EXERCISE 7.3

Name the type of triangle formed by the points A (–5, 6), B (–4, –2) and C (7, 5). Solution:

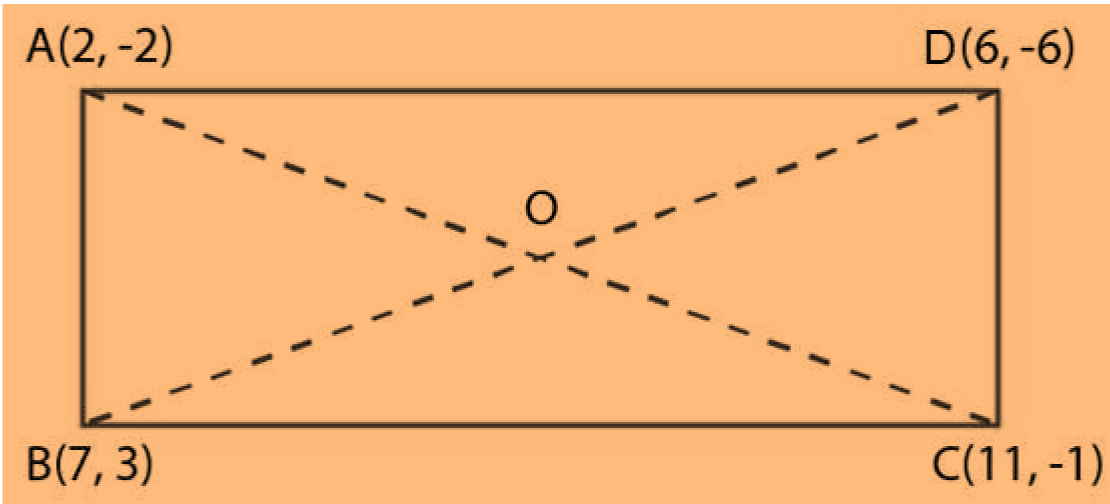
The points are A (–5, 6), B (–4, –2) and C (7, 5)
Using distance formula,
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$AB = \sqrt{(-4+5)^2 + (-2-6)^2}$
$= \sqrt{1+64}$
$= \sqrt{65}$
$BC = \sqrt{(7+4)^2 + (5+2)^2}$
$= \sqrt{121 + 49}$
$= \sqrt{170}$
$AC = \sqrt{(7+5)^2 + (5-6)^2}$
$= \sqrt{144 + 1}$
$= \sqrt{145}$
Since all sides are of different length, ABC is a scalene triangle.

1. Find the points on the *x*–axis which are at a distance of 2√5 from the point (7, –4). How many such points are there?
Solution:

Let coordinates of the point=(x, 0)(given that the point lies on x axis)
$x_1=7.\ y_1=-4$
$x_2=x.\ y_2=0$
Distance $=\sqrt{(x_2-x_1)^2+ (y_2-y_1)^2}$
According to the question,
$2\sqrt{5}=\sqrt{(x-7)^2+ (0-(-4))^2}$
Squaring L.H.S and R.H.S
$20=x^2+49-14x+16$
$20=x^2+65-14x$
$0=x^2-14x+45$
$0=x^2-9x-5x+45$
$0=x(x-9)-5(x-9)$
$0=(x-9)\ (x-5)$
$x-9 =0.\ x-5= 0$
$x=9$ or $x=5$
Therefore, coordinates of points.....(9,0)or(5,0)

2. What type of a quadrilateral do the points A (2, –2), B (7, 3), C (11, –1) and D (6, –6) taken in that order, form?
Solution:

The points are A (2, –2), B (7, 3), C (11, –1) and D (6, –6)



Using distance formula,
d = √ $((x_2 - x_1)^2 + (y_2 - y_1)^2)$

$AB = \sqrt{(7-2)^2 + (3+2)^2}$

$= \sqrt{(5)^2 + (5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$BC = \sqrt{(11-7)^2 + (-1-3)^2}$

