

EXERCISE 15.3

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1. In Fig. 35, $\angle CBX$ is an exterior angle of $\triangle ABC$ at B. Name

- (i) The interior adjacent angle
- (ii) The interior opposite angles to exterior $\angle CBX$

Also, name the interior opposite angles to an exterior angle at A.

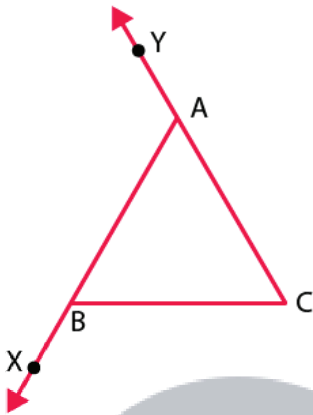


Fig. 35

Solution:

(i) The interior adjacent angle is $\angle ABC$

(ii) The interior opposite angles to exterior $\angle CBX$ is $\angle BAC$ and $\angle ACB$

Also the interior angles opposite to exterior $\angle BAY$ are $\angle ABC$ and $\angle ACB$.

2. In the fig. 36, two of the angles are indicated. What are the measures of $\angle ACX$ and $\angle ACB$?

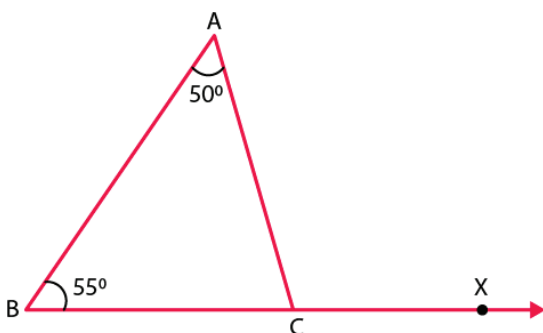


Fig. 36

Solution:

Given that in $\triangle ABC$, $\angle A = 50^\circ$ and $\angle B = 55^\circ$

We know that the sum of angles in a triangle is 180°

Therefore we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$50^\circ + 55^\circ + \angle C = 180^\circ$$

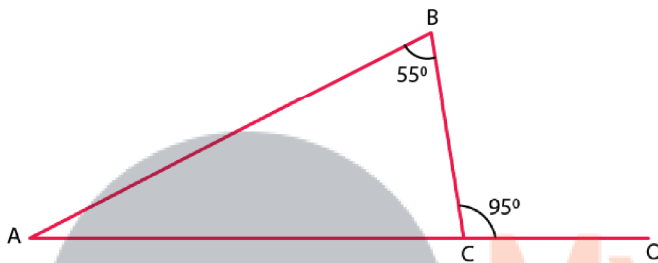
$$\angle C = 75^\circ$$

$$\angle ACB = 75^\circ$$

$$\angle ACX = 180^\circ - \angle ACB = 180^\circ - 75^\circ = 105^\circ$$

3. In a triangle, an exterior angle at a vertex is 95° and its one of the interior opposite angles is 55° . Find all the angles of the triangle.

Solution:



We know that the sum of interior opposite angles is equal to the exterior angle. Hence, for the given triangle, we can say that:

$$\angle ABC + \angle BAC = \angle BCO$$

$$55^\circ + \angle BAC = 95^\circ$$

$$\angle BAC = 95^\circ - 55^\circ$$

$$\angle BAC = 40^\circ$$

We also know that the sum of all angles of a triangle is 180° .

Hence, for the given $\triangle ABC$, we can say that:

$$\angle ABC + \angle BAC + \angle BCA = 180^\circ$$

$$55^\circ + 40^\circ + \angle BCA = 180^\circ$$

$$\angle BCA = 180^\circ - 95^\circ$$

$$\angle BCA = 85^\circ$$

4. One of the exterior angles of a triangle is 80° , and the interior opposite angles are equal to each other. What is the measure of each of these two angles?

Solution:

Let us assume that A and B are the two interior opposite angles.

We know that $\angle A$ is equal to $\angle B$.

We also know that the sum of interior opposite angles is equal to the exterior angle.

