

Exercise 4(A)

1. Find the square of:

(i) $2a + b$

(ii) $3a + 7b$

(iii) $3a - 4b$

(iv) $3a/2b - 2b/3a$

Solution:

Using the identities,

$$(a + b)^2 = a^2 + b^2 + 2ab \text{ and}$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned} \text{(i) } (2a + b)^2 &= (2a)^2 + b^2 + 2(2a)(b) \\ &= 4a^2 + b^2 + 4ab \end{aligned}$$

$$\begin{aligned} \text{(ii) } (3a + 7b)^2 &= (3a)^2 + (7b)^2 + 2(3a)(7b) \\ &= 9a^2 + 49b^2 + 42ab \end{aligned}$$

$$\begin{aligned} \text{(iii) } (3a - 4b)^2 &= (3a)^2 + (4b)^2 - 2(3a)(4b) \\ &= 9a^2 + 16b^2 - 24ab \end{aligned}$$

$$\begin{aligned} \text{(iv) } (3a/2b - 2b/3a)^2 &= (3a/2b)^2 + (2b/3a)^2 - 2(3a/2b)(2b/3a) \\ &= 9a^2/4b^2 + 4b^2/9a^2 - 2 \end{aligned}$$

2. Use identities to evaluate:

(i) $(101)^2$

(ii) $(502)^2$

(iii) $(97)^2$

(iv) $(998)^2$

Solution:

Using the identities,

$$(a + b)^2 = a^2 + b^2 + 2ab \text{ and}$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\begin{aligned} \text{(i) } (101)^2 &= (100 + 1)^2 \\ &= 100^2 + 1^2 + 2 \times 100 \times 1 \\ &= 10000 + 1 + 200 \\ &= 10201 \end{aligned}$$

$$\begin{aligned} \text{(ii) } (502)^2 &= (500 + 2)^2 \\ &= 500^2 + 2^2 + 2 \times 500 \times 2 \\ &= 250000 + 4 + 2000 \\ &= 252004 \end{aligned}$$

$$\text{(iii) } (97)^2 = (100 - 3)^2$$

$$\begin{aligned}
 &= 100^2 + 3^2 - 2 \times 100 \times 3 \\
 &= 10000 + 9 - 600 \\
 &= 9409
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) } (998)^2 &= (1000 - 2)^2 \\
 &= 1000^2 + 2^2 - 2 \times 1000 \times 2 \\
 &= 1000000 + 4 - 4000 \\
 &= 996004
 \end{aligned}$$

3. Evaluate:

(i) $(7x/8 + 4y/5)^2$

(ii) $(2x/7 - 7y/4)^2$

Solution:

Using the identities,

$$(a + b)^2 = a^2 + b^2 + 2ab \text{ and}$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

(i) We have, $(7x/8 + 4y/5)^2$
 $= (7x/8)^2 + (4y/5)^2 + 2 \times (7x/8) \times (4y/5)$
 $= 49x^2/64 + 16y^2/25 + 7xy/5$

(ii) We have, $(2x/7 - 7y/4)^2$
 $= (2x/7)^2 + (7y/4)^2 - 2 \times (2x/7) \times (7y/4)$
 $= 4x^2/49 + 49y^2/16 - xy$

4. Evaluate:

(i) $(a/2b + 2b/a)^2 - (a/2b - 2b/a)^2 - 4$

(ii) $(4a + 3b)^2 - (4a - 3b)^2 + 48ab$

Solution:

Using the identities,

$$(a + b)^2 = a^2 + b^2 + 2ab \text{ and}$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

(i) Given expression, $\left(\frac{a}{2b} + \frac{2b}{a}\right)^2 - \left(\frac{a}{2b} - \frac{2b}{a}\right)^2 - 4$

On expanding the term using the identity, we have

$$\begin{aligned}
 (a/2b + 2b/a)^2 &= (a/2b)^2 + (2b/a)^2 + 2 \times (a/2b) \times (2b/a) \\
 &= a^2/4b + 4b^2/a^2 + 2
 \end{aligned}$$

Next, expanding the second term using the identity, we have

$$\begin{aligned}
 (a/2b - 2b/a)^2 &= (a/2b)^2 + (2b/a)^2 - 2 \times (a/2b) \times (2b/a) \\
 &= a^2/4b + 4b^2/a^2 - 2
 \end{aligned}$$

Now, using these results in the given expression

$$\begin{aligned}(a/2b + 2b/a)^2 - (a/2b - 2b/a)^2 - 4 &= (a^2/4b + 4b^2/a^2 + 2) - (a^2/4b + 4b^2/a^2 - 2) - 4 \\ &= a^2/4b + 4b^2/a^2 + 2 - a^2/4b - 4b^2/a^2 + 2 - 4 \\ &= 0\end{aligned}$$

