$\Rightarrow 2I = \sin(\log x).x - \cos(\log x).x + c$

$\Rightarrow I = \frac{\sin(\log x).x - \cos(\log x).x}{2} + c$

$\Rightarrow I = \frac{1}{2}x.\sin(\log x) - x.\frac{1}{2}\cos(\log x) + c$

Ans) B $\frac{1}{2}x.\sin(\log x) - x.\frac{1}{2}\cos(\log x) + c$

42. Question

Mark (√) against the correct answer in each of the following:

$\int (\sin^{-1}x)^2 dx = ?$

A. $\frac{2\sin^{-1}x}{\sqrt{1-x^2}} + C$

B. $\frac{1}{3}(\sin^{-1}x)^3 + \frac{1}{\sqrt{1-x^2}} + C$

C. $x(\sin^{-1}x)^2 + (\sin^{-1}x)\sqrt{1-x^2} + 2x + C$

D. $x(\sin^{-1}x)^2 + 2(\sin^{-1}x)\sqrt{1-x^2} - 2x + C$

Answer

To find: Value of $\int (\sin^{-1}x)^2 dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int (\sin^{-1}x)^2 dx$ ... (i)

Putting sint = x, ⇒ t = $\sin^{-1}x$

⇒ dx = cost dt

When x = sint then $\sqrt{1-x^2} = \cos t$

$I = \int (\sin^{-1}x)^2 dx$

$\Rightarrow I = \int (\sin^{-1}(\sin t))^2 \cos t\, dt$

$\Rightarrow I = \int t^2 \cos t\, dt$

Taking 1st function as t^2 and second function as cost

$\Rightarrow I = \left[t^2 \int \cos t\, dt - \int \left(\frac{dt^2}{dt} \int \cos t\, dt \right) dt \right]$

$\Rightarrow I = \left[t^2 \sin t - \int (2t \sin t) dt \right]$

$\Rightarrow I = \left[t^2 \sin t - 2\int (t \sin t) dt\right]$

Taking 1^{st} function as t and second function as sint

$\Rightarrow I = t^2 \sin t - 2\left[\int (t \sin t) dt\right]$

$\Rightarrow I = t^2 \sin t - 2\left[t\int \sin t\ dt - \int\left(\frac{dt}{dt}\int \sin t\ dt\right) dt\right]$

$\Rightarrow I = t^2 \sin t - 2\left[t(-\cos t) - \int (-\cos t) dt\right]$

$\Rightarrow I = t^2 \sin t - 2[-t\cos t - (-\sin t) + c]$

$\Rightarrow I = t^2 \sin t + 2t\cos t - 2\sin t + c$

$\Rightarrow I = x\left(\sin^{-1} x\right)^2 + 2 \sin^{-1} x\ \sqrt{1-x^2} - 2x + c$

Ans) D $x\left(\sin^{-1} x\right)^2 + 2 \sin^{-1} x\ \sqrt{1-x^2} - 2x + c$

43. Question

Mark (√) against the correct answer in each of the following:

$\int e^x \left\{\frac{1}{x} - \frac{1}{x^2}\right\} dx = ?$

A. $e^x \left\{\log x + \frac{1}{x}\right\} + C$

B. $xe^x - e^x + C$

C. $e^x . \frac{1}{x} + C$

D. none of these

Answer

To find: Value of $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ ... (i)

Here $f(x) = \frac{1}{x}$

$\Rightarrow f'(x) = -\frac{1}{x^2}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \frac{1}{x} + c$

Ans) C $e^x \frac{1}{x} + c$

44. Question

Mark (√) against the correct answer in each of the following:

$\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx = ?$

A. $\frac{-e^x}{x^2} + C$

B. $\frac{e^x}{x^2} + C$

C. $e^x \left(\frac{-1}{x} + \frac{1}{x^2}\right) + C$

D. none of these

Answer

To find: Value of $\int e^x \left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x \left(\frac{1}{x^2} - \frac{2}{x^3}\right) dx$ ... (i)

Here $f(x) = \frac{1}{x^2}$

$\Rightarrow f'(x) = -\frac{2}{x^3}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \frac{1}{x^2} + c$

Ans) B $e^x \frac{1}{x^2} + c$

45. Question

Mark (√) against the correct answer in each of the following:

$\int e^x \left\{\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right\} dx = ?$