Therefore from the figure we have,

$$\angle A + \angle B = 80^\circ$$

$$\angle A + \angle A = 80^\circ \text{ (because } \angle A = \angle B)$$

$$2\angle A = 80^\circ$$

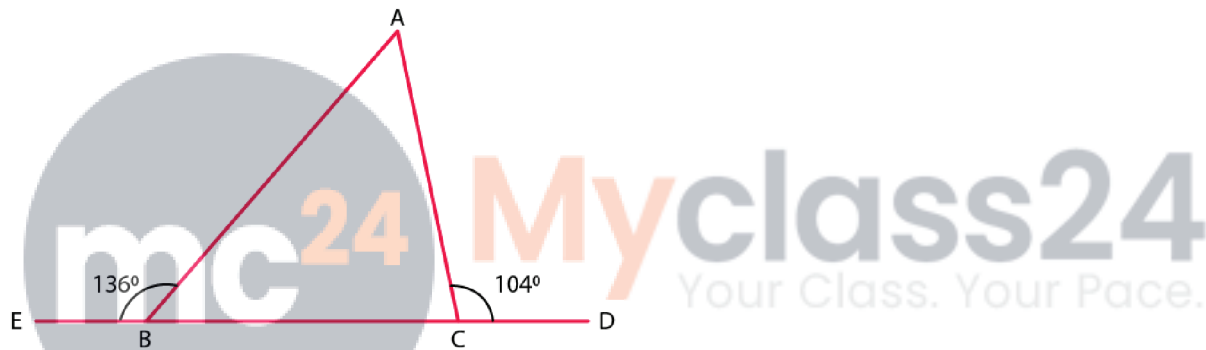
$$\angle A = 80/2 = 40^\circ$$

$$\angle A = \angle B = 40^\circ$$

Thus, each of the required angles is of 40° .

5. The exterior angles, obtained on producing the base of a triangle both ways are 104° and 136° . Find all the angles of the triangle.

Solution:



In the given figure, $\angle ABE$ and $\angle ABC$ form a linear pair.

$$\angle ABE + \angle ABC = 180^\circ$$

$$\angle ABC = 180^\circ - 136^\circ$$

$$\angle ABC = 44^\circ$$

We can also see that $\angle ACD$ and $\angle ACB$ form a linear pair.

$$\angle ACD + \angle ACB = 180^\circ$$

$$\angle ACB = 180^\circ - 104^\circ$$

$$\angle ACB = 76^\circ$$

We know that the sum of interior opposite angles is equal to the exterior angle.

Therefore, we can write as

$$\angle BAC + \angle ABC = 104^\circ$$

$$\angle BAC = 104^\circ - 44^\circ = 60^\circ$$

Thus,

$$\angle ACE = 76^\circ \text{ and } \angle BAC = 60^\circ$$

6. In Fig. 37, the sides BC, CA and BA of a $\triangle ABC$ have been produced to D, E and F respectively. If $\angle ACD = 105^\circ$ and $\angle EAF = 45^\circ$; find all the angles of the $\triangle ABC$.

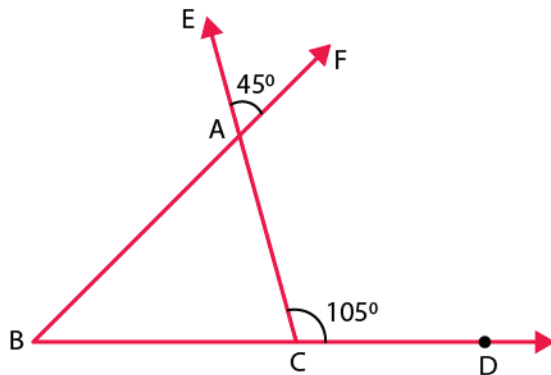


Fig. 37

Solution:

In a $\triangle ABC$, $\angle BAC$ and $\angle EAF$ are vertically opposite angles.

Hence, we can write as

$$\angle BAC = \angle EAF = 45^\circ$$

Considering the exterior angle property, we have

$$\angle BAC + \angle ABC = \angle ACD = 105^\circ$$

On rearranging we get

$$\angle ABC = 105^\circ - 45^\circ = 60^\circ$$

We know that the sum of angles in a triangle is 180°

$$\angle ABC + \angle ACB + \angle BAC = 180^\circ$$

$$\angle ACB = 75^\circ$$

Therefore, the angles are 45° , 60° and 75° .

