

NCERT Solutions for Class-XII Maths

Chapter-7.3

NCERT Maths Class 12

1. $\sin^2(2x + 5)$

1.
$$\sin^2(2x + 5) = \frac{1 - \cos^2(2x + 5)}{2} = \frac{1 - \cos(4x + 10)}{2}$$
$$= \int \sin^2(2x + 5) dx = \int \frac{1 - \cos(4x + 10)}{2} dx$$
$$= \frac{1}{2} \int 1 dx - \frac{1}{2} \int \cos(4x + 10) dx$$
$$= \frac{1}{2} x - \frac{1}{2} \left(\frac{\sin(4x + 10)}{4} \right) + C$$
$$= \frac{1}{2} x - \frac{1}{8} \sin(4x + 10) + C$$

2. $\sin^3 x \cdot \cos^4 x$

2.
$$\int \sin^3 x \cos^4 x dx = \frac{1}{2} \int \{ \sin(3x + 4x) + \sin(3x - 4x) \} dx$$
$$= \frac{1}{2} \int \{ \sin 7x + \sin(-x) \} dx$$
$$= \frac{1}{2} \int \{ \sin 7x - \sin x \} dx$$
$$= \frac{1}{2} \int \sin 7x dx - \frac{1}{2} \int \sin x dx$$
$$= \frac{1}{2} \left(\frac{-\cos 7x}{7} \right) - \frac{1}{2} (-\cos x) + C$$
$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

3. $\cos 2x \cos 4x \cos 6x$

3. It is known that, $\cos A \cos B = \frac{1}{2} \{ \cos(A + B) + \cos(A - B) \}$

$$\therefore \int \cos 2x (\cos 4x \cos 6x) dx = \int \cos 2x \left[\frac{1}{2} \{ \cos(4x + 6x) + \cos(4x - 6x) \} \right] dx$$
$$= \frac{1}{2} \int \{ \cos 2x \cos 10x + \cos 2x \cos(-2x) \} dx$$

$$\begin{aligned}
&= \frac{1}{2} \int \{ \cos 2x \cos 10x + \cos^2 2x \} dx \\
&= \frac{1}{2} \int \left[\left\{ \frac{1}{2} \cos(2x + 10x) + \cos(2x - 10x) \right\} + \left(\frac{1 + \cos 4x}{2} \right) \right] dx \\
&= \frac{1}{4} \int (\cos 12x + \cos 8x + 1 + \cos 4x) dx \\
&= \frac{1}{4} \left[\frac{\sin 12x}{12} + \frac{\sin 8x}{8} + x + \frac{\sin 4x}{4} \right] + C
\end{aligned}$$

4. $\sin^3(2x + 1)$

4. Let $I = \sin^3(2x+1)$

$$\Rightarrow \int \sin^3(2x + 1) dx = \int \sin^2(2x + 1) \cdot \sin(2x + 1) dx$$

