

EXERCISE 24.1

Find the equation of the circle with:

(i) Centre $(-2, 3)$ and radius 4.

(ii) Centre (a, b) and radius $\sqrt{a^2 + b^2}$.

(iii) Centre $(0, -1)$ and radius 1.

(iv) Centre $(a \cos \alpha, a \sin \alpha)$ and radius a .

(v) Centre (a, a) and radius $\sqrt{2} a$.

Solution:

(i) Centre $(-2, 3)$ and radius 4.

Given:

The radius is 4 and the centre $(-2, 3)$

By using the formula,

The equation of the circle with centre (p, q) and radius 'r' is $(x - p)^2 + (y - q)^2 = r^2$

Where, $p = -2, q = 3, r = 4$

Now by substituting the values in the above equation, we get

$$(x - p)^2 + (y - q)^2 = r^2$$

$$(x - (-2))^2 + (y - 3)^2 = 4^2$$

$$(x + 2)^2 + (y - 3)^2 = 16$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$$

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

\therefore The equation of the circle is $x^2 + y^2 + 4x - 6y - 3 = 0$

(ii) Centre (a, b) and radius $\sqrt{a^2 + b^2}$.

Given:

The radius is $\sqrt{a^2 + b^2}$ and the centre (a, b)

By using the formula,

The equation of the circle with centre (p, q) and radius 'r' is $(x - p)^2 + (y - q)^2 = r^2$

Where, $p = a, q = b, r = \sqrt{a^2 + b^2}$

Now by substituting the values in the above equation, we get

$$(x - p)^2 + (y - q)^2 = r^2$$

$$(x - a)^2 + (y - b)^2 = (\sqrt{a^2 + b^2})^2$$

$$x^2 - 2ax + a^2 + y^2 - 2by + b^2 = a^2 + b^2$$

$$x^2 + y^2 - 2ax - 2by = 0$$

\therefore The equation of the circle is $x^2 + y^2 - 2ax - 2by = 0$

(iii) Centre (0, -1) and radius 1.

Given:

The radius is 1 and the centre (0, -1)

By using the formula,

The equation of the circle with centre (p, q) and radius 'r' is $(x - p)^2 + (y - q)^2 = r^2$

Where, $p = 0, q = -1, r = 1$

Now by substituting the values in the above equation, we get

$$(x - p)^2 + (y - q)^2 = r^2$$

$$(x - 0)^2 + (y - (-1))^2 = 1^2$$

$$(x - 0)^2 + (y + 1)^2 = 1$$

$$x^2 + y^2 + 2y + 1 = 1$$

$$x^2 + y^2 + 2y = 0$$

\therefore The equation of the circle is $x^2 + y^2 + 2y = 0$.

(iv) Centre ($a \cos \alpha, a \sin \alpha$) and radius a.

Given:

The radius is 'a' and the centre ($a \cos \alpha, a \sin \alpha$)

By using the formula,

The equation of the circle with centre (p, q) and radius 'r' is $(x - p)^2 + (y - q)^2 = r^2$

Where, $p = a \cos \alpha, q = a \sin \alpha, r = a$

Now by substituting the values in the above equation, we get

$$(x - p)^2 + (y - q)^2 = r^2$$

$$(x - a \cos \alpha)^2 + (y - a \sin \alpha)^2 = a^2$$

$$x^2 - (2a \cos \alpha)x + a^2 \cos^2 \alpha + y^2 - (2a \sin \alpha)y + a^2 \sin^2 \alpha = a^2$$

We know that $\sin^2 \theta + \cos^2 \theta = 1$

So,

$$x^2 - (2a \cos \alpha)x + y^2 - 2a \sin \alpha y + a^2 = a^2$$

$$x^2 + y^2 - (2a \cos \alpha)x - (2a \sin \alpha)y = 0$$

\therefore The equation of the circle is $x^2 + y^2 - (2a \cos \alpha)x - (2a \sin \alpha)y = 0$.

(v) Centre (a, a) and radius $\sqrt{2} a$.

Given:

The radius is $\sqrt{2} a$ and the centre (a, a)

By using the formula,

The equation of the circle with centre (p, q) and radius 'r' is $(x - p)^2 + (y - q)^2 = r^2$

Where, $p = a, q = a, r = \sqrt{2} a$

Now by substituting the values in the above equation, we get

$$(x - p)^2 + (y - q)^2 = r^2$$

$$(x - a)^2 + (y - a)^2 = (\sqrt{2} a)^2$$

$$x^2 - 2ax + a^2 + y^2 - 2ay + a^2 = 2a^2$$

$$x^2 + y^2 - 2ax - 2ay = 0$$

∴ The equation of the circle is $x^2 + y^2 - 2ax - 2ay = 0$.