7. In Fig. 38, AC perpendicular to CE and $\angle A : \angle B : \angle C = 3 : 2 : 1$. Find the value of $\angle ECD$.

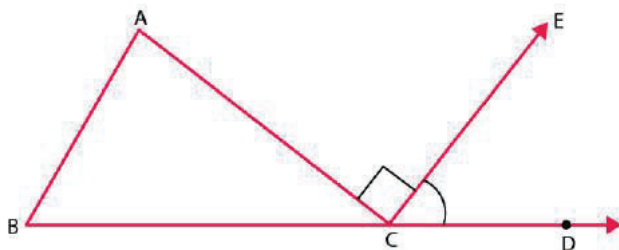


Fig. 38

Solution:

In the given triangle, the angles are in the ratio 3 : 2 : 1.

Let the angles of the triangle be $3x$, $2x$ and x .

We know that sum of angles in a triangle is 180°

$$3x + 2x + x = 180^\circ$$

$$6x = 180^\circ$$

$$x = 30^\circ$$

$$\text{Also, } \angle ACB + \angle ACE + \angle ECD = 180^\circ$$

$$x + 90^\circ + \angle ECD = 180^\circ \quad (\angle ACE = 90^\circ)$$

$$\text{We know that } x = 30^\circ$$

Therefore

$$\angle ECD = 60^\circ$$

8. A student when asked to measure two exterior angles of $\triangle ABC$ observed that the exterior angles at A and B are of 103° and 74° respectively. Is this possible? Why or why not?

Solution:

We know that sum of internal and external angle is equal to 180°

$$\text{Internal angle at A} + \text{External angle at A} = 180^\circ$$

$$\text{Internal angle at A} + 103^\circ = 180^\circ$$

$$\text{Internal angle at A} = 77^\circ$$

$$\text{Internal angle at B} + \text{External angle at B} = 180^\circ$$

$$\text{Internal angle at B} + 74^\circ = 180^\circ$$

$$\text{Internal angle at B} = 106^\circ$$

$$\text{Sum of internal angles at A and B} = 77^\circ + 106^\circ = 183^\circ$$

It means that the sum of internal angles at A and B is greater than 180° , which cannot be possible.

9. In Fig.39, AD and CF are respectively perpendiculars to sides BC and AB of $\triangle ABC$. If $\angle FCD = 50^\circ$, find $\angle BAD$

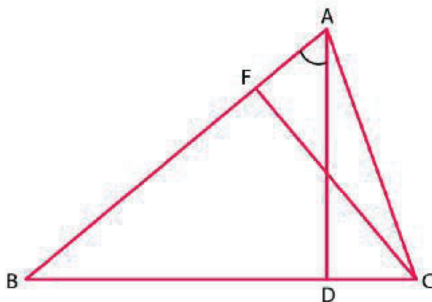


Fig. 39

Solution:

We know that the sum of all angles of a triangle is 180°

Therefore, for the given $\triangle FCB$, we have

$$\angle FCB + \angle CBF + \angle BFC = 180^\circ$$

$$50^\circ + \angle CBF + 90^\circ = 180^\circ$$

$$\angle CBF = 180^\circ - 50^\circ - 90^\circ = 40^\circ$$

Using the above steps for $\triangle ABD$, we can say that:

$$\angle ABD + \angle BDA + \angle BAD = 180^\circ$$

$$\angle BAD = 180^\circ - 90^\circ - 40^\circ = 50^\circ$$

10. In Fig.40, measures of some angles are indicated. Find the value of x.

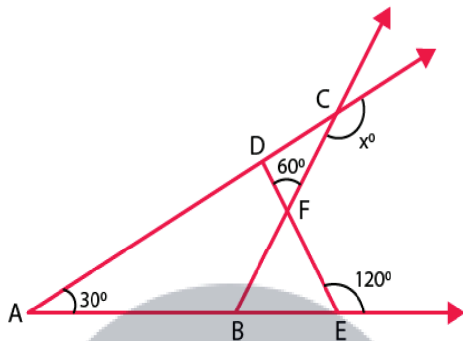


Fig. 40

Solution:

We know that the sum of the angles of a triangle is 180°

From the figure we have,

$$\angle AED + 120^\circ = 180^\circ \text{ (Linear pair)}$$

$$\angle AED = 180^\circ - 120^\circ = 60^\circ$$

We know that the sum of all angles of a triangle is 180° .