$= \sqrt{(4)^2 + (-4)^2}$

$= \sqrt{16 + 16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

CD = $\sqrt{(6 - 11)^2 + (-6 + 1)^2}$

$= \sqrt{(-5)^2 + (-5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50}$

$= 5\sqrt{2}$

DA = $\sqrt{(2 - 6)^2 + (-2 + 6)^2}$

$= \sqrt{(-4)^2 + (4)^2}$

$= \sqrt{16 + 16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

Finding diagonals AC and BD, we get,

AC = $\sqrt{(11 - 2)^2 + (-1 + 2)^2}$

$= \sqrt{(9)^2 + (1)^2}$

$= \sqrt{81 + 1}$

$= \sqrt{82}$

And BD = $\sqrt{(6 - 7)^2 + (-6 - 3)^2}$

$= \sqrt{(-1)^2 + (-9)^2}$

$= \sqrt{1 + 81}$

$= \sqrt{82}$

3. Find the value of *a*, if the distance between the points A (–3, –14) and B (*a*, –5) is 9 units.
Solution:

Distance between two points (x_1,y_1) (x_2,y_2) is :

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Distance between A (–3, –14) and B (a, –5) is :

$=\sqrt{[(a+3)^2+(-5+14)^2]} =9$

Squaring on L.H.S and R.H.S.

$(a+3)^2+81=81$

$(a+3)^2=0$

$(a+3)(a+3)=0$

$a+3=0$

$a= -3$

4. Find a point which is equidistant from the points A (–5, 4) and B (–1, 6)? How many such points are there?
Solution:

Let the point be P

According to the question,

P is equidistant from A (–5, 4) and B (–1, 6)

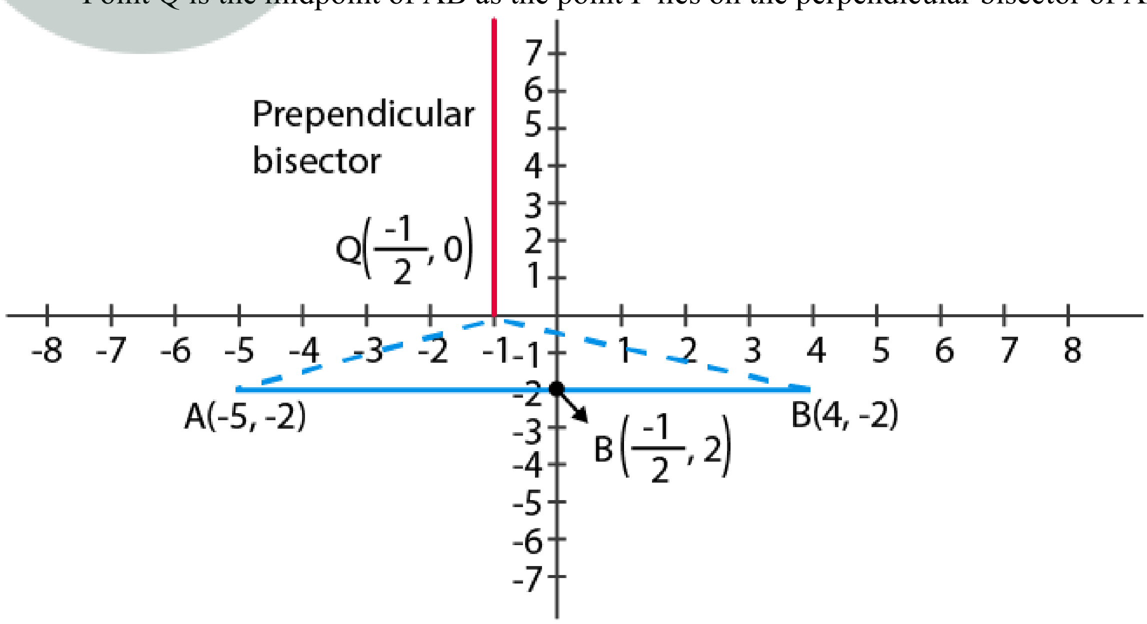
Then the point P $= ((x_1+x_2)/2, (y_1+y_2)/2)$

$= ((-5-1)/2, (6+4)/2)$

$= (-3 , 5)$

5. Find the coordinates of the point Q on the x–axis which lies on the perpendicular bisector of the line segment joining the points A (–5, –2) and B(4, –2). Name the type of triangle formed by the points Q, A and B.
Solution:

Point Q is the midpoint of AB as the point P lies on the perpendicular bisector of AB.



