EXERCISE 22.2

1. Find the locus of a point equidistant from the point (2, 4) and the y-axis.

Solution:

Let P (h, k) be any point on the locus and let A (2, 4) and B (0, k).

Then, PA = PB

$PA^2 = PB^2$

By using distance formula:

Distance of (h, k) from (2, 4) = $\sqrt{(h-2)^2 + (k-4)^2}$

Distance of (h, k) from (0, k) = $\sqrt{(h-0)^2 + (k-k)^2}$

So both the distances are same.

$\therefore \sqrt{(h-2)^2 + (k-4)^2} = \sqrt{(h-0)^2 + (k-k)^2}$

By squaring on both the sides we get,

$(h-2)^2 + (k-4)^2 = (h-0)^2 + (k-k)^2$

$h^2 + 4 - 4h + k^2 - 8k + 16 = h^2 + 0$

$k^2 - 4h - 8k + 20 = 0$

Replace (h, k) with (x, y)

∴ The locus of point equidistant from (2, 4) and y-axis is

$y^2 - 4x - 8y + 20 = 0$

2. Find the equation of the locus of a point which moves such that the ratio of its distance from (2, 0) and (1, 3) is 5: 4.

Solution:

Let P (h, k) be any point on the locus and let A (2, 0) and B (1, 3).

So then, PA/ BP = 5/4

$PA^2 = BP^2 = 25/16$

Distance of (h, k) from (2, 0) = $\sqrt{(h-2)^2 + (k-0)^2}$

Distance of (h, k) from (1, 3) = $\sqrt{(h-1)^2 + (k-3)^2}$

So,

$$\frac{\sqrt{(h-2)^2 + (k-0)^2}}{\sqrt{(h-1)^2 + (k-3)^2}} = \frac{5}{4}$$

By squaring on both the sides we get,

$16\{(h-2)^2 + k^2\} = 25\{(h-1)^2 + (k-3)^2\}$

$16\{h^2 + 4 - 4h + k^2\} = 25\{h^2 - 2h + 1 + k^2 - 6k + 9\}$

$9h^2 + 9k^2 + 14h - 150k + 186 = 0$

Replace (h, k) with (x, y)

∴ The locus of a point which moves such that the ratio of its distance from (2, 0) and (1, 3) is 5: 4 which is

$9x^2 + 9y^2 + 14x - 150y + 186 = 0$

3. A point moves as so that the difference of its distances from (ae, 0) and (-ae, 0) is 2a, prove that the equation to its locus is

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ **, where $b^2 = a^2(e^2 - 1)$.**

Solution:

Let P (h, k) be any point on the locus and let A (ae, 0) and B (-ae, 0).

Where, PA – PB = 2a

Distance of (h, k) from (ae, 0) = $\sqrt{(h - ae)^2 + (k - 0)^2}$

Distance of (h, k) from (-ae, 0) = $\sqrt{(h - (-ae))^2 + (k - 0)^2}$

So,

$$\sqrt{(h - ae)^2 + (k - 0)^2} - \sqrt{(h - (-ae))^2 + (k - 0)^2} = 2a$$

$$\sqrt{(h - ae)^2 + (k - 0)^2} = 2a + \sqrt{(h + ae)^2 + (k - 0)^2}$$

By squaring on both the sides we get:

$$(h - ae)^2 + (k - 0)^2 = \left\{2a + \sqrt{(h + ae)^2 + (k - 0)^2}\right\}^2$$

$$\Rightarrow h^2 + a^2e^2 - 2aeh + k^2 = 4a^2 + \{(h + ae)^2 + k^2\} + 4a\sqrt{(h + ae)^2 + (k - 0)^2}$$