A. $e^x \cdot \frac{1}{\sqrt{1-x^2}} + C$

B. $e^x \sin^{-1} x + C$

C. $\frac{-e^x}{\sin^{-1} x} + C$

D. none of these

Answer

To find: Value of $\int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right) dx$ ... (i)

Here $f(x) = \sin^{-1} x$

$\Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \sin^{-1} x + c$

Ans) B $e^x \sin^{-1} x + c$

46. Question

Mark (√) against the correct answer in each of the following:

$\int e^x (\tan x + \log \sec x) dx = ?$

A. $e^x \log \sec x + C$

B. $e^x \tan x + C$

C. $e^x (\log \cos x) + C$

D. none of these

Answer

To find: Value of $\int e^x(\tan x + \log(\sec x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int e^x(\tan x + \log(\sec x))dx$ ... (i)

$\Rightarrow I = \int e^x(\tan x - \log(\cos x))dx$

Here f(x) = -log(cosx)

$\Rightarrow f^{'}(x) = \tan x$

$\Rightarrow I = \int e^x \left(f(x) + f^{'}(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x \log(\cos x) + c$

$\Rightarrow I = e^x \log(\sec x) + c$

Ans) A $e^x \log(\sec x) + c$

47. Question

Mark (√) against the correct answer in each of the following:

$\int e^x (\tan x + \log \sec x) dx = ?$

A. e^x log sec x + C

B. e^x tan x + C

C. e^x (log cos x) + C

D. none of these

Answer

To find: Value of $\int e^x (tanx + log(secx)) dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f^{'}(x) \int g(x)dx \right] dx$

We have, $I = \int e^x (\tan x + \log(\sec x)) dx$ ... (i)

$\Rightarrow I = \int e^x (\tan x - \log(\cos x)) dx$

Here f(x) = -log(cosx)

$\Rightarrow f^{'}(x) = \tan x$

$\Rightarrow I = \int e^x \left(f(x) + f^{'}(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x \log(\cos x) + c$

$\Rightarrow I = e^x \log(\sec x) + c$

Ans) A $e^x \log(\sec x) + c$

48. Question

Mark (√) against the correct answer in each of the following:

$\int e^x (\cot x + \log \sin x) dx = ?$

A. e^x log (sec x + tan x) + C

B. $e^x \sec x + C$

C. $e^x \log \tan x + C$

D. none of these

Answer

To find: Value of $\int e^x(cotx + log(sinx))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\cot x + \log(\sin x))dx$ ... (i)

Here $f(x) = \log(\sin x)$

$\Rightarrow f'(x) = \cot x$

$\Rightarrow I = \int e^x\left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \log(\sin x) + c$

Ans) D None of these

49. Question

Mark (√) against the correct answer in each of the following:

$\int e^x\left\{\tan^{-1}x + \frac{1}{(1+x^2)}\right\}dx = ?$

A. $e^x \cdot \frac{1}{(1+x^2)} + C$

B. $e^x \tan^{-1} x + C$

C. $-e^x \cot^{-1} x + C$

D. none of these

Answer

To find: Value of $\int e^x\left(\tan^{-1}x + \frac{1}{(1+x)^2}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\tan^{-1}x + \frac{1}{(1+x)^2}\right)dx$ ... (i)

Here $f(x) = \tan^{-1}x$

$\Rightarrow f'(x) = \frac{1}{(1+x)^2}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x(\tan^{-1} x) + c$

Ans) B $e^x(\tan^{-1} x) + c$

50. Question

Mark (√) against the correct answer in each of the following:

$\int e^x (\tan x - \log \cos x) dx = ?$

A. $e^x \tan x + C$

B. $e^x \log \cos x + C$

C. $e^x \log \sec x + C$

D. none of these

Answer

To find: Value of $\int e^x(\tan x - \log(\cos x))dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(\tan x - \log(\cos x))dx$ ... (i)

Here $f(x) = -\log(\cos x)$

$\Rightarrow f'(x) = \tan x$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = -e^x \log(\cos x) + c$

$\Rightarrow I = e^x \log(\sec x) + c$

Ans) C $e^x \log(\sec x) + c$

51. Question

Mark (√) against the correct answer in each of the following:

$\int e^x \left(\cot x - \operatorname{cosec}^2 x\right) dx = ?$

A. $-e^x \operatorname{cosec}^2 x + C$

B. $e^x \cot x + C$

C. $-e^x \cot x + C$

D. None of these

Answer

To find: Value of $\int e^x(cotx - cosec^2x)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(cotx - cosec^2x)dx$ ... (i)