2. Find the centre and radius of each of the following circles:

(i) $(x - 1)^2 + y^2 = 4$

(ii) $(x + 5)^2 + (y + 1)^2 = 9$

(iii) $x^2 + y^2 - 4x + 6y = 5$

(iv) $x^2 + y^2 - x + 2y - 3 = 0$

Solution:

(i) $(x - 1)^2 + y^2 = 4$

Given:

The equation $(x - 1)^2 + y^2 = 4$

We need to find the centre and the radius.

By using the standard equation formula,

$$(x - a)^2 + (y - b)^2 = r^2 \dots (1)$$

Now let us convert given circle's equation into the standard form.

$$(x - 1)^2 + y^2 = 4$$

$$(x - 1)^2 + (y - 0)^2 = 2^2 \dots (2)$$

By comparing equation (2) with (1), we get

Centre = (1, 0) and radius = 2

∴ The centre of the circle is (1, 0) and the radius is 2.

(ii) $(x + 5)^2 + (y + 1)^2 = 9$

Given:

The equation $(x + 5)^2 + (y + 1)^2 = 9$

We need to find the centre and the radius.

By using the standard equation formula,

$$(x - a)^2 + (y - b)^2 = r^2 \dots (1)$$

Now let us convert given circle's equation into the standard form.

$$(x + 5)^2 + (y + 1)^2 = 9$$

$$(x - (-5))^2 + (y - (-1))^2 = 3^2 \dots (2)$$

By comparing equation (2) with (1), we get

Centre = (-5, -1) and radius = 3

∴ The centre of the circle is (-5, -1) and the radius is 3.

(iii) $x^2 + y^2 - 4x + 6y = 5$

Given:

The equation $x^2 + y^2 - 4x + 6y = 5$

We need to find the centre and the radius.

By using the standard equation formula,

$$(x - a)^2 + (y - b)^2 = r^2 \dots (1)$$

Now let us convert given circle's equation into the standard form.

$$x^2 + y^2 - 4x + 6y = 5$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 5 + 4 + 9$$

$$(x - 2)^2 + (y + 3)^2 = 18$$

$$(x - 2)^2 + (y - (-3))^2 = (3\sqrt{2})^2 \dots (2)$$

By comparing equation (2) with (1), we get

$$\text{Centre} = (2, -3) \text{ and radius} = 3\sqrt{2}$$

\therefore The centre of the circle is (2, -3) and the radius is $3\sqrt{2}$.

(iv) $x^2 + y^2 - x + 2y - 3 = 0$

Given:

$$\text{The equation } x^2 + y^2 - x + 2y - 3 = 0$$

We need to find the centre and the radius.

By using the standard equation formula,

$$(x - a)^2 + (y - b)^2 = r^2 \dots (1)$$

Now let us convert given circle's equation into the standard form.

$$x^2 + y^2 - x + 2y - 3 = 0$$

$$(x^2 - x + \frac{1}{4}) + (y^2 + 2y + 1) - 3 - \frac{1}{4} - 1 = 0$$

$$(x - \frac{1}{2})^2 + (y + 1)^2 = \frac{17}{4} \dots (2)$$

By comparing equation (2) with (1), we get

$$\text{Centre} = (\frac{1}{2}, -1) \text{ and radius} = \sqrt{17/2}$$

\therefore The centre of the circle is $(\frac{1}{2}, -1)$ and the radius is $\sqrt{17/2}$.

3. Find the equation of the circle whose centre is (1, 2) and which passes through the point (4, 6).

Solution:

Given:

Centre is (1, 2) and which passes through the point (4, 6).

Where, $p = 1, q = 2$

We need to find the equation of the circle.

By using the formula,

$$(x - p)^2 + (y - q)^2 = r^2$$

$$(x - 1)^2 + (y - 2)^2 = r^2$$

It passes through the point (4, 6)

$$(4 - 1)^2 + (6 - 2)^2 = r^2$$

$$3^2 + 4^2 = r^2$$

$$9 + 16 = r^2$$

$$25 = r^2$$

$$r = \sqrt{25}$$

$$= 5$$

So $r = 5$ units

We know that the equation of the circle with centre (p, q) and having radius 'r' is given

$$\text{by: } (x - p)^2 + (y - q)^2 = r^2$$

By substitute the values in the above equation, we get

$$(x - 1)^2 + (y - 2)^2 = 5^2$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 = 25$$

$$x^2 + y^2 - 2x - 4y - 20 = 0.$$

\therefore The equation of the circle is $x^2 + y^2 - 2x - 4y - 20 = 0$.