$$\Rightarrow h^2 + a^2e^2 - 2aeh + k^2$$

$$= 4a^2 + h^2 + 2aeh + a^2e^2 + k^2 + 4a\sqrt{(h + ae)^2 + (k - 0)^2}$$

$$-4aeh - 4a^2 = 4a\sqrt{(h + ae)^2 + (k - 0)^2}$$

$$-4a(eh + a) = 4a\sqrt{(h + ae)^2 + (k - 0)^2}$$

Now again let us square on both the sides we get,

$(eh + a)^2 = (h + ae)^2 + (k - 0)^2$

$e^2h^2 + a^2 + 2aeh = h^2 + a^2e^2 + 2aeh + k^2$

$h^2(e^2 - 1) - k^2 = a^2(e^2 - 1)$

$$\frac{h^2}{a^2} - \frac{k^2}{a^2(e^2 - 1)} = 1$$

$$\frac{h^2}{a^2} - \frac{k^2}{b^2} = 1 \ [where, b^2 = a^2(e^2 - 1)]$$

Now let us replace (h, k) with (x, y)
The locus of a point such that the difference of its distances from (ae, 0) and (-ae, 0) is 2a.
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Where $b^2 = a^2 (e^2 - 1)$
Hence proved.

4. Find the locus of a point such that the sum of its distances from (0, 2) and (0, -2) is 6.
Solution:
Let P (h, k) be any point on the locus and let A (0, 2) and B (0, -2).
Where, PA – PB = 6
Distance of (h, k) from (0, 2) = $\sqrt{(h-0)^2 + (k-2)^2}$
Distance of (h, k) from (0, -2) = $\sqrt{(h-0)^2 + (k-(-2))^2}$
So,
$\sqrt{(h)^2 + (k-2)^2} + \sqrt{(h)^2 + (k+2)^2} = 6$
$\sqrt{(h)^2 + (k-2)^2} = 6 - \sqrt{(h)^2 + (k+2)^2}$
By squaring on both the sides we get,
$h^2 + (k-2)^2 = \left\{6 - \sqrt{h^2 + (k+2)^2}\right\}^2$
$\Rightarrow h^2 + 4 - 4k + k^2 = 36 + \{h^2 + k^2 + 4k + 4\} - 12\sqrt{h^2 + (k+2)^2}$
$\Rightarrow -8k - 36 = -12\sqrt{h^2 + (k+2)^2}$
$\Rightarrow -4(2k+9) = -12\sqrt{h^2 + (k+2)^2}$
Now, again let us square on both the sides we get,
$(2k+9)^2 = \left\{3\sqrt{h^2 + (k+2)^2}\right\}^2$
$4k^2 + 81 + 36k = 9(h^2 + k^2 + 4k + 4)$
$9h^2 + 5k^2 = 45$
By replacing (h, k) with (x, y)
∴ The locus of a point is
$9x^2 + 5y^2 = 45$

5. Find the locus of a point which is equidistant from (1, 3) and x-axis.
Solution:
Let P (h, k) be any point on the locus and let A (1, 3) and B (h, 0).
Where, PA = PB

Distance of (h, k) from (1, 3) = $\sqrt{(h-1)^2 + (k-3)^2}$

Distance of (h, k) from (h, 0) = $\sqrt{(h-h)^2 + (k-0)^2}$

It is given that both distance are same.

So,

$\sqrt{(h-1)^2 + (k-3)^2} = \sqrt{(h-h)^2 + (k-0)^2}$

Now, let us square on both the sides we get,

$(h-1)^2 + (k-3)^2 = (h-h)^2 + (k-0)^2$

$h^2 + 1 - 2h + k^2 - 6k + 9 = k^2 + 0$

$h^2 - 2h - 6k + 10 = 0$

By replacing (h, k) with (x, y),

∴ The locus of point equidistant from (1, 3) and x-axis is

$x^2 - 2x - 6y + 10 = 0$

6. Find the locus of a point which moves such that its distance from the origin is three times is distance from x-axis.

Solution:

Let P (h, k) be any point on the locus and let A (0, 0) and B (h, 0).

Where, PA = 3PB

Distance of (h, k) from (0, 0) = $\sqrt{(h-0)^2 + (k-0)^2}$

Distance of (h, k) from (h, 0) = $\sqrt{(h-h)^2 + (k-0)^2}$

So, where PA = 3PB

∴ $\sqrt{(h-0)^2 + (k-0)^2} = 3\sqrt{(h-h)^2 + (k-0)^2}$

Now by squaring on both the sides we get,

$h^2 + k^2 = 9k^2$

$h^2 = 8k^2$

By replacing (h, k) with (x, y)

∴ The locus of point is $x^2 = 8y^2$