$$= \int (1 - \cos^2(2x + 1)) \sin(2x + 1) dx$$

Let $\cos(2x + 1) = t$

$$\Rightarrow -2 \sin(2x + 1) dx = dt$$

$$\Rightarrow \sin(2x + 1) dx = \frac{-dt}{2}$$

$$\Rightarrow I = \frac{-1}{2} \int (1 - t^2) dt$$

$$= \frac{-1}{2} \left\{ t - \frac{t^3}{3} \right\}$$

$$= \frac{-1}{2} \left\{ \cos(2x + 1) - \frac{\cos^3(2x + 1)}{3} \right\}$$

$$= \frac{-\cos(2x + 1)}{2} + \frac{\cos^3(2x + 1)}{6} + C$$

5. $\sin^3 x \cos^3 x$

5. Let $I = \int \sin^3 x \cos^3 x$

$$= \int \cos^3 x \cdot \sin^2 x \cdot \sin x \cdot dx$$

$$= \int \cos^3 x (1 - \cos^2 x) \sin x \cdot dx$$

Let $\cos x = t$

$$\Rightarrow -\sin x \cdot dx = dt$$

$$\Rightarrow I = -\int t^3 (1 - t^2) dt$$

$$= -\int (t^3 - t^5) dt$$

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$$\begin{aligned}
&= -\left\{\frac{t^4}{4} - \frac{t^6}{6}\right\} + C \\
&= -\left\{\frac{\cos^4 x}{4} - \frac{\cos^6 x}{6}\right\} + C \\
&= \frac{\cos^6 x}{6} - \frac{\cos^4 x}{4} + C
\end{aligned}$$

6. $\sin x \sin 2x \sin 3x$

6. $\int \sin 2x \sin 3x dx = \int \sin x \cdot \frac{1}{2} [\cos(2x - 3x) - \cos(2x + 3x)] dx$

$$\begin{aligned}
&= \frac{1}{2} \int \{\sin x \cos(-x) - \sin x \cos 5x\} dx \\
&= \frac{1}{2} \int \{\sin x \cos x - \sin x \cos 5x\} dx \\
&= \frac{1}{2} \int \frac{\sin 2x}{2} dx - \frac{1}{2} \int \sin x \cos 5x dx \\
&= \frac{1}{4} \left[\frac{-\cos 2x}{2} \right] - \frac{1}{2} \int \left\{ \frac{1}{2} \sin(x + 5x) + \sin(x - 5x) \right\} dx \\
&= \frac{-\cos 2x}{8} - \frac{1}{4} \int (\sin 6x + \sin(-4x)) dx \\
&= \frac{-\cos 2x}{8} - \frac{1}{4} \left[\frac{-\cos 6x}{3} + \frac{\cos 4x}{4} \right] + C \\
&= \frac{1}{8} \left[\frac{\cos 6x}{3} - \frac{\cos 4x}{2} - \cos 2x \right] + C
\end{aligned}$$

7. $\sin 4x \sin 8x$

7. It is known that, $\sin A \sin B = \frac{1}{2} \cos(A - B) - \cos(A + B)$

$$\begin{aligned}
\therefore \int \sin 4x \sin 8x dx &= \int \left\{ \frac{1}{2} \cos(4x - 8x) - \cos(4x + 8x) \right\} dx \\
&= \frac{1}{2} \int (\cos(-4x) - \cos 12x) dx \\
&= \frac{1}{2} \left[\frac{\sin 4x}{4} - \frac{\sin 12x}{12} \right]
\end{aligned}$$

8. $\frac{1 - \cos x}{1 + \cos x}$

$$8. \quad \frac{1 - \cos x}{1 + \cos x} = \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = 2 \tan^2 \frac{x}{2} = \left(\sec^2 \frac{x}{2} - 1 \right)$$

$$\therefore \int \frac{1 - \cos x}{1 + \cos x} dx = \int \left(\sec^2 \frac{x}{2} - 1 \right) dx$$

$$= \left[\frac{\tan \frac{x}{2}}{\frac{1}{2}} - x \right] + C$$

$$= 2 \tan \frac{x}{2} - x + C$$

$$9. \quad \frac{\cos x}{1 + \cos x}$$

$$9. \quad \frac{\cos x}{1 + \cos x} = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}}$$

$$= \frac{1}{2} \left[1 - \tan^2 \frac{x}{2} \right]$$

$$\therefore \int \frac{\cos x}{1 + \cos x} dx = \frac{1}{2} \int \left(1 - \tan^2 \frac{x}{2} \right) dx$$