Therefore, for $\triangle ADE$, we have

$$\angle ADE + \angle AED + \angle DAE = 180^\circ$$

$$60^\circ + \angle ADE + 30^\circ = 180^\circ$$

$$\angle ADE = 180^\circ - 60^\circ - 30^\circ = 90^\circ$$

From the given figure, we have

$$\angle FDC + 90^\circ = 180^\circ \text{ (Linear pair)}$$

$$\angle FDC = 180^\circ - 90^\circ = 90^\circ$$

Using the same steps for $\triangle CDF$, we get

$$\angle CDF + \angle DCF + \angle DFC = 180^\circ$$

$$90^\circ + \angle DCF + 60^\circ = 180^\circ$$

$$\angle DCF = 180^\circ - 60^\circ - 90^\circ = 30^\circ$$

Again from the figure we have

$$\angle DCF + x = 180^\circ \text{ (Linear pair)}$$

$$30^\circ + x = 180^\circ$$

$$x = 180^\circ - 30^\circ = 150^\circ$$

11. In Fig. 41, ABC is a right triangle right angled at A. D lies on BA produced and DE perpendicular to BC intersecting AC at F. If $\angle AFE = 130^\circ$, find

- (i) $\angle BDE$
- (ii) $\angle BCA$
- (iii) $\angle ABC$

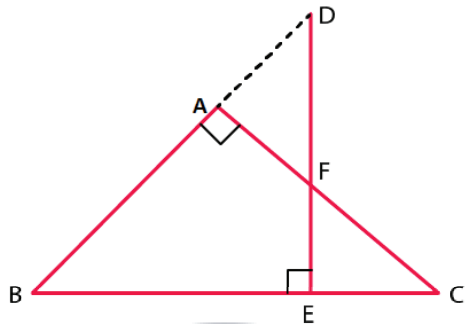


Fig. 41

Solution:

(i) Here,

$$\angle BAF + \angle FAD = 180^\circ \text{ (Linear pair)}$$

$$\angle FAD = 180^\circ - \angle BAF = 180^\circ - 90^\circ = 90^\circ$$

Also from the figure,

$$\angle AFE = \angle ADF + \angle FAD \text{ (Exterior angle property)}$$

$$\angle ADF + 90^\circ = 130^\circ$$

$$\angle ADF = 130^\circ - 90^\circ = 40^\circ$$

$$\angle BDE = 40^\circ$$

(ii) We know that the sum of all the angles of a triangle is 180° .

Therefore, for $\triangle BDE$, we have

$$\angle BDE + \angle BED + \angle DBE = 180^\circ$$

$$\angle DBE = 180^\circ - \angle BDE - \angle BED$$

$$\angle DBE = 180^\circ - 40^\circ - 90^\circ = 50^\circ \text{ Equation (i)}$$

Again from the figure we have,

$$\angle FAD = \angle ABC + \angle ACB \text{ (Exterior angle property)}$$

$$90^\circ = 50^\circ + \angle ACB$$

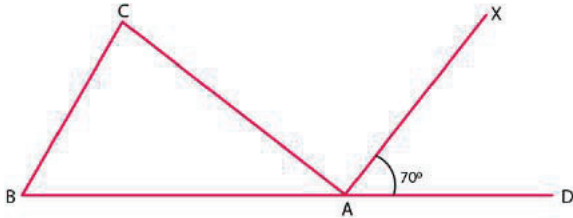
$$\angle ACB = 90^\circ - 50^\circ = 40^\circ$$

(iii) From equation we have

$$\angle ABC = \angle DBE = 50^\circ$$

12. ABC is a triangle in which $\angle B = \angle C$ and ray AX bisects the exterior angle DAC. If $\angle DAX = 70^\circ$. Find $\angle ACB$.