4. Find the equation of the circle passing through the point of intersection of the lines $x + 3y = 0$ and $2x - 7y = 0$ and whose centre is the point of intersection of the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0$.

Solution:

Let us find the points of intersection of the lines.

On solving the lines $x + 3y = 0$ and $2x - 7y = 0$, we get the point of intersection to be $(0, 0)$

On solving the lines $x + y + 1 = 0$ and $x - 2y + 4 = 0$, we get the point of intersection to be $(-2, 1)$

We have circle with centre $(-2, 1)$ and passing through the point $(0, 0)$.

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

$$\text{So, the equation is } (x - p)^2 + (y - q)^2 = r^2$$

$$\text{Where, } p = -2, q = 1$$

$$(x + 2)^2 + (y - 1)^2 = r^2 \dots (1)$$

Equation (1) passes through $(0, 0)$

$$\text{So, } (0 + 2)^2 + (0 - 1)^2 = r^2$$

$$4 + 1 = r^2$$

$$5 = r^2$$

$$r = \sqrt{5}$$

We know that the equation of the circle with centre (p, q) and having radius 'r' is given

$$\text{by: } (x - p)^2 + (y - q)^2 = r^2$$

By substitute the values in the above equation, we get

$$(x - (-2))^2 + (y - 1)^2 = (\sqrt{5})^2$$

$$(x + 2)^2 + (y - 1)^2 = 5$$

$$x^2 + 4x + 4 + y^2 - 2y + 1 = 5$$

$$x^2 + y^2 + 4x - 2y = 0$$

∴ The equation of the circle is $x^2 + y^2 + 4x - 2y = 0$.

5. Find the equation of the circle whose centre lies on the positive direction of y - axis at a distance 6 from the origin and whose radius is 4.

Solution:

It is given that the centre lies on the positive y - axis at a distance of 6 from the origin, we get the centre (0, 6).

We have a circle with centre (0, 6) and having radius 4.

We know that the equation of the circle with centre (p, q) and having radius 'r' is given

by: $(x - p)^2 + (y - q)^2 = r^2$

Where, $p = 0, q = 6, r = 4$

Now by substituting the values in the equation, we get

$$(x - 0)^2 + (y - 6)^2 = 4^2$$

$$x^2 + y^2 - 12y + 36 = 16$$

$$x^2 + y^2 - 12y + 20 = 0.$$

∴ The equation of the circle is $x^2 + y^2 - 12y + 20 = 0$.

6. If the equations of two diameters of a circle are $2x + y = 6$ and $3x + 2y = 4$ and the radius is 10, find the equation of the circle.

Solution:

It is given that the circle has the radius 10 and has diameters $2x + y = 6$ and $3x + 2y = 4$.

We know that the centre is the intersection point of the diameters.

On solving the diameters, we get the centre to be (8, -10).

We have a circle with centre (8, -10) and having radius 10.

By using the formula,

We know that the equation of the circle with centre (p, q) and having radius 'r' is given

by: $(x - p)^2 + (y - q)^2 = r^2$

Where, $p = 8, q = -10, r = 10$

Now by substituting the values in the equation, we get

$$(x - 8)^2 + (y - (-10))^2 = 10^2$$

$$(x - 8)^2 + (y + 10)^2 = 100$$

$$x^2 - 16x + 64 + y^2 + 20y + 100 = 100$$

$$x^2 + y^2 - 16x + 20y + 64 = 0.$$

∴ The equation of the circle is $x^2 + y^2 - 16x + 20y + 64 = 0$.

7. Find the equation of the circle

(i) which touches both the axes at a distance of 6 units from the origin.

(ii) Which touches x - axis at a distance of 5 from the origin and radius 6 units.

(iii) Which touches both the axes and passes through the point (2, 1).

(iv) Passing through the origin, radius 17 and ordinate of the centre is - 15.

Solution:

(i) which touches both the axes at a distance of 6 units from the origin.

A circle touches the axes at the points $(\pm 6, 0)$ and $(0, \pm 6)$.

So, a circle has a centre $(\pm 6, \pm 6)$ and passes through the point $(0, 6)$.