$$= \frac{1}{2} \int \left(1 - \sec^2 \frac{x}{2} + 1 \right) dx$$

$$= \frac{1}{2} \int \left(2 - \sec^2 \frac{x}{2} \right) dx$$

$$= \frac{1}{2} \left[2x - \frac{\tan \frac{x}{2}}{\frac{1}{2}} \right] + C$$

$$= x - \tan \frac{x}{2} + C$$

$$10. \quad \sin^4 x$$

$$10. \quad \sin^4 x = \sin^2 x \sin^2 x$$

$\left[\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \text{ and } \cos x = 2 \cos^2 \frac{x}{2} - 1 \right]$
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$$= \left(\frac{1 - \cos 2x}{2} \right) \left(\frac{1 - \cos 2x}{2} \right)$$

$$= \frac{1}{4} (1 - \cos 2x)^2$$

$$= \frac{1}{4} [1 + \cos^2 2x - 2\cos 2x]$$

$$= \frac{1}{4} \left[1 + \left(\frac{1 + \cos 4x}{2} \right) - 2\cos 2x \right]$$

$$= \frac{1}{4} \left[1 + \frac{1}{2} + \frac{1}{2} \cos 4x - 2\cos 2x \right]$$

$$= \frac{1}{4} \left[\frac{3}{2} + \frac{1}{2} \cos 4x - 2\cos 2x \right]$$

$$\text{Now, } \int \sin^4 x dx = \frac{1}{4} \int \left[\frac{3}{2} + \frac{1}{2} \cos 4x - 2\cos 2x \right] dx$$

$$= \frac{1}{4} \left[\frac{3}{2} x + \frac{1}{2} \left(\frac{\sin 4x}{4} \right) - \frac{2\sin 2x}{2} \right] + C$$

$$= \frac{1}{8} \left[3x + \left(\frac{\sin 4x}{4} \right) - 2\sin 2x \right] + C$$

$$= \frac{3x}{8} - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + C$$

11. $\cos^4 2x$

11. $\cos^4 2x = (\cos^2 2x)^2$

$$= \left(\frac{1 + \cos 4x}{2} \right)^2$$

$$= \frac{1}{4} [1 + \cos^2 4x + 2\cos 4x]$$

$$= \frac{1}{4} \left[1 + \left(\frac{1 + \cos 8x}{2} \right) + 2\cos 4x \right]$$

$$= \frac{1}{4} \left[1 + \frac{1}{2} + \frac{\cos 8x}{2} + 2\cos 4x \right]$$

$$= \frac{1}{4} \left[\frac{3}{2} + \frac{\cos 8x}{2} + 2\cos 4x \right]$$

$$\therefore \int \cos^4 2x dx = \int \left(\frac{3}{8} + \frac{\cos 8x}{8} + \frac{\cos 4x}{2} \right) dx$$

$$= \frac{3}{8} x + \frac{\sin 8x}{64} + \frac{\sin 4x}{8} + C$$

$$12. \frac{\sin^2 x}{1 + \cos x}$$

$$12. \frac{\sin^2 x}{1 + \cos x} = \frac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}} = \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$= 2\sin^2\frac{x}{2} = 1 - \cos x$$

$$\int \frac{\sin^2 x}{1 + \cos x} dx = \int (1 - \cos x) dx$$

$$= x - \sin x + C$$

$$13. \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}$$

$$13. \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} = \frac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}} \quad \left[\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2} \right]$$

$$= \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\left[\cos\left(\frac{x+\alpha}{2} + \frac{x-\alpha}{2}\right) + \cos\frac{x+\alpha}{2} - \frac{x-\alpha}{2}\right]$$

$$= 2[\cos(x) + \cos\alpha]$$

$$= 2\cos x + 2\cos \alpha$$

$$\therefore \int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx = \int 2\cos x + 2\cos \alpha$$

$$= 2[\sin x + x\cos \alpha] + C$$

$$14. \frac{\cos x - \sin x}{1 + \sin 2x}$$

$$14. \frac{\cos x - \sin x}{1 + \sin 2x}$$

$$\begin{aligned}
&= \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2\sin x \cos x} \\
&= \frac{\cos x - \sin x}{(\sin x + \cos x)^2} \\
&\text{Let } \sin x + \cos x = t \\
&(\cos x - \sin x) dx = dt \\
\Rightarrow \int \frac{\cos x - \sin x}{1 + \sin 2x} dx &= \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx \\
&= \int \frac{dt}{t^2} \\
&= -t^{-1} + C \\
&= -\frac{1}{t} + C \\
&= \frac{-1}{\sin x + \cos x} + C
\end{aligned}$$

