Exercise 6(D)

1. Solve the pairs of equations:
9/x – 4/y = 8
13/x + 7/y = 101
Solution:

Given equations,
9/x – 4/y = 8 … (1)
13/x + 7/y = 101 … (2)

Performing (1)×7 + (2)×4, we get
63/x – 28/y = 56
52/x + 28/y = 202

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115/x = 460
x = 115/460
⇒ x = 1/4
Now, substituting value of x in (i), we get
9 × (4/1) – (4/y) = 8
-(4/y) = - 28
⇒ y = 1/7
Therefore, the solution is x = ¼ and y = 1/7.

2. Solve the pairs of equations:
(3/x) + (2/y) = 10
(9/x) – (7/y) = 10.5
Solution:

Given equations,
(3/x) + (2/y) = 10 … (i)
(9/x) – (7/y) = 10.5 … (ii)
On multiplying equation (i) by 3, we get
(9/x) + (6/y) = 30 …(iii)
Now, on subtracting (ii) from (iii), we get
13/y = 19.5
y = 13/19.5
⇒ y = 2/3
On substituting the value of y in (i), we get
(3/x) + (2 × 3)/2 = 10
(3/x) + 3 = 10
3/x = 7
⇒ x = 3/7
Therefore, the solution is x = ¼ and y = 1/7.

3. 5x + (8/y) = 19

3x – (4/y) = 7

Solution:

Given equations,
5x + (8/y) = 19 … (i)
3x – (4/y) = 7 … (ii)
On multiplying equation (ii) by 2, we get
6x – (8/y) = 14 … (iii)
On adding (i) and (iii), we get
11x = 33
⇒ x = 3
Substituting x = 3 in equation (1), we get
5(3) + (8/y) = 19
(8/y) = 19-15
y = (8/4)
⇒ y = 2
Therefore, the solution is x = 3 and y = 2.

4. Solve: 4x + (6/y) = 15 and 3x – (4/y) = 7. Hence, find 'a' if y = ax - 2

Solution:

Given equations,
4x + (6/y) = 15 … (i)
3x – (4/y) = 7 … (ii)
On multiplying (i) by 4 and (ii) by 6
16x + (24/y) = 60 … (iii)
18x – (24/y) = 42 … (iv)
Now, on adding (iii) and (iv), we get
34x = 102
⇒ x = 3
On substituting x = 3 in (i), we get
4(3) + (6/y) = 15
6/y = 15 – 12
y = 6/3
⇒ y = 2
Now, y = ax – 2
2 = a(3) – 2
2 = 3a – 2
3a = 4
∴ a = 4/3

5. Solve: (3/x) – (2/y) = 0 and (2/x) + (5/y) = 19. Hence, find 'a' if y = ax + 3.

Solution:

Given equations,
(3/x) – (2/y) = 0 … (1)

(2/x) + (5/y) = 19 … (2)
Performing (1)×5 and (2)×2, we get
(15/x) – (10/y) = 0
(4/x) + (10/y) = 38

19/x = 38
x = 38/19
⇒ x = 1/2
Now, substituting the value of x in (1), we get
3(1/2) – (2/y) = 0
y = 1/3
So, y = ax + 3
(1/3) = a(1/2) + 3
a/2 = (-8/3)
∴ a = -16/3

6. Solve:
(i) 20/(x + y) + 3/(x - y) = 7
8/(x - y) – 15/(x + y) = 5
(ii) 34/(3x + 4y) + 15/(3x – 2y) = 5
25/(3x - 2y) – 8.50/(3x + 4y) = 4.5
Solution:

(i) Given equations,
20/(x + y) + 3/(x - y) = 7 … (1)
8/(x + y) – 15(x + y) = 5 … (2)
Performing (1)×8 - (2)×3, we get
160/(x + y) + 24/(x - y) = 56
-45/(x + y) + 24(x - y) = 15
(-) (-) (-)

205/(x + y) = 41
x + y = 205/41
⇒ x + y = 5 … (3)
Using (3) in (1), we get
(20/5) + 3/(x - y) = 7
3/(x - y) = 3
⇒ x – y = 1 … (4)
Now, adding (3) and (4), we get
x + y = 5
x – y = 1

2x = 6
⇒ x = 3
On substituting the value of x in (3), we get
3 + y = 5

⇒ y = 2
∴ The solution is x = 3 and y = 2

(ii) Let's assume a = 3x + 4y and b = 3x – 2y
Then, the given equations will become
(34/a) + (15/b) = 5 … (i)
-(8.50/a) + (25/b) = 4.5 … (ii)
Performing (ii)×4 + (i), we get
-(34/a) + (100/b) = 18
34/a + 15/b = 5
(+) (+) (+)
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115/b = 23
So, b = 115/23
⇒ b = 5
Now, we have 3x – 2y = 5 …(iii)
And, on substituting b = 5 in equation (i), we get
(34/a) + (15/5) = 5
2a = 34
⇒ a = 17
So, 3x + 4y = 17 …(iv)
On subtracting equation (iv) from equation (iii), we get
3x – 2y = 5
3x + 4y = 17
(-) (-) (-)
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-6y = -12
y = 2
Now, on substituting y = 2 in equation (iii), we get
3x – 2(2) = 5
3x = 9
⇒ x = 3
∴ The solution is x = 3 and y = 2.