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

So, the equation is $(x - p)^2 + (y - q)^2 = r^2$

Where, $p = 6, q = 6$

$$(x - 6)^2 + (y - 6)^2 = r^2 \dots (1)$$

Equation (1) passes through $(0, 6)$

$$\text{So, } (0 - 6)^2 + (6 - 6)^2 = r^2$$

$$36 + 0 = r^2$$

$$r = \sqrt{36}$$

$$= 6$$

We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation, we get

$$(x \pm 6)^2 + (y \pm 6)^2 = (6)^2$$

$$x^2 \pm 12x + 36 + y^2 \pm 12y + 36 = 36$$

$$x^2 + y^2 \pm 12x \pm 12y + 36 = 0$$

\therefore The equation of the circle is $x^2 + y^2 \pm 12x \pm 12y + 36 = 0$.

(ii) Which touches x - axis at a distance of 5 from the origin and radius 6 units.

A circle touches the x - axis at the points $(\pm 5, 0)$.

Let us assume the centre of the circle is $(\pm 5, a)$.

We have a circle with centre $(5, a)$ and passing through the point $(5, 0)$ and having radius 6.

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

So, the equation is $(x - p)^2 + (y - q)^2 = r^2$

Where, $p = 5, q = a$

$$(x - 5)^2 + (y - a)^2 = r^2 \dots (1)$$

Equation (1) passes through $(5, 0)$

$$\text{So, } (5 - 5)^2 + (0 - a)^2 = r^2$$

$$0 + 36 = r^2$$

$$r = \sqrt{36}$$

$$= 6$$

We have got the centre at $(\pm 5, \pm 6)$ and having radius 6 units.

We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation, we get

$$(x \pm 5)^2 + (y \pm 6)^2 = (6)^2$$

$$x^2 \pm 10x + 25 + y^2 \pm 12y + 36 = 36$$

$$x^2 + y^2 \pm 10x \pm 12y + 25 = 0.$$

\therefore The equation of the circle is $x^2 + y^2 \pm 10x \pm 12y + 25 = 0$.

(iii) Which touches both the axes and passes through the point $(2, 1)$.

Let us assume the circle touches the x-axis at the point $(a, 0)$ and y-axis at the point $(0, a)$.

Then the centre of the circle is (a, a) and radius is a .

Its equation will be $(x - p)^2 + (y - q)^2 = r^2$

By substituting the values we get

$$(x - a)^2 + (y - a)^2 = a^2 \dots (1)$$

So now, equation (1) passes through P $(2, 1)$

By substituting the values we get

$$(2 - a)^2 + (1 - a)^2 = a^2$$

$$4 - 4a + a^2 + 1 - 2a + a^2 = a^2$$

$$5 - 6a + a^2 = 0$$

$$(a - 5)(a - 1) = 0$$

So, $a = 5$ or 1

Case (i)

We have got the centre at $(5, 5)$ and having radius 5 units.

We know that the equation of the circle with centre (p, q) and having radius 'r' is given

by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation we get

$$(x - 5)^2 + (y - 5)^2 = 5^2$$

$$x^2 - 10x + 25 + y^2 - 10y + 25 = 25$$

$$x^2 + y^2 - 10x - 10y + 25 = 0.$$

\therefore The equation of the circle is $x^2 + y^2 - 10x - 10y + 25 = 0$.

Case (ii)

We have got the centre at $(1, 1)$ and having a radius 1 unit.

We know that the equation of the circle with centre (p, q) and having radius 'r' is given

by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation we get

$$(x - 1)^2 + (y - 1)^2 = 1^2$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 1$$

$$x^2 + y^2 - 2x - 2y + 1 = 0$$

∴ The equation of the circle is $x^2 + y^2 - 2x - 2y + 1 = 0$.

(iv) Passing through the origin, radius 17 and ordinate of the centre is - 15.

Let us assume the abscissa as 'a'

We have a circle with centre (a, - 15) and passing through the point (0, 0) and having radius 17.

We know that the radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

By using the distance formula,

$$r = \sqrt{(a - 0)^2 + (-15 - 0)^2}$$

$$17 = \sqrt{(a - 0)^2 + (-15 - 0)^2}$$

$$17^2 = a^2 + (-15)^2$$

$$289 = a^2 + 225$$

$$a^2 = 64$$

$$|a| = \sqrt{64}$$

$$|a| = 8$$

$$a = \pm 8 \dots (1)$$

We have got the centre at (± 8 , - 15) and having radius 17 units.

We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation, we get

$$(x \pm 8)^2 + (y - 15)^2 = 17^2$$

$$x^2 \pm 16x + 64 + y^2 - 30y + 225 = 289$$

$$x^2 + y^2 \pm 16x - 30y = 0.$$

∴ The equation of the circle is $x^2 + y^2 \pm 16x - 30y = 0$.