15. $\tan^3 2x \sec x$

15. $\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$
 $= (\sec^2 2x - 1) \tan 2x \sec 2x$
 $= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$
 $= \int \tan^3 2x \sec 2x dx = \int \sec^2 2x \tan 2x \sec 2x dx - \int \tan 2x \sec 2x dx$
 $= \int \sec^2 2x \tan 2x \sec 2x dx - \frac{\sec 2x}{2} + C$

Let $\sec 2x = t$

$\therefore 2 \sec 2x \tan 2x dx = dt$

$\therefore \int \tan^3 2x \sec 2x dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C$

$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$

$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$

16. $\tan^4 x$

16. $\tan^4 x = \tan^2 x \cdot \tan^2 x$

$= (\sec^2 x - 1) \tan^2 x$

$= \sec^2 x \tan^2 x - \tan^2 x$

$$= \sec^2 x \tan^2 x - (\sec^2 x - 1)$$

$$= \sec^2 x \tan^2 x - \sec^2 x + 1$$

$$\text{Now, } \int \tan^4 x dx = \int \sec^2 x \tan^2 x dx - \int \sec^2 x dx - \int 1 dx$$

$$= \int \sec^2 x \tan^2 x dx - \tan x + x + C$$

Now, let $\tan x = t$

$$\Rightarrow \sec^2 x dx = dt$$

$$\int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

$$\int \tan^4 x dx = \frac{1}{3} \tan^3 x - \tan x + x + C$$

$$17. \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$$

$$17. \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

$$= \tan x \sec x + \cot x \operatorname{cosec} x$$

$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\tan x \sec x + \cot x \operatorname{cosec} x) dx$$

$$= \sec x - \operatorname{cosec} x + C$$

$$18. \frac{\cos 2x + 2\sin^2 x}{\cos^2 x}$$

$$18. \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} = \frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\text{Now, } \int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \sec^2 x dx$$

$$= \tan x + C$$

$$19. \frac{1}{\sin x \cos^3 x}$$

$$19. \frac{1}{\sin x \cos^3 x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x}$$

$$= \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x \cos x}$$

$$= \tan x \sec^2 x + \frac{1 \cos^2 x}{\sin x \cos x}$$

$$= \tan x \sec^2 x + \frac{\sec^2 x}{\tan x}$$

$$\therefore \int \frac{1}{\sin x \cos^3 x} dx = \int \tan x \sec^2 x dx + \int \frac{\sec^2 x}{\tan x} dx$$

$$\text{Let } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\Rightarrow \int \frac{1}{\sin x \cos^3 x} dx = \int t dt + \int \frac{1}{t} dt$$

$$= \frac{t^2}{2} + \log |t| + C$$

$$= \frac{1}{2} \tan^2 x + \log |\tan x| + C$$

$$20. \frac{\cos 2x}{(\cos x + \sin x)^2}$$

$$20. \frac{\cos 2x}{(\cos x + \sin x)^2} = \frac{\cos 2x}{\cos^2 x + \sin^2 x + 2 \sin x \cos x} = \frac{\cos 2x}{1 + \sin 2x}$$

$$\text{Now, } \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \int \frac{\cos 2x}{1 + \sin 2x} dx$$

$$\text{Let } 1 + \sin 2x = t$$

$$2 \cos 2x dx = dt$$

$$\text{Thus, } \int \frac{\cos 2x}{(\cos x + \sin x)^2} dx = \frac{1}{2} \int \frac{1}{t} dt$$