(ii) Given expression, $(4a + 3b)^2 - (4a - 3b)^2 + 48ab$

On expanding the term using the identity, we have

$$\begin{aligned}(4a + 3b)^2 &= (4a)^2 + (3b)^2 + 2 \times (4a) \times (3b) \\ &= 16a^2 + 9b^2 + 24ab\end{aligned}$$

Next, expanding the second term using the identity, we have

$$\begin{aligned}(4a - 3b)^2 &= (4a)^2 + (3b)^2 - 2 \times (4a) \times (3b) \\ &= 16a^2 + 9b^2 - 24ab\end{aligned}$$

Now, using these results in the given expression

$$\begin{aligned}(4a + 3b)^2 - (4a - 3b)^2 + 48ab &= (16a^2 + 9b^2 + 24ab) - (16a^2 + 9b^2 - 24ab) + 48ab \\ &= 16a^2 + 9b^2 + 24ab - 16a^2 - 9b^2 + 24ab + 48ab \\ &= 96ab\end{aligned}$$

5. If $a + b = 7$ and $ab = 10$; find $a - b$.

Solution:

Using the identities,

$$(a + b)^2 = a^2 + b^2 + 2ab \text{ and}$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

Rewriting the above equation as

$$\begin{aligned}(a - b)^2 &= a^2 + b^2 + 2ab - 4ab \\ &= (a + b)^2 - 4ab \dots (i)\end{aligned}$$

We have,

$$a + b = 7 \text{ and } ab = 10$$

So, using these in equation (i), we get

$$\begin{aligned}(a - b)^2 &= (7)^2 - (4 \times 10) \\ &= 49 - 40 \\ &= 9\end{aligned}$$

Then,

$$\begin{aligned}(a - b) &= \sqrt{9} \\ &= \pm 3\end{aligned}$$

6. If $a - b = 7$ and $ab = 18$; find $a + b$.

Solution:

Using the identities,

$$(a - b)^2 = a^2 + b^2 - 2ab \text{ and}$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

Rewriting the above equation as

$$\begin{aligned}(a + b)^2 &= a^2 + b^2 - 2ab + 4ab \\ &= (a - b)^2 + 4ab \dots (i)\end{aligned}$$

We have,

$$a - b = 7 \text{ and } ab = 18$$

So, using these in equation (i), we get

$$\begin{aligned}(a + b)^2 &= (7)^2 + (4 \times 18) \\ &= 49 + 72 \\ &= 121\end{aligned}$$

Then,

$$\begin{aligned}(a + b) &= \sqrt{121} \\ &= \pm 11\end{aligned}$$

7. If $x + y = 7/2$ and $xy = 5/2$; find:

(i) $x - y$

(ii) $x^2 - y^2$

Solution:

Using the identities,

$$(x + y)^2 = x^2 + y^2 + 2xy \text{ and}$$

$$(x - y)^2 = x^2 + y^2 - 2xy$$

Rewriting the above equation as

$$\begin{aligned}(x - y)^2 &= x^2 + y^2 + 2xy - 4xy \\ &= (x + y)^2 - 4xy \dots (1)\end{aligned}$$

We have,

$$x + y = 7/2 \text{ and } xy = 5/2$$

So, using these in equation (1), we get

$$\begin{aligned}(x - y)^2 &= (7/2)^2 - (4 \times 5/2) \\ &= 49/4 - 10 \\ &= 9/4\end{aligned}$$

Then,

$$\begin{aligned}(x - y) &= \sqrt{9/4} \\ &= \pm 3/2 \dots (2)\end{aligned}$$

(ii) We know that,

$$x^2 - y^2 = (x + y)(x - y)$$

Substituting values in RHS using given and (2), we get

$$\begin{aligned}x^2 - y^2 &= (7/2)(\pm 3/2) \\ &= \pm 21/4\end{aligned}$$

8. If $a - b = 0.9$ and $ab = 0.36$; find:

(i) $a + b$

(ii) $a^2 - b^2$

Solution:

Using the identities,

$$(a - b)^2 = a^2 + b^2 - 2ab \text{ and}$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

Rewriting the above equation as

$$(a + b)^2 = a^2 + b^2 - 2ab + 4ab$$

$$= (a - b)^2 + 4ab \dots (1)$$

We have,

$$a - b = 0.9 \text{ and } ab = 0.36$$

So, using these in equation (1), we get

$$\begin{aligned} (a + b)^2 &= (0.9)^2 + (4 \times 0.36) \\ &= 0.81 + 1.44 \\ &= 2.25 \end{aligned}$$