Here f(x) = cotx

$\Rightarrow f'(x) = -cosec^2x$

$\Rightarrow I = \int e^x\left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x cotx + c$

Ans) B $e^x cotx + c$

52. Question

Mark (√) against the correct answer in each of the following:

$\int e^x(\sin x + \cos x)dx = ?$

A. $e^x \sin x + C$

B. $e^x \cos x + C$

C. $e^x \tan x + C$

D. None of these

Answer

To find: Value of $\int e^x(sinx + cosx)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x(sinx + cosx)dx$ ... (i)

Here f(x) = sinx

$\Rightarrow f'(x) = cosx$

$\Rightarrow I = \int e^x\left(f(x) + f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x sinx + c$

Ans) A $e^x sinx + c$

53. Question

Mark (√) against the correct answer in each of the following:

$\int e^x \sec x(1 + \tan x)dx = ?$

A. $e^x(1 + \tan x) + C$

B. $e^x \sec x + C$

C. $e^x \tan x + C$

D. none of these

Answer

To find: Value of $\int e^x secx\,(1 + tanx)\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x secx\,(1+tanx)dx$ ... (i)

$I = \int e^x(secx+secxtanx)dx$

Here $f(x) = secx$

$\Rightarrow f'(x) = secxtanx$

$\Rightarrow I = \int e^x\left(f(x)+f'(x)\right)dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x secx + c$

Ans) B $e^x secx + c$

54. Question

Mark (√) against the correct answer in each of the following:

$\int e^x\left(\frac{1+x\log x}{x}\right)dx = ?$

A. $e^x \cdot \frac{1}{x} + C$

B. $e^x \log x + C$

C. $x e^x \log x + C$

D. None of these

Answer

To find: Value of $\int e^x\left(\frac{1+x\,logx}{x}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\frac{1+x\log x}{x}\right)dx$ ... (i)

$I = \int e^x\left(\frac{1}{x}+\log x\right)dx$

Here $f(x) = \log x$

$\Rightarrow f'(x) = \frac{1}{x}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \log x + c$

Ans) B $e^x \log x + c$

55. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x \cdot \frac{x}{(1+x)^2} dx = ?$$

A. $e^x \cdot \frac{1}{(1+x)} + C$

B. $e^x \cdot \frac{1}{x} + C$

C. $e^x \cdot \frac{x}{(1+x)} + C$

D. None of these

Answer

To find: Value of $\int e^x \frac{x}{(1+x)^2} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int e^x \frac{x}{(1+x)^2} dx$ ... (i)

$I = \int e^x \left(\frac{x+1-1}{(1+x)^2}\right) dx$

$\Rightarrow I = \int e^x \left(\frac{1}{(1+x)} - \frac{1}{(1+x)^2}\right) dx$

Here $f(x) = \frac{1}{(1+x)}$

$\Rightarrow f'(x) = -\frac{1}{(1+x)^2}$

$\Rightarrow I = \int e^x \left(f(x) + f'(x)\right) dx$

$\Rightarrow I = e^x f(x) + c$

$\Rightarrow I = e^x \frac{1}{(1+x)} + c$

Ans) A $e^x\frac{1}{(1+x)}+c$

56. Question

Mark (√) against the correct answer in each of the following:

$$\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx=?$$

A. $e^x \sin\frac{x}{2}+C$

B. $e^x \cos\frac{x}{2}+C$

C. $e^x \tan\frac{x}{2}+C$

D. None of these

Answer

To find: Value of $\int e^x\left(\frac{1+sinx}{1+cosx}\right)dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$ ... (i)

$$I = \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$$

$$\Rightarrow I = \int e^x\left(\frac{1}{1+\cos x}+\frac{\sin x}{1+\cos x}\right)dx$$

$$\Rightarrow I = \int e^x\left(\frac{1}{2\cos^2\frac{x}{2}}+\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)dx$$

$$\Rightarrow I = \int e^x\left(\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right)dx$$

Here $f(x) = \tan\frac{x}{2}$

$$\Rightarrow f'(x)=\frac{1}{2}\sec^2\frac{x}{2}$$

$$\Rightarrow I=\int e^x\left(f(x)+f'(x)\right)dx$$

$$\Rightarrow I=e^x f(x)+c$$

$$\Rightarrow I= e^x\tan\frac{x}{2}+c$$

Ans) C $e^x\tan\frac{x}{2}+c$