

EXERCISE 19.27

Evaluate the following integrals:

1. $\int e^{ax} \cos bx \, dx$

Solution:

Let $I = \int e^{ax} \cos bx \, dx$

Integrating by parts, we get

$$I = e^{ax} \frac{\sin bx}{b} - a \int e^{ax} \frac{\sin bx}{b} \, dx$$

Taking $1/b$ as common and a/b as common we get

$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \int e^{ax} \sin bx \, dx$$

Now again by using integration by parts, we get

$$= \frac{1}{b} e^{ax} \sin bx - \frac{a}{b} \left[-e^{ax} \frac{\cos bx}{b} + a \int e^{ax} \frac{\cos bx}{b} \, dx \right]$$

$$= \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx \, dx$$

By computing,

$$I = \frac{e^{ax}}{b^2} [b \sin bx + a \cos bx] - \frac{a^2}{b^2} I + c$$

$$= \frac{e^{ax}}{a^2 + b^2} [b \sin bx + a \cos bx] + c$$

2. $\int e^{ax} \sin(bx + c) \, dx$

Solution:

$$\begin{aligned} \text{Let } I &= \int e^{ax} \sin(bx + c) dx \\ &= -e^{ax} \frac{\cos(bx + c)}{b} + \int a e^{ax} \frac{\cos(bx + c)}{b} dx \end{aligned}$$

Now taking common

$$= -\frac{1}{b} e^{ax} \cos(bx + c) + \frac{a}{b} \int e^{ax} \cos(bx + c)$$

On integrating we get

$$I = \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} - \frac{a^2}{b^2} I + c_1$$

By computing the above equation can be written as

$$I = \left\{ \frac{a^2 + b^2}{b^2} \right\} - \frac{e^{ax}}{b^2} \{a \sin(bx + c) - b \cos(bx + c)\} + c_1$$

$$= \frac{e^{ax}}{a^2 + b^2} \{a \sin(bx + c) - b \cos(bx + c)\} + c_1$$

3. $\int \cos(\log x) dx$

Solution:

$$\text{Let } I = \int \cos(\log x) dx$$

Let $\log x = t$

$$\frac{1}{x} dx = dt$$

$$dx = x dt$$

$$= \int e^t \cos t dt$$

$$\text{We know that, } \int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \cos bx + b \sin bx\}$$

Hence, $a=1, b=1$

$$\text{So, } I = \frac{e^t}{2} [\cos t + \sin t] + c$$

Hence,

$$\int \cos(\log x) dx = \frac{e^{\log x}}{2} \{\cos(\log x) + \sin(\log x)\} + c$$

$$I = \frac{x}{2} \{\cos(\log x) + \sin(\log x)\} + c$$

4. $\int e^{2x} \cos(3x + 4) dx$

Solution:

$$\text{Let } I = \int e^{2x} \cos(3x + 4) dx$$

Integrating by parts

$$\begin{aligned} I &= e^{2x} \frac{\sin(3x + 4)}{3} - \int 2e^{2x} \frac{\sin(3x + 4)}{3} dx \\ &= \frac{1}{3} e^{2x} \sin(3x + 4) - \frac{2}{3} \int e^{2x} \sin(3x + 4) dx \\ &= \frac{1}{3} e^{2x} \sin(3x + 4) - \frac{2}{3} \left\{ -e^{2x} \frac{\cos(3x + 4)}{3} + \int 2e^{2x} \frac{\cos(3x + 4)}{3} dx \right\} \end{aligned}$$

$$I = \frac{1}{3} e^{2x} \sin(3x + 4) + \frac{2}{9} e^{2x} \cos(3x + 4) - \frac{4}{9} I$$

Hence,

$$I = \frac{e^{2x}}{13} [2 \cos(3x + 4) + 3 \sin(3x + 4)] + c$$

5. $\int e^{2x} \sin x \cos x dx$

Solution:

$$\text{Let } I = \int e^{2x} \sin x \cos x dx$$

$$= \frac{1}{2} \int e^{2x} 2 \sin x \cos x dx$$

$$= \frac{1}{2} \int e^{2x} \sin 2x dx$$

We know that,

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} \{a \sin bx - b \cos bx\} + c$$

$$= \frac{e^{2x}}{8} \{2 \sin 2x - 2 \cos 2x\} + c$$

$$I = \frac{1}{2} \frac{e^{2x}}{8} \{2 \sin 2x - 2 \cos 2x\} + c$$

$$I = \frac{e^{2x}}{8} \{\sin 2x - \cos 2x\} + c$$

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