{1}{2}e^0$

$= \frac{1}{2}(e-1)$

16. Evaluate $\int_1^2 \frac{5x^2}{x^2+4x+3}$

16. Let $I = \int_1^2 \frac{5x^2}{x^2+4x+3}$

Dividing $5x^2$ by $x^2 + 4x + 3$ we get 5 as quotient and $-(20x + 15)$ as remainder

$\text{So, } I = \int_1^2 \left(5 - \frac{20x+15}{x^2+4x+3}\right) dx$

$\Rightarrow I = \int_1^2 5dx - \int_1^2 \frac{20x+15}{x^2+4x+3} = 5[x]_1^2 - \int_1^2 \frac{20x+15}{x^2+4x+3}$

$\Rightarrow I = 5(2-1) - \int_1^2 \frac{20x+15}{x^2+4x+3}$

$\Rightarrow I = 5 - I_1$

$I_1 = \int_1^2 \frac{20x+15}{x^2+4x+3}$

Adding and subtracting 25 in the numerator

$I_1 = \int_1^2 \frac{20x+15+25-25}{x^2+4x+3} dx = \int_1^2 \frac{20x+40}{x^2+4x+3} dx - \int_1^2 \frac{25}{x^2+4x+3} dx$

$I_1 = 10\int_1^2 \frac{2x+4}{x^2+4x+3} dx - 25\int_1^2 \frac{1}{x^2+4x+3} dx$

Let $x^2 + 4x + 3 = t$

$(2x + 4)dx = dt$

$$\Rightarrow I_1 = 10\int \frac{dt}{t} - 25\int \frac{1}{x^2+4x+3+1-1}dx = 10\log t + 25\int \frac{1}{x^2+4x+4-1}dx$$

$$I_1 = 10\log t - 25\int \frac{1}{(x+2)^2 - 1^2}dx \qquad \left[\int \frac{1}{x}dx = \log x\right]$$

$$I_1 = 10\log\ t - 25\left[\frac{1}{2}\log\left(\frac{x+2-1}{x+2+1}\right)\right] \qquad \left[\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\frac{x-a}{x+a} + c\right]$$

$$I_1 = 10\left[\log\left(x^2+4x+3\right)\right]_1^2 - \frac{25}{2}\left[\log\left(\frac{x+1}{x+3}\right)\right]_1^2$$

$$I_1 = 10\left[\log\left(2^2+4\times 2+3\right) - \log\left(1^2+4\times 1+3\right)\right] - \frac{25}{2}\left[\log\left(\frac{2+1}{2+3}\right) - \log\left(\frac{1+1}{1+3}\right)\right]$$

$$I_1 = 10\left[\log 15 - \log 8\right] - \frac{25}{2}\left[\log\frac{3}{5} - \log\frac{2}{4}\right]$$

$$I_1 = 10\left[\log(5\times 3) - \log(4\times 2)\right] - \frac{25}{2}\left[\log 3 - \log 5 - \log 2 + \log 4\right]$$

$$I_1 = 10\log 5 + 10\log 3 - 10\log 4 - 10\log 2 - \frac{25}{2}\log 3 + \frac{25}{2}\log 5 + \frac{25}{2}\log 2 - \frac{25}{2}\log 4$$

$$I_1 = \left(10+\frac{25}{2}\right)\log 5 - \left(10+\frac{25}{2}\right)\log 4 + \left(10-\frac{25}{2}\right)\log 3 + \left(-10+\frac{25}{2}\right)\log 2$$

$$I_1 = \frac{45}{2}\log 5 - \frac{45}{2}\log 4 - \frac{5}{2}\log 3 + \frac{5}{2}\log 2 = \frac{45}{2}\log\frac{5}{4} - \frac{5}{2}\log\frac{3}{2}$$

$\because I = 5 - I_1$

Substituting I_1 in I we get

$$I = 5 - \frac{45}{2}\log\frac{5}{4} - \frac{5}{2}\log\frac{3}{2}$$

$$\therefore \int_1^2 \frac{5x^2}{x^2+4x+3} = 5 - \frac{45}{2}\log\frac{5}{4} - \frac{5}{2}\log\frac{3}{2}$$

17. $\int_0^{\frac{x}{4}}\left(2\sec^2 x + x^3 + 2\right)dx$

17. Let $I = \int_0^{\frac{\pi}{4}}(2\sec^2 x + x^3 + 2)dx$

$$\int\left(2\sec^2 x + x^3 + 2\right)dx = 2\tan x + \frac{x^4}{4} + 2x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= \left\{\left(2\tan\frac{\pi}{4} + \frac{1}{4} + \left(\frac{\pi}{4}\right)^4 + 2\left(\frac{\pi}{4}\right)\right) - (2\tan 0 + 0 + 0)\right\}$$