Solution:



Given that ABC is a triangle in which $\angle B = \angle C$
Also given that AX bisects the exterior angle DAC

$\angle CAX = \angle DAX$ (AX bisects $\angle CAD$)

$\angle CAX = 70^\circ$ [given]

$\angle CAX + \angle DAX + \angle CAB = 180^\circ$

$70^\circ + 70^\circ + \angle CAB = 180^\circ$

$\angle CAB = 180^\circ - 140^\circ$

$\angle CAB = 40^\circ$

$\angle ACB + \angle CBA + \angle CAB = 180^\circ$ (Sum of the angles of $\triangle ABC$)

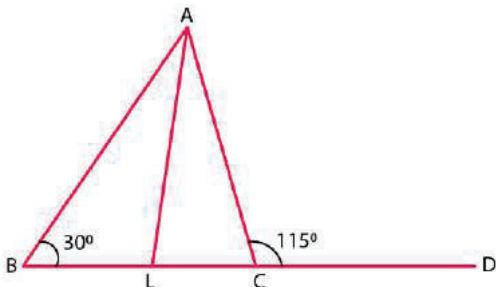
$\angle ACB + \angle ACB + 40^\circ = 180^\circ$ ($\angle C = \angle B$)

$2\angle ACB = 180^\circ - 40^\circ$

$\angle ACB = 140/2$

$\angle ACB = 70^\circ$

13. The side BC of $\triangle ABC$ is produced to a point D. The bisector of $\angle A$ meets side BC in L. If $\angle ABC = 30^\circ$ and $\angle ACD = 115^\circ$, find $\angle ALC$



Solution:

Given that $\angle ABC = 30^\circ$ and $\angle ACD = 115^\circ$

From the figure, we have

$\angle ACD$ and $\angle ACL$ make a linear pair.

$$\angle ACD + \angle ACB = 180^\circ$$

$$115^\circ + \angle ACB = 180^\circ$$

$$\angle ACB = 180^\circ - 115^\circ$$

$$\angle ACB = 65^\circ$$

We know that the sum of all angles of a triangle is 180° .

Therefore, for $\triangle ABC$, we have

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$30^\circ + \angle BAC + 65^\circ = 180^\circ$$

$$\angle BAC = 85^\circ$$

$$\angle LAC = \angle BAC/2 = 85/2$$

Using the same steps for $\triangle ALC$, we get

$$\angle ALC + \angle LAC + \angle ACL = 180^\circ$$

$$\angle ALC + 85/2 + 65^\circ = 180^\circ$$

We know that $\angle ACL = \angle ACB$

$$\angle ALC = 180^\circ - 85/2 - 65^\circ$$

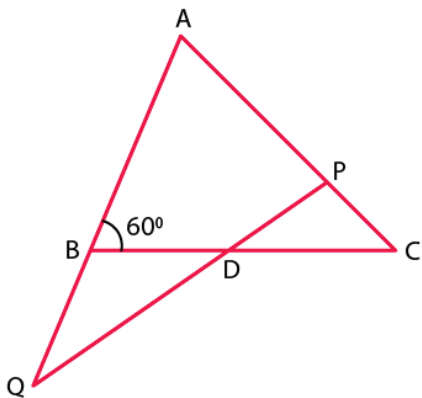
$$\angle ALC = 72\frac{1}{2}^\circ$$

14. D is a point on the side BC of $\triangle ABC$. A line PDQ through D, meets side AC in P and AB produced at Q. If $\angle A = 80^\circ$, $\angle ABC = 60^\circ$ and $\angle PDC = 15^\circ$, find

(i) $\angle AQD$

(ii) $\angle APD$

Solution:



From the figure we have

$\angle ABD$ and $\angle QBD$ form a linear pair.

$$\angle ABC + \angle QBC = 180^\circ$$

$$60^\circ + \angle QBC = 180^\circ$$

$$\angle QBC = 120^\circ$$

$$\angle PDC = \angle BDQ \text{ (Vertically opposite angles)}$$

$$\angle BDQ = 15^\circ$$

(i) In $\triangle QBD$:

$$\angle QBD + \angle QDB + \angle BQD = 180^\circ \text{ (Sum of angles of } \triangle QBD)$$

$$120^\circ + 15^\circ + \angle BQD = 180^\circ$$

$$\angle BQD = 180^\circ - 135^\circ$$

$$\angle BQD = 45^\circ$$

$$\angle AQD = \angle BQD = 45^\circ$$

(ii) In $\triangle AQP$:

$$\angle QAP + \angle AQP + \angle APQ = 180^\circ \text{ (Sum of angles of } \triangle AQP)$$

