

## EXERCISE 32.5

Find the standard deviation for the following distribution:

x:	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f:	1	5	12	22	17	9	4

**Solution:**

By using the formula for standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

$$\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}$$

So,

$$\text{Mean} = \frac{4.5+14.5+24+34.5+44.4+54.5+64.5}{7} = 34.4$$

$X_i$	$F_i$	$d_i = (x_i - \text{mean})$	$u_i = \frac{x_i - \text{mean}}{10}$	$f_i u_i$	$u_i^2$	$f_i u_i^2$
4.5	1	-30	-3	-3	9	9
14.5	5	-20	-2	-10	4	20
24	12	-10	-1	-12	1	12
34.5	22	0	0	0	0	0
44.5	17	10	1	17	1	17
54.5	9	20	2	18	4	36
64.5	4	30	3	12	9	36
	$\sum f_i = 70$			$\sum u_i f_i = 22$		$\sum u_i^2 f_i = 130$

Now,

$$N = 70, \sum u_i f_i = 22, \sum u_i^2 f_i = 130$$

$$\text{Var}(X) = h^2 \left[ \frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left( \frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\text{Var}(X) = 10^2 \left[ \frac{1}{70} \times 130 - \left( \frac{1}{70} \times 22 \right)^2 \right]$$

$$= 100 \left[ \frac{130}{70} - \left( \frac{22}{70} \right)^2 \right]$$

$$= 100 \left[ \frac{13}{7} - \frac{121}{1225} \right]$$

$$= 100 [1.857 - 0.0987]$$

$$= 100 [1.7583]$$

$$\text{Var}(X) = 175.83$$

$$\begin{aligned}\text{Standard Deviation, } \sigma &= \sqrt{\text{Var}(X)} \\ &= \sqrt{175.83} \\ &= 13.26\end{aligned}$$

∴ The standard deviation is 13.26

1. Table below shows the frequency f with which 'x' alpha particles were radiated from a diskette

x:	0	1	2	3	4	5	6	7	8	9	10	11	12
f:	51	203	383	525	532	408	273	139	43	27	10	4	2

Calculate the mean and variance.

**Solution:**

By using the formula to find mean,

$$\begin{aligned}\text{Mean} &= \frac{\sum f_i x_i}{N} \\ &= \frac{10078}{2600} = 3.88\end{aligned}$$

$X_i$	$F_i$	$F_i X_i$	$(X_i - \bar{X})$	$(X_i - \bar{X})^2$	$F_i (X_i - \bar{X})^2$
0	51	0	-3.88	15.05	767.55
1	203	203	-2.88	8.29	1682.87
2	383	766	-1.88	3.53	1351.99
3	525	1575	-0.88	0.77	404.25
4	532	2128	0.12	0.014	7.448
5	408	2040	1.12	1.25	510
6	273	1638	2.12	4.49	1225.77
7	139	973	3.12	9.73	1352.47
8	42	344	4.12	16.97	729.71
9	27	243	5.12	26.21	707.67
10	10	100	6.12	37.45	374.5
11	4	44	7.12	50.69	202.76
12	2	24	8.12	65.93	131.86
	$N=2600$	$\sum f_i x_i = 10078$			$\sum f_i (x_i - \bar{X})^2 = 9448.848$

Now,

$$N = 2600$$

$$\text{Variance}(X) = \frac{\sum f_i (x_i - \bar{X})^2}{N}$$

$$\sigma^2 = \frac{9448.848}{2600} = 3.63$$

∴ The mean is 3.88 and variance is 3.63

2. Find the mean, and standard deviation for the following data:

(i)

Year render:	10	20	30	40	50	60
No. of persons (cumulative)	15	32	51	78	97	109

**Solution:**

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

$X_i$	$F_i$	$f_i$	$u_i = \frac{x_i - \text{mean}}{10}$	$f_i u_i$	$U_i^2$	$f_i u_i^2$
10	15	15	-2.5	-37.5	6.25	93.75
20	32	17	-1.5	-25.5	2.25	38.25
30	51	19	-0.5	-9.5	0.25	4.75
40	78	27	0.5	13.5	0.25	6.75
50	97	19	1.5	28.5	2.25	42.75
60	109	12	2.5	30	6.25	75
		$\sum f_i = 109$		$\sum u_i f_i = -0.5$		$\sum u_i^2 f_i = 261.2$