$$= 2\tan\frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2}$$

$$= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$$

18. Evaluate $\int_0^{\pi}(\sin^2\frac{x}{2} - \cos^2\frac{x}{2})dx$

18. Let I = $\int_0^{\pi}(\sin^2\frac{x}{2} - \cos^2\frac{x}{2})dx$

$$\text{We know}, \cos x = \sin^2\frac{x}{2} - \cos^2\frac{x}{2}$$

$$\text{Substituting } \cos x = \sin^2\frac{x}{2} - \cos^2\frac{x}{2} \text{ in I, we get}$$

$$\Rightarrow I = \int_0^{\pi}\cos x dx = [\sin x]_0^{\pi} \qquad \left[\int \cos x dx = \sin x + c\right]$$

$$\Rightarrow I = \sin\pi - \sin 0 = 0 - 0 = 0$$

$$\therefore \int_0^{\pi}(\sin^2\frac{x}{2} - \cos^2\frac{x}{2})dx = 0$$

19. $\int_0^2 \frac{6x+3}{x^2+4}dx$

19. Let $I = \int_0^2 \frac{6x+3}{x^2+4}dx$

$$\int \frac{6x+3}{x^2+4}dx = 3\int \frac{2x+1}{x^2+4}dx$$

$$= 3\int \frac{2x}{x^2+4}dx + 3\int \frac{1}{x^2+4}dx$$

$$= 3\log(x^2+4) + \frac{3}{2}\tan^{-1}\frac{x}{2} = F(x)$$

By the fundamental theorem of calculus, we obtain

$$I = F(2) - F(0)$$

$$= \left\{3\log\left(2^2+4\right) + \frac{3}{2}\tan^{-1}\left(\frac{2}{3}\right)\right\} - \left\{3\log(0+4) + \frac{3}{2}\tan^{-1}\left(\frac{0}{2}\right)\right\}$$

$$= 3\log 8 + \frac{3}{2}\tan^{-1}1 - 3\log 4 - \frac{3}{2}\tan^{-1}0$$

$$3\log 8 + \frac{3}{2}\left(\frac{\pi}{4}\right) - 3\log 4 - 0$$

$$= 3\log\left(\frac{8}{4}\right) + \frac{3\pi}{8}$$

$$= 3\log 2 + \frac{3\pi}{8}$$

20. $\int_0^1\left(xe^x+\sin\frac{\pi x}{4}\right)dx$

20. $Let\ I=\int_0^1(xe^x+\sin\frac{\pi x}{4})dx$

$\Rightarrow I=\int_0^1 xe^x dx+\int_0^1\sin\frac{\pi x}{4}dx$

$I = I_1 + I_2$

$I_1=x\int e^x dx-\int\left\{\left(\frac{d}{dx}x\right)\int e^x dx\right\}dx$

$\Rightarrow I_1=xe^x-\int e^x dx$ $\qquad\left[\int e^x dx=e^x+c\right]$

$\Rightarrow I_1=\left[xe^x-e^x\right]_0^1=\left[\left(1\times e^1-e^1\right)-\left(0\times e^0-e^0\right)\right]$

$\Rightarrow I_1 = e - e - 0 + 1$

$\Rightarrow I_1 = 1$

$I_2=\int_0^1\sin\frac{\pi x}{4}dx$ $\qquad\left[\int\sin x dx=-\cos x\right]$

$\Rightarrow I_2=\left[-\frac{\cos\frac{\pi x}{4}}{\frac{\pi}{4}}\right]_0^1=-\frac{4}{\pi}\left[\cos\frac{\pi}{4}\times 1-\cos\frac{\pi}{4}\times 0\right]=-\frac{4}{\pi}\left[\cos\frac{\pi}{4}-\cos 0\right]$

$\Rightarrow I_2=\frac{4}{\pi}\left(1-\frac{1}{\sqrt{2}}\right)=\frac{4}{\pi}-\frac{2\sqrt{2}}{\pi}$

Since, $I = I_1 + I_2$

$\therefore I=1+\frac{4}{\pi}-\frac{2\sqrt{2}}{\pi}$

$\therefore\int_0^1(xe^x+\sin\frac{\pi x}{4})dx=1+\frac{4}{\pi}-\frac{2\sqrt{2}}{\pi}$

21. $\int_1^{\sqrt{3}}\frac{dx}{1+x^2}$

(a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{6}$ (d) $\frac{\pi}{12}$ equals

21. $\int\frac{dx}{1+x^2}=\tan^{-1}x=F(x)$

By second fundamental theorem of calculus, we obtain

$\int_1^{\sqrt{3}}\frac{dx}{1+x^2}=F\left(\sqrt{3}\right)-F(1)$

$=\tan^{-1}\sqrt{3}\tan^{-1}1$

$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{\pi}{12}$

Hence, the correct Answer is D.

22. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$

(c) $\frac{\pi}{24}$ (d) $\frac{\pi}{4}$

22. $\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2 + (3x)^2}$

Put $3x = t \Rightarrow 3dx = dt$

$\therefore \int \frac{dx}{(2)^2 + (3x)^2} = \frac{1}{3} \int \frac{dt}{(2)^2 + t^2}$

$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]$

$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$

$= F(x)$