

## EXERCISE 5.2

1. Find the values of the other five trigonometric functions in each of the following:

(i)  $\cot x = 12/5$ ,  $x$  in quadrant III

(ii)  $\cos x = -1/2$ ,  $x$  in quadrant II

(iii)  $\tan x = 3/4$ ,  $x$  in quadrant III

(iv)  $\sin x = 3/5$ ,  $x$  in quadrant I

**Solution:**

(i)  $\cot x = 12/5$ ,  $x$  in quadrant III

In third quadrant,  $\tan x$  and  $\cot x$  are positive.  $\sin x$ ,  $\cos x$ ,  $\sec x$ ,  $\operatorname{cosec} x$  are negative.

By using the formulas,

$$\begin{aligned}\tan x &= 1/\cot x \\ &= 1/(12/5) \\ &= 5/12\end{aligned}$$

$$\begin{aligned}\operatorname{cosec} x &= -\sqrt{1 + \cot^2 x} \\ &= -\sqrt{1 + (12/5)^2} \\ &= -\sqrt{(25+144)/25} \\ &= -\sqrt{(169/25)} \\ &= -13/5\end{aligned}$$

$$\begin{aligned}\sin x &= 1/\operatorname{cosec} x \\ &= 1/(-13/5) \\ &= -5/13\end{aligned}$$

$$\begin{aligned}\cos x &= -\sqrt{1 - \sin^2 x} \\ &= -\sqrt{1 - (-5/13)^2} \\ &= -\sqrt{(169-25)/169} \\ &= -\sqrt{(144/169)} \\ &= -12/13\end{aligned}$$

$$\begin{aligned}\sec x &= 1/\cos x \\ &= 1/(-12/13) \\ &= -13/12\end{aligned}$$

$$\therefore \sin x = -5/13, \cos x = -12/13, \tan x = 5/12, \operatorname{cosec} x = -13/5, \sec x = -13/12$$

(ii)  $\cos x = -1/2$ ,  $x$  in quadrant II

In second quadrant,  $\sin x$  and  $\operatorname{cosec} x$  are positive.  $\tan x$ ,  $\cot x$ ,  $\cos x$ ,  $\sec x$  are negative.

By using the formulas,

$$\begin{aligned}\sin x &= \sqrt{1 - \cos^2 x} \\ &= \sqrt{1 - (-1/2)^2} \\ &= \sqrt{(4-1)/4} \\ &= \sqrt{3/4} \\ &= \sqrt{3}/2\end{aligned}$$

$$\begin{aligned}\tan x &= \sin x / \cos x \\ &= (\sqrt{3}/2) / (-1/2) \\ &= -\sqrt{3}\end{aligned}$$

$$\begin{aligned}\cot x &= 1/\tan x \\ &= 1/(-\sqrt{3}) \\ &= -1/\sqrt{3}\end{aligned}$$

$$\begin{aligned}\operatorname{cosec} x &= 1/\sin x \\ &= 1/(\sqrt{3}/2) \\ &= 2/\sqrt{3}\end{aligned}$$

$$\begin{aligned}\sec x &= 1/\cos x \\ &= 1/(-1/2) \\ &= -2\end{aligned}$$

$$\therefore \sin x = \sqrt{3}/2, \tan x = -\sqrt{3}, \operatorname{cosec} x = 2/\sqrt{3}, \cot x = -1/\sqrt{3}, \sec x = -2$$

(iii)  $\tan x = 3/4$ ,  $x$  in quadrant III

In third quadrant,  $\tan x$  and  $\cot x$  are positive.  $\sin x$ ,  $\cos x$ ,  $\sec x$ ,  $\operatorname{cosec} x$  are negative.

By using the formulas,

$$\begin{aligned}\sin x &= \sqrt{1 - \cos^2 x} \\ &= -\sqrt{1 - (-4/5)^2} \\ &= -\sqrt{(25-16)/25} \\ &= -\sqrt{9/25} \\ &= -3/5\end{aligned}$$

$$\begin{aligned}\cos x &= 1/\sec x \\ &= 1/(-5/4) \\ &= -4/5\end{aligned}$$

$$\cot x = 1/\tan x$$

$$\begin{aligned} &= 1/(3/4) \\ &= 4/3 \end{aligned}$$

$$\begin{aligned} \operatorname{cosec} x &= 1/\sin x \\ &= 1/(-3/5) \\ &= -5/3 \end{aligned}$$

$$\begin{aligned} \sec x &= -\sqrt{(1 + \tan^2 x)} \\ &= -\sqrt{(1+(3/4)^2)} \\ &= -\sqrt{(16+9)/16} \\ &= -\sqrt{(25/16)} \\ &= -5/4 \end{aligned}$$

$$\therefore \sin x = -3/5, \cos x = -4/5, \operatorname{cosec} x = -5/3, \sec x = -5/4, \cot x = 4/3$$

(iv)  $\sin x = 3/5$ ,  $x$  in quadrant I

In first quadrant, all trigonometric ratios are positive.