Now,

$$N = 109, \quad \sum u_i f_i = -0.5, \quad \sum u_i^2 f_i = 261.2$$

$$\text{Mean, } \bar{X} = A + h \left( \frac{\sum u_i f_i}{N} \right)$$

$$\bar{X} = 35 + 10 \left( \frac{-0.5}{109} \right)$$

$$\bar{X} = 34.96$$

$$\text{Var}(X) = h^2 \left[ \frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left( \frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\begin{aligned} \text{Var}(X) &= 100 \left[ \frac{261.25}{109} - \frac{0.25}{11881} \right] \\ &= 100 \times 2.396 \end{aligned}$$

$$\text{Variance} = 239.6$$

$$\text{Standard Deviation, } \sigma = \sqrt{239.6}$$

$$= 15.47 \text{ years}$$

$\therefore$  The standard deviation is 15.47

(ii)

Marks:	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency:	1	6	6	8	8	2	2	3	0	2	1	0	0	0	1

**Solution:**

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

$x_i$	$f_i$	$f_i x_i$	$f_i x_i^2$
2	1	2	4
3	6	18	54
4	6	24	96
5	8	40	200
6	8	48	288
7	2	14	98
8	2	16	128
9	3	27	243
10	0	0	0
11	2	22	242
12	1	12	144
13	0	0	0
14	0	0	0
15	0	0	0
16	1	16	256
	N=40	Total=239	Total=1753

Now,

$$N = 40, \sum x_i f_i = 239, \sum x_i^2 f_i = 1753$$

$$\text{Mean, } \bar{X} = \left( \frac{\sum x_i f_i}{N} \right)$$

$$\bar{X} = \frac{239}{40}$$

$$= 5.975$$

$$\text{Var}(X) = \frac{1753}{40} - (5.97)^2$$

$$\text{Variance} = 8.12$$

$$\text{Standard Deviation, } \sigma = \sqrt{8.12}$$

$$= 2.85 \text{ years}$$

∴ The standard deviation is 2.85

**3. Find the standard deviation for the following data:**

(i)

<b>x:</b>	<b>3</b>	<b>8</b>	<b>13</b>	<b>18</b>	<b>23</b>
<b>f:</b>	<b>7</b>	<b>10</b>	<b>15</b>	<b>10</b>	<b>6</b>

**Solution:**

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

$X_i$	$F_i$	$F_i X_i$	$(x_i - \bar{X})$	$(x_i - \bar{X})^2$	$(x_i - \bar{X})^2 f$
3	7	21	-9.79	95.84	670.88
8	10	80	-4.79	22.94	229.4
13	15	195	0.21	0.04	0.6
18	10	180	5.21	27.14	271.4
23	6	138	10.21	104.24	625.44
	$\sum f_i = 48$	$\sum f_i x_i = 614$			$\sum (x_i - \bar{X})^2 f = 1797.32$

Now,  $N = 48$

$$\text{Var}(X) = \frac{\sum (x_i - \bar{X})^2 f}{\sum f_i}$$

$$\text{Var}(X) = \frac{1797.32}{48}$$

Variance = 37.44

$$\text{Standard Deviation, } \sigma = \sqrt{37.44} = 6.12$$

∴ The standard deviation is 6.12

(ii)

<b>x:</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>
<b>f:</b>	<b>4</b>	<b>9</b>	<b>16</b>	<b>14</b>	<b>11</b>	<b>6</b>

**Solution:**

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

$x_i$	$f_i$	$f_i x_i$	$f_i x_i^2$
2	4	8	16
3	9	27	81
4	16	64	256
5	14	70	350
6	11	66	396
7	6	42	294
	$N=60$	Total = 277	Total=1393

Now,

$$N = 60, \sum x_i f_i = 277, \sum x_i^2 f_i = 1393$$

$$\text{Mean, } \bar{X} = \left( \frac{\sum x_i f_i}{N} \right)$$

$$\bar{X} = \frac{277}{60}$$

$$= 4.62$$

$$\text{Var}(X) = \frac{1393}{60} - (4.62)^2$$

$$\text{Variance} = 1.88$$

$$\text{Standard Deviation, } \sigma = \sqrt{1.88}$$

$$= 1.37$$

∴ The standard deviation is 1.37



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