Then,

$$\begin{aligned} (a + b) &= \sqrt{2.25} \\ &= \pm 1.5 \dots (2) \end{aligned}$$

(ii) We know that,

$$a^2 - b^2 = (a + b)(a - b)$$

Substituting values in RHS using given and (2), we get

$$\begin{aligned} a^2 - b^2 &= (\pm 1.5)(0.9) \\ &= \pm 1.35 \end{aligned}$$

9. If $a - b = 4$ and $a + b = 6$; find

(i) $a^2 + b^2$

(ii) ab

Solution:

Given, $a - b = 4$ and $a + b = 6$

We know that,

$$(a - b)^2 = a^2 + b^2 - 2ab \text{ and}$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

Now, rewriting the above equation as

$$(a + b)^2 = a^2 + b^2 - 2ab + 4ab$$

$$\Rightarrow (a + b)^2 = (a - b)^2 + 4ab$$

Substituting the values in the above equation, we get

$$(6)^2 = (4)^2 + 4ab$$

$$36 = 16 + 4ab$$

$$4ab = 36 - 16$$

$$4ab = 20$$

$$ab = 20/4$$

(ii) Thus, $ab = 5$

Now, in the identity: $(a + b)^2 = a^2 + b^2 + 2ab$

$$(a + b)^2 = (a^2 + b^2) + 2ab$$

Let's substitute the values of the known terms,

$$(6)^2 = (a^2 + b^2) + 2 \times (5)$$

$$36 = (a^2 + b^2) + 10$$

$$a^2 + b^2 = 36 - 10$$

(i) Thus, $a^2 + b^2 = 26$

10. If $a + 1/a = 6$ and $a \neq 0$ find:

- (i) $a - 1/a$
 (ii) $a^2 - 1/a^2$

Solution:

Using the identities,

$$(a - b)^2 = a^2 + b^2 - 2ab \text{ and}$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

(i) Now,

$$\begin{aligned} (a + 1/a)^2 &= a^2 + 1/a^2 + (2 \times a \times 1/a) \\ &= a^2 + 1/a^2 + 2 \dots (1) \end{aligned}$$

Substituting the value of $(a + 1/a)$ in the equation (1), we get

$$6^2 = a^2 + 1/a^2 + 2$$

$$36 = a^2 + 1/a^2 + 2$$

$$a^2 + 1/a^2 = 36 - 2 = 34 \dots (2)$$

Similarly,

$$\begin{aligned} (a - 1/a)^2 &= a^2 + 1/a^2 - (2 \times a \times 1/a) \\ &= (a^2 + 1/a^2) - 2 \\ &= 34 - 2 \dots [\text{From (2)}] \\ &= 32 \end{aligned}$$

$$\Rightarrow (a - 1/a)^2 = 32$$

$$a - 1/a = \pm\sqrt{32}$$

$$= \pm 4\sqrt{2} \dots (3)$$

$$\text{Thus, } a - 1/a = \pm 4\sqrt{2}$$

(ii) We know that,

$$a^2 - 1/a^2 = (a - 1/a)(a + 1/a)$$

Using the given and (3) in the above equation,

$$a^2 - 1/a^2 = (\pm 4\sqrt{2})(6)$$

$$= \pm 24\sqrt{2}$$

$$\text{Thus, } a^2 - 1/a^2 = \pm 24\sqrt{2}$$

11. If $a - 1/a = 8$ and $a \neq 0$, find:

(i) $a + 1/a$

(ii) $a^2 - 1/a^2$

Solution:

Using the identities,

$$(a - b)^2 = a^2 + b^2 - 2ab \text{ and}$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

(i) Now,

$$(a - 1/a)^2 = a^2 + 1/a^2 - (2 \times a \times 1/a)$$

$$= a^2 + 1/a^2 - 2 \dots (1)$$

Substituting the value of $(a + 1/a)$ in the equation (1), we get

$$8^2 = a^2 + 1/a^2 - 2$$

$$64 = a^2 + 1/a^2 - 2$$

$$a^2 + 1/a^2 = 64 + 2 = 66 \dots (2)$$

Similarly,

$$(a + 1/a)^2 = a^2 + 1/a^2 + (2 \times a \times 1/a)$$

$$= (a^2 + 1/a^2) + 2$$

$$= 66 + 2 \dots \text{[From (2)]}$$

$$= 68$$

$$\Rightarrow (a + 1/a)^2 = 68$$

$$a + 1/a = \sqrt{68}$$

$$= \pm 2\sqrt{17} \dots (3)$$

$$\text{Thus, } a + 1/a = \pm 2\sqrt{17}$$

(ii) We know that,

$$a^2 - 1/a^2 = (a - 1/a)(a + 1/a)$$

Using the given and (3) in the above equation,

$$a^2 - 1/a^2 = (8)(\pm 2\sqrt{17})$$

$$= \pm 16\sqrt{17}$$

$$\text{Thus, } a^2 - 1/a^2 = \pm 16\sqrt{17}$$

12. If $a^2 - 3a + 1 = 0$, and $a \neq 0$; find:

(i) $a + 1/a$

(ii) $a^2 + 1/a^2$

Solution:

(i) Given equation,

$$a^2 - 3a + 1 = 0$$

$$a^2 + 1 = 3a$$

$$(a^2 + 1)/a = 3$$

$$\Rightarrow a + 1/a = 3 \dots (1)$$

(ii) We know that,

$$(a + b)^2 = a^2 + b^2 + 2ab$$

Now,

$$(a + 1/a)^2 = a^2 + 1/a^2 + 2(a)(1/a)$$

$$= a^2 + 1/a^2 + 2$$

Using (1) in the above equation, we get

$$(3)^2 = a^2 + 1/a^2 + 2$$

$$9 = a^2 + 1/a^2 + 2$$

$$a^2 + 1/a^2 = 9 - 2$$

Thus, $a^2 + 1/a^2 = 7$

13. If $a^2 - 5a - 1 = 0$ and $a \neq 0$; find:

(i) $a - 1/a$

(ii) $a + 1/a$

(iii) $a^2 - 1/a^2$

Solution:

(i) Given, $a^2 - 5a - 1 = 0$

Rewriting the equation, we get

$$a^2 - 1 = 5a$$

$$(a^2 - 1)/a = 5$$

$$\text{Hence, } a - 1/a = 5 \dots (1)$$

(ii) We know that,

$$(a + 1/a)^2 = a^2 + 1/a^2 + 2$$

Manipulating the above as,

$$(a + 1/a)^2 = a^2 + 1/a^2 - 2 + 4$$

$$(a + 1/a)^2 = (a - 1/a)^2 + 4$$

Now, using (1) in the above

$$(a + 1/a)^2 = (5)^2 + 4$$

$$(a + 1/a)^2 = 25 + 4 = 29$$

$$\text{Hence, } a + 1/a = \pm\sqrt{29} \dots (2)$$

(iii) We know that,

$$a^2 - 1/a^2 = (a + 1/a)(a - 1/a)$$

Now, using (1) and (2) in the above equation, we get

$$a^2 - 1/a^2 = (5) \times (\pm\sqrt{29})$$

$$\text{Hence, } a^2 - 1/a^2 = \pm 5\sqrt{29}$$

14. If $3a + 4b = 16$ and $ab = 4$; find the value of $9a^2 + 16b^2$.

Solution:

Given, $3a + 4b = 16$ and $ab = 4$

Required to find: value of $9a^2 + 16b^2$

We know that,

$$(a + b)^2 = a^2 + b^2 + 2ab$$

Now, the square of $(3a + 4b)$ will be

$$\begin{aligned}(3a + 4b)^2 &= (3a)^2 + (4b)^2 + 2 \times (3a) \times (4b) \\ &= 9a^2 + 16b^2 + 24ab\end{aligned}$$

And, given $3a + 4b = 16$

So, by squaring on both the sides

$$(3a + 4b)^2 = 16^2$$

$$9a^2 + 16b^2 + 24ab = 256$$

$$\begin{aligned}9a^2 + 4b^2 + 24(4) &= 256 && \text{[Given } ab = 4\text{]} \\9a^2 + 4b^2 &= 256 - 96 \\ \Rightarrow 9a^2 + 4b^2 &= 160\end{aligned}$$

15. The number a is 2 more than the number b. If the sum of the squares of a and b is 34, then find the product of a and b.

Solution:

Given, a is 2 more than b

$$\Rightarrow a = b + 2$$

And, sum of squares of a and b is 34

$$\Rightarrow a^2 + b^2 = 34$$

Let's replace $a = (b + 2)$ in the above equation and solve for b

Then,

$$(b + 2)^2 + b^2 = 34$$

$$2b^2 + 4b - 30 = 0$$

$$b^2 + 2b - 15 = 0$$

$$(b + 5)(b - 3) = 0$$

So,

$$b = -5 \text{ or } 3$$

Now,

$$\text{For } b = -5, a = -5 + 2 = -3$$

$$\text{For } b = 3, a = 3 + 2 = 5$$

Thus, the product of a and b is 15 in both cases.  Myclass24 Your Class. Your Pace.

16. The difference between two positive numbers is 5 and the sum of their squares is 73. Find the product of these numbers.

Solution:

Let's assume the two positive numbers as a and b

Given, the difference between them is 5 and the sum of their squares is 73

So, we have

$$a - b = 5 \dots \text{(i) and}$$

$$a^2 + b^2 = 73 \dots \text{(ii)}$$

On squaring (i) on both sides, we get

$$(a - b)^2 = 5^2$$

$$(a^2 + b^2) - 2ab = 25$$

$$73 - 2ab = 25 \dots \text{[Using (ii), given]}$$

So,

$$2ab = 73 - 25 = 48$$

$$ab = 24$$

Therefore, the product of numbers is 24.