$$= \frac{1}{2} \log |t| + C$$

$$= \frac{1}{2} \log |1 + \sin 2x| + C$$

$$= \frac{1}{2} \log |(\cos x + \sin x)^2| + C$$

$$= \log |\sin x + \cos x| + C$$

$$21. \sin^{-1}(\cos x)$$

$$21. \sin^{-1}(\cos x)$$

$$\text{Let } \cos x = t$$

$$\text{Then, } \sin x = \sqrt{1-t^2}$$

$$\Rightarrow (-\sin x)dx = dt$$

$$dx = \frac{-dt}{\sin x}$$

$$dx = \frac{-dt}{\sqrt{1-t^2}}$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int \sin^{-1}t \left(\frac{-dt}{\sqrt{1-t^2}} \right)$$

$$= -\int \frac{\sin^{-1}t}{\sqrt{1-t^2}} dt$$

Let $\sin^{-1}t = u$

$$\Rightarrow \frac{1}{\sqrt{1-t^2}} dt = du$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int 4du$$

$$= -\frac{u^2}{2} + C$$

$$= \frac{-(\sin^{-1}t)^2}{2} + C$$

$$= \frac{-[\sin^{-1}(\cos x)]^2}{2} + C$$

It is known that,

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$\therefore \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x) = \left(\frac{\pi}{2} - x \right)$$

Substituting in equation (1), we obtain

$$\int \sin^{-1}(\cos x)dx = \frac{-\left[\frac{\pi}{2} - x \right]^2}{2} + C$$

$$= \frac{1}{2} \left(\frac{\pi^2}{2} + x^2 - \pi x \right) + C$$

$$= \frac{\pi^2}{8} - \frac{x^2}{2} + \frac{1}{2} \pi x + C$$

$$= \frac{\pi x}{2} - \frac{x^2}{2} + \left(C - \frac{\pi^2}{8} \right)$$

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$$= \frac{\pi x}{2} - \frac{x^2}{2} + C_1$$

22. $\frac{1}{\cos(x-a)\cos(x-b)}$

$$\begin{aligned} 22. \quad \frac{1}{\cos(x-a)\cos(x-b)} &= \frac{1}{\sin(a-b)} \left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)} \right] \\ &= \frac{1}{\sin(a-b)} \left[\frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)} \right] \\ &= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right] \\ &= \frac{1}{\sin(a-b)} [\tan(x-b) - \tan(x-a)] \end{aligned}$$

Now, $\int \frac{1}{\cos(x-a)\cos(x-b)} dx = \frac{1}{\sin(a-b)} [\tan x - b - \tan x - a] dx$

$$= \frac{1}{\sin(a-b)} [-\log |\cos(x-b)| + \log |\cos(x-a)|]$$

$$= \frac{1}{\sin(a-b)} \left[\log \left| \frac{\cos(x-a)}{\cos(x-b)} \right| \right] + C$$

23. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

- (a) $\tan x + \cot x + C$ (b) $\tan x + \operatorname{cosec} x + C$
 (c) $-\tan x + \cot x + C$ (d) $\tan x + \sec x + C$

23. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$

$$= \int (\sec^2 x - \operatorname{cosec}^2 x) dx$$

$$= \tan x + \cot x + C$$

Hence, the correct answer is A.

24. $\int \frac{e^x(1+x)}{\cos^2(e^x)} dx$ equals

- (a) $-\cot(e^x) + C$ (b) $\tan(xe^x) + C$
 (c) $\tan(e^x) + C$ (d) $\cot(e^x) + C$

24. Let $ex^x = t$

$$(e^x \cdot x + e^x \cdot 1) dx = dt$$

$$e^x(x + 1) = dt$$

$$\text{Now, } \int \frac{e^x(1+x)}{\cos^2(e^x x)} dx = \int \frac{dt}{\cos^2 t}$$

$$= \int \sec^2 t \cdot dt = \tan t + C = \tan(e^x \cdot x) + C$$



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