7. Solve:
(i) x + y = 2xy
x - y = 6xy
(ii) x+ y = 7xy
2x - 3y = -xy
Solution:

(i) Given equations,
x + y = 2xy ... (i)
x – y = 6xy … (ii)
_____ [Addition]

2x = 8xy
2 = 8y
⇒ y = ¼
On substituting the value of y in (i), we get
x + ¼ = 2 × (1/4)
1/2x = -1/4
⇒ x = -½
∴ The solution is x = -½ and y = ¼.

(ii) Given equations,
x + y = 7xy ... (1)
2x - 3 = -xy ... (2)
Performing (1)×3 + (2), we get
3x + 3y = 21xy
2x – 3y = - xy

5x = 20xy
So, y = 5x/20x
y = 1/4
Now, on substituting the value of y in (i), we get
x + 1/4 = 7 × (1/4)
¼ = ¾ x
⇒ x = 1/3
∴ The solution is x = 1/3 and y = ¼.

8. Solve:
$a/x – b/y = 0$
$ab^2/x + a^2b/y = a^2 + b^2$
Solution:

Given equations are $a/x – b/y = 0$ and $ab^2/x + a^2b/y = a^2 + b^2$
On taking 1/x = u and 1/y = v, the above system of equations become
au – bv + 0 = 0
$ab^2u + a^2bv – (a^2 + b^2) = 0$
By cross-multiplication, we have
$u/(-b \times (-a^2 + b^2) – a^2b \times 0) = -v/[a \times (-a^2 + b^2) – ab^2 \times 0] = 1/(a \times a^2b - ab^2 \times -b)$
$u/b(a^2 + b^2) = -v/[-a(a^2 + b^2)] = 1/(a^3b + ab^3)$
$u/b(a^2 + b^2) = v/[a(a^2 + b^2)] = 1/[ab(a^2 + b^2)]$
Hence,
$u = [b(a^2 + b^2)]/ [ab(a^2 + b^2)]$ and $v = a(a^2 + b^2)/ [ab(a^2 + b^2)]$
u = 1/a and v = 1/b
1/x = 1/a and 1/y = 1/b
Therefore, the solution is x = a and y = b.

9. Solve:

2xy/(x + y) = 3/2
xy/(2x - y) = - 3/10;
x + y ≠ 0 and 2x – y ≠ 0
Solution:

Given equations,
2xy/(x + y) = 3/2
(x + y)/xy = 4/3
⇒ (1/x) + (1/y) = 4/3 … (1)
And, xy/(2x - y) = -3/10
(2x - y)/xy = -10/3
⇒ -(1/x) + (2/y) = -10/3 … (2)
Let's assume 1/x = u and 1/y = v
Then equations (1) and (2) become,
u + v = 4/3 and -u + 2v = -10/3
Now, on multiplying and adding the above equations by 3, we get
3u + 3v = 4
-3u + 6v = -10
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9v = -6
⇒ v = -(6/9) = -(2/3)
So, 1/y = -(2/3)
⇒ y = -(3/2)
On substituting y = -(3/2) in (1), we get
1/x – 2/3 = 4/3
1/x = 6/3 = 2
⇒ x = 1/2
Therefore, the solution is x = ½ and y = -3/2.

10. Solve:
3/2x + 2/3y = -1/3
3/4x + 1/2y = -1/8
Solution:

Given equations are 3/2x + 2/3y = -1/3 and 3/4x + 1/2y = -1/8
Let's assume 1/x = u and 1/y = v
Then the system of equations become,
3u/2 + 2v/3 = -1/3 and 3u/4 + v/2 = -1/8
(9u + 4v)/6 = -1/3 and (3u + 2v)/4 = -1/8
⇒ 9u + 4v = -2 and 3u + 2v = -1/2
On multiplying the equations by 3 and 8 respectively, we get
27u + 12v = -6 and 24u + 16v = -4
⇒ 27u + 12v + 6 = 0 and 24u + 16v + 4 = 0
By cross-multiplication method, we have
u/(12 × 4 – 16 × 6) = -v/(27 × 4 – 24 × 6) = 1/ (27 × 16 – 24 × 12)

u/(48 – 96) = -v/(108 – 144) = 1/(432 – 288)
u/-48 = -v/-36 = 1/144
u/-48 = v/36 = 1/144
⇒ u = -48/144 = -1/3 and v = 36/144 = ¼
So, 1/x = -1/3 and 1/y = ¼
⇒ x = -3 and y = 4
Therefore, the solutions is x = -3 and y = 4.