8. Find the equation of the circle which has its centre at the point (3, 4) and touches the straight line $5x + 12y - 1 = 0$.

Solution:

It is given that we need to find the equation of the circle with centre (3, 4) and touches the straight line $5x + 12y - 1 = 0$.

We know that the perpendicular distance from the point (x_1, y_1) on to the line $ax + by + c = 0$ is given by

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Let us assume 'r' be the radius of the circle.

$$\begin{aligned} r &= \frac{|5(3) + 12(4) - 1|}{\sqrt{5^2 + 12^2}} \\ &= \frac{|15 + 48 - 1|}{\sqrt{25 + 144}} \\ &= \frac{|62|}{\sqrt{169}} \\ &= \frac{62}{13} \end{aligned}$$

We have a circle with centre (3, 4) and having a radius 62/13.

We know that the equation of the circle with centre (p, q) and having radius 'r' is given

by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation, we get

$$(x - 3)^2 + (y - 4)^2 = (62/13)^2$$

$$x^2 - 6x + 9 + y^2 - 8y + 16 = 3844/169$$

$$169x^2 + 169y^2 - 1014x - 1352y + 4225 = 3844$$

$$169x^2 + 169y^2 - 1014x - 1352y + 381 = 0$$

$$\therefore \text{The equation of the circle is } 169x^2 + 169y^2 - 1014x - 1352y + 381 = 0.$$

9. Find the equation of the circle which touches the axes and whose centre lies on $x - 2y = 3$.

Solution:

Let us assume the circle touches the axes at (a, 0) and (0, a) and we get the radius to be |a|.

We get the centre of the circle as (a, a). This point lies on the line $x - 2y = 3$

$$a - 2(a) = 3$$

$$-a = 3$$

$$a = -3$$

$$\text{Centre} = (a, a) = (-3, -3) \text{ and radius of the circle } (r) = |-3| = 3$$

We have circle with centre (-3, -3) and having radius 3.

We know that the equation of the circle with centre (p, q) and having radius 'r' is given

by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation, we get

$$(x - (-3))^2 + (y - (-3))^2 = 3^2$$

$$(x + 3)^2 + (y + 3)^2 = 9$$

$$x^2 + 6x + 9 + y^2 + 6y + 9 = 9$$

$$x^2 + y^2 + 6x + 6y + 9 = 0$$

$$\therefore \text{The equation of the circle is } x^2 + y^2 + 6x + 6y + 9 = 0.$$

10. A circle whose centre is the point of intersection of the lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ passes through the origin. Find its equation.

Solution:

It is given that the circle has the centre at the intersection point of the lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$ and passes through the origin

On solving the lines $2x - 3y + 4 = 0$ and $3x + 4y - 5 = 0$, we get the point of

intersection to be $\left(\frac{-1}{17}, \frac{22}{17}\right)$

We have circle with centre $\left(\frac{-1}{17}, \frac{22}{17}\right)$ and passing through the point $(0, 0)$.

We know that radius of the circle is the distance between the centre and any point on the radius. So, we find the radius of the circle.

By using the distance formula,

We know that the distance between the two points (x_1, y_1) and (x_2, y_2)

is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Let us assume r be the radius of the circle.

$$r = \sqrt{\left(\frac{-1}{17} - 0\right)^2 + \left(\frac{22}{17} - 0\right)^2}$$

$$r = \sqrt{\left(-\frac{1}{17}\right)^2 + \left(\frac{22}{17}\right)^2}$$

$$r = \sqrt{\frac{1}{289} + \frac{484}{289}}$$

$$r = \sqrt{\frac{485}{289}}$$

$$r = \frac{\sqrt{485}}{17}$$

We know that the equation of the circle with centre (p, q) and having radius 'r' is given by: $(x - p)^2 + (y - q)^2 = r^2$

Now by substituting the values in the equation, we get

$$\left(x - \left(\frac{-1}{17}\right)\right)^2 + \left(y - \frac{22}{17}\right)^2 = \left(\frac{\sqrt{485}}{17}\right)^2$$

$$\left(x + \frac{1}{17}\right)^2 + \left(y - \frac{22}{17}\right)^2 = \frac{485}{289}$$

$$x^2 + \frac{2x}{17} + \frac{1}{289} + y^2 - \frac{44y}{17} + \frac{484}{289} = \frac{485}{289}$$

$$17x^2 + 17y^2 + 2x - 44y = 0$$

∴ The equation of the circle is $17x^2 + 17y^2 + 2x - 44y = 0$.