So, by using the formulas,

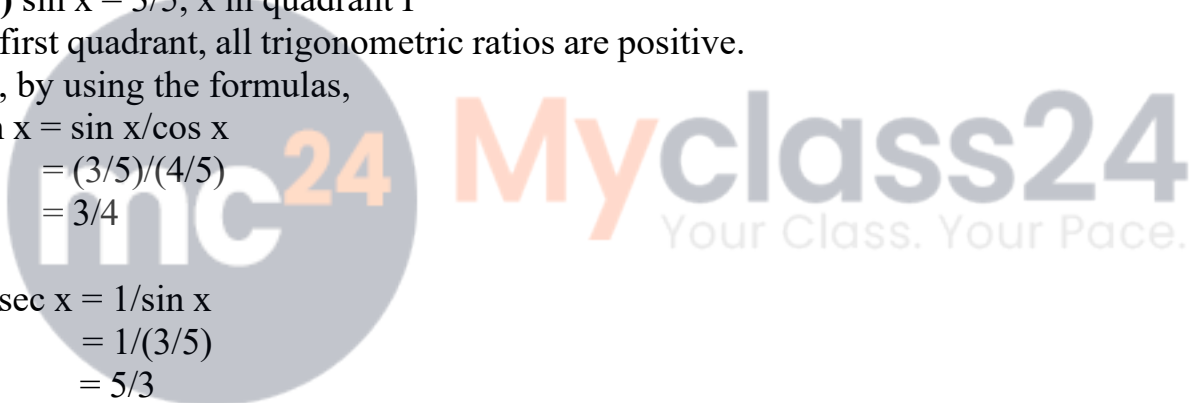
$$\begin{aligned} \tan x &= \sin x/\cos x \\ &= (3/5)/(4/5) \\ &= 3/4 \end{aligned}$$

$$\begin{aligned} \operatorname{cosec} x &= 1/\sin x \\ &= 1/(3/5) \\ &= 5/3 \end{aligned}$$

$$\begin{aligned} \cos x &= \sqrt{(1-\sin^2 x)} \\ &= \sqrt{(1 - (-3/5)^2)} \\ &= \sqrt{(25-9)/25} \\ &= \sqrt{(16/25)} \\ &= 4/5 \end{aligned}$$

$$\begin{aligned} \sec x &= 1/\cos x \\ &= 1/(4/5) \\ &= 5/4 \end{aligned}$$

$$\begin{aligned} \cot x &= 1/\tan x \\ &= 1/(3/4) \\ &= 4/3 \end{aligned}$$



$$\therefore \cos x = 4/5, \tan x = 3/4, \operatorname{cosec} x = 5/3, \sec x = 5/4, \cot x = 4/3$$

**2. If  $\sin x = 12/13$  and lies in the second quadrant, find the value of  $\sec x + \tan x$ .**

**Solution:**

Given:

$\sin x = 12/13$  and  $x$  lies in the second quadrant.

We know, in second quadrant,  $\sin x$  and  $\operatorname{cosec} x$  are positive and all other ratios are negative.

By using the formulas,

$$\begin{aligned}\cos x &= \sqrt{1 - \sin^2 x} \\ &= -\sqrt{1 - (12/13)^2} \\ &= -\sqrt{1 - (144/169)} \\ &= -\sqrt{(169 - 144)/169} \\ &= -\sqrt{(25/169)} \\ &= -5/13\end{aligned}$$

We know,

$$\tan x = \sin x / \cos x$$

$$\sec x = 1 / \cos x$$

Now,

$$\begin{aligned}\tan x &= (12/13) / (-5/13) \\ &= -12/5\end{aligned}$$

$$\begin{aligned}\sec x &= 1 / (-5/13) \\ &= -13/5\end{aligned}$$

$$\begin{aligned}\sec x + \tan x &= -13/5 + (-12/5) \\ &= (-13 - 12)/5 \\ &= -25/5 \\ &= -5\end{aligned}$$

$$\therefore \sec x + \tan x = -5$$

**3. If  $\sin x = 3/5$ ,  $\tan y = 1/2$  and  $\pi/2 < x < \pi < y < 3\pi/2$  find the value of  $8 \tan x - \sqrt{5} \sec y$ .**

**Solution:**

Given:

$$\sin x = 3/5, \tan y = 1/2 \text{ and } \pi/2 < x < \pi < y < 3\pi/2$$

We know that,  $x$  is in second quadrant and  $y$  is in third quadrant.

In second quadrant,  $\cos x$  and  $\tan x$  are negative.

In third quadrant,  $\sec y$  is negative.