By mid point formula:

$(x_1 + x_2)/2 = (-5+4)/2$

$= -½$

$x = -½$

Given that, P lies on x axis, so y=0

P(x,y)= (-½ , 0)

Therefore, it is an isosceles triangle

6. Find the value of m if the points (5, 1), (–2, –3) and (8, 2m) are collinear.

Solution:

The points A(5, 1), B(–2, –3) and C(8, 2m) are collinear.

i.e., Area of ΔABC = 0

$½ [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]=0$

½ [5(-3 - 2m) + (- 2)(2m - 1) + 8(1 - (- 3))]=0

½ (-15 – 10m – 4m + 2 + 32) = 0

½ (-14m + 19) = 0

m = 19/14

7. If the point A (2, – 4) is equidistant from P (3, 8) and Q (–10, y), find the values of y. Also find distance PQ.

Solution:

Given points are $A(2, -4)$, $P(3, 8)$ and $Q(-10, y)$

According to the question,

$$PA = QA$$

$$\sqrt{(2-3)^2 + (-4-8)^2} = \sqrt{(2+10)^2 + (-4-y)^2}$$

$$\sqrt{(-1)^2 + (-12)^2} = \sqrt{(12)^2 + (4+y)^2}$$

$$\sqrt{1+144} = \sqrt{144+16+y^2+8y}$$

$$\sqrt{145} = \sqrt{160+y^2+8y}$$

On squaring both sides, we get

$$145 = 160 + y^2 + 8y$$

$$y^2 + 8y + 160 - 145 = 0$$

$$y^2 + 8y + 15 = 0$$

$$y^2 + 5y + 3y + 15 = 0$$

$$y(y+5) + 3(y+5) = 0$$

$\Rightarrow$ $(y+5)(y+3) = 0$

$\Rightarrow$ $y + 5 = 0 \Rightarrow y = -5$

and $y + 3 = 0 \Rightarrow y = -3$

$\therefore$ $y = -3, -5$

Now, $PQ = \sqrt{(-10-3)^2 + (y-8)^2}$

For $y = -3$ $PQ = \sqrt{(-13)^2 + (-3-8)^2} = \sqrt{169+121} = \sqrt{290}$ units

and for $y = -5$ $PQ = \sqrt{(-13)^2 + (-5-8)^2} = \sqrt{169+169} = \sqrt{338}$ units

Hence, values of y are -3 and -5, $PQ = \sqrt{290}$ and $\sqrt{338}$

8. Find the area of the triangle whose vertices are (–8, 4), (–6, 6) and (–3, 9).
Solution:

Given vertices are:

$(x_1, y_1) = (-8, 4)$
$(x_2, y_2) = (-6, 6)$
$(x_3, y_3) = (-3, 4)$

Area of triangle = $(½)\ (x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$
= (½) (-8(6 - 4) + -6(4 - 4) + -3(4 - 6))
= (½) (-8(2) + -6(0) + -3(-2))
= (½) (-16 + 6)
= (½) (-10)
= 5 units.

9. In what ratio does the x–axis divide the line segment joining the points (– 4, – 6) and (–1, 7)? Find the coordinates of the point of division.
Solution:

Let the ratio in which x-axis divides the line segment joining (–4, –6) and (–1, 7) = 1: k.
Then,
x-coordinate becomes (-1 - 4k) / (k + 1)
y-coordinate becomes (7 - 6k) / (k + 1)
Since P lies on x-axis, y coordinate = 0
(7 - 6k) / (k + 1) = 0
7 - 6k = 0
k = 7/6
Therefore, the ratio is 1:7/6 = 6:7
Hence, the coordinates of P are (-34/13, 0)