By using the formula,

$$\cos x = -\sqrt{1-\sin^2 x}$$

$$\tan x = \sin x / \cos x$$

Now,

$$\begin{aligned}\cos x &= -\sqrt{1-\sin^2 x} \\ &= -\sqrt{1-(3/5)^2} \\ &= -\sqrt{1-9/25} \\ &= -\sqrt{(25-9)/25} \\ &= -\sqrt{16/25} \\ &= -4/5\end{aligned}$$

$$\begin{aligned}\tan x &= \sin x / \cos x \\ &= (3/5) / (-4/5) \\ &= 3/5 \times -5/4 \\ &= -3/4\end{aligned}$$

$$\begin{aligned}\text{We know that } \sec y &= -\sqrt{1+\tan^2 y} \\ &= -\sqrt{1+(1/2)^2} \\ &= -\sqrt{1+1/4} \\ &= -\sqrt{(4+1)/4} \\ &= -\sqrt{5/4} \\ &= -\sqrt{5}/2\end{aligned}$$

$$\begin{aligned}\text{Now, } 8 \tan x - \sqrt{5} \sec y &= 8(-3/4) - \sqrt{5}(-\sqrt{5}/2) \\ &= -6 + 5/2 \\ &= (-12+5)/2 \\ &= -7/2\end{aligned}$$

$$\therefore 8 \tan x - \sqrt{5} \sec y = -7/2$$

**4. If  $\sin x + \cos x = 0$  and  $x$  lies in the fourth quadrant, find  $\sin x$  and  $\cos x$ .**

**Solution:**

Given:

$\sin x + \cos x = 0$  and  $x$  lies in fourth quadrant.

$$\sin x = -\cos x$$

$$\sin x / \cos x = -1$$

So,  $\tan x = -1$  (since,  $\tan x = \sin x / \cos x$ )

We know that, in fourth quadrant,  $\cos x$  and  $\sec x$  are positive and all other ratios are negative.

By using the formulas,

$$\sec x = \sqrt{1 + \tan^2 x}$$

$$\cos x = 1/\sec x$$

$$\sin x = -\sqrt{1 - \cos^2 x}$$

Now,

$$\begin{aligned}\sec x &= \sqrt{1 + \tan^2 x} \\ &= \sqrt{1 + (-1)^2} \\ &= \sqrt{2}\end{aligned}$$

$$\begin{aligned}\cos x &= 1/\sec x \\ &= 1/\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sin x &= -\sqrt{1 - \cos^2 x} \\ &= -\sqrt{1 - (1/\sqrt{2})^2} \\ &= -\sqrt{1 - (1/2)} \\ &= -\sqrt{(2-1)/2} \\ &= -\sqrt{1/2} \\ &= -1/\sqrt{2}\end{aligned}$$

$$\therefore \sin x = -1/\sqrt{2} \text{ and } \cos x = 1/\sqrt{2}$$

**5. If  $\cos x = -3/5$  and  $\pi < x < 3\pi/2$  find the values of other five trigonometric functions**

**and hence evaluate  $\frac{\operatorname{cosec} x + \cot x}{\sec x - \tan x}$**

**Solution:**

Given:

$$\cos x = -3/5 \text{ and } \pi < x < 3\pi/2$$

We know that in the third quadrant,  $\tan x$  and  $\cot x$  are positive and all other ratios are negative.

By using the formulas,

$$\sin x = -\sqrt{1 - \cos^2 x}$$

$$\tan x = \sin x / \cos x$$

$$\cot x = 1/\tan x$$

$$\sec x = 1/\cos x$$

$$\operatorname{Cosec} x = 1/\sin x$$

Now,

$$\begin{aligned}\sin x &= -\sqrt{1 - \cos^2 x} \\ &= -\sqrt{1 - (-3/5)^2} \\ &= -\sqrt{1 - 9/25} \\ &= -\sqrt{(25-9)/25}\end{aligned}$$

$$\begin{aligned} &= -\sqrt{16/25} \\ &= -4/5 \end{aligned}$$

$$\begin{aligned} \tan x &= \sin x / \cos x \\ &= (-4/5) / (-3/5) \\ &= -4/5 \times -5/3 \\ &= 4/3 \end{aligned}$$

$$\begin{aligned} \cot x &= 1 / \tan x \\ &= 1 / (4/3) \\ &= 3/4 \end{aligned}$$

$$\begin{aligned} \sec x &= 1 / \cos x \\ &= 1 / (-3/5) \\ &= -5/3 \end{aligned}$$

$$\begin{aligned} \operatorname{cosec} x &= 1 / \sin x \\ &= 1 / (-4/5) \\ &= -5/4 \end{aligned}$$

$$\begin{aligned} \therefore \frac{\operatorname{cosec} x + \cot x}{\sec x - \tan x} &= [(-5/4) + (3/4)] / [(-5/3) - (4/3)] \\ &= [(-5+3)/4] / [(-5-4)/3] \\ &= [-2/4] / [-9/3] \\ &= [-1/2] / [-3] \\ &= 1/6 \end{aligned}$$