$$80^\circ + 45^\circ + \angle APQ = 180^\circ$$

$$\angle APQ = 55^\circ$$

$$\angle APD = \angle APQ = 55^\circ$$

15. Explain the concept of interior and exterior angles and in each of the figures given below. Find x and y (Fig. 42)

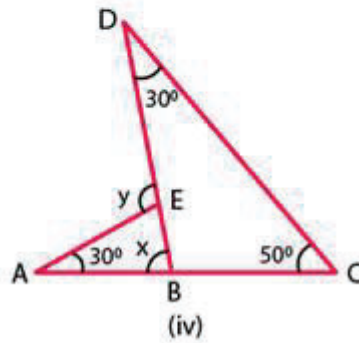
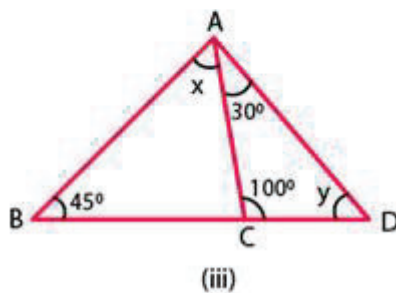
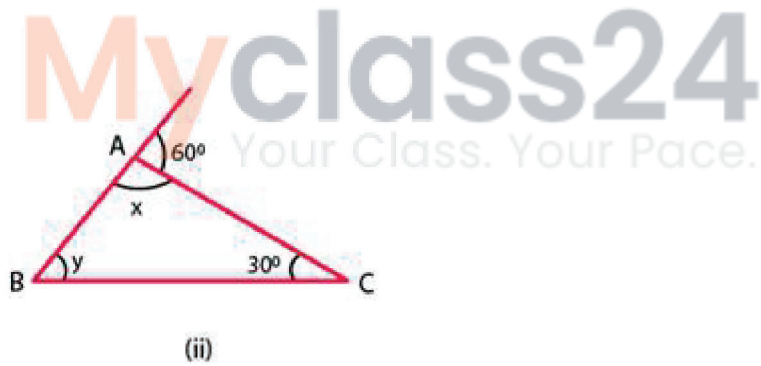
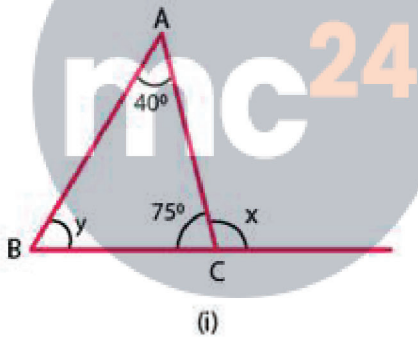


Fig. 42

Solution:

The interior angles of a triangle are the three angle elements inside the triangle.

The exterior angles are formed by extending the sides of a triangle, and if the side of a

triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

Using these definitions, we will obtain the values of x and y .

(i) From the given figure, we have

$$\angle ACB + x = 180^\circ \text{ (Linear pair)}$$

$$75^\circ + x = 180^\circ$$

$$x = 105^\circ$$

We know that the sum of all angles of a triangle is 180°

Therefore, for $\triangle ABC$, we can say that:

$$\angle BAC + \angle ABC + \angle ACB = 180^\circ$$

$$40^\circ + y + 75^\circ = 180^\circ$$

$$y = 65^\circ$$

(ii) From the figure, we have

$$x + 80^\circ = 180^\circ \text{ (Linear pair)}$$

$$x = 100^\circ$$

In $\triangle ABC$, we have

We also know that the sum of angles of a triangle is 180°

$$x + y + 30^\circ = 180^\circ$$

$$100^\circ + 30^\circ + y = 180^\circ$$

$$y = 50^\circ$$

(iii) We know that the sum of all angles of a triangle is 180° .

Therefore, for $\triangle ACD$, we have

$$30^\circ + 100^\circ + y = 180^\circ$$

$$y = 50^\circ$$

Again from the figure we can write as

$$\angle ACB + 100^\circ = 180^\circ$$

$$\angle ACB = 80^\circ$$

Using the above rule for $\triangle ACB$, we can say that:

$$x + 45^\circ + 80^\circ = 180^\circ$$

$$x = 55^\circ$$

(iv) We know that the sum of all angles of a triangle is 180° .

Therefore, for $\triangle DBC$, we have

$$30^\circ + 50^\circ + \angle DBC = 180^\circ$$

$$\angle DBC = 100^\circ$$

From the figure we can say that
 $x + \angle DBC = 180^\circ$ is a Linear pair
 $x = 80^\circ$

From the exterior angle property we have
 $y = 30^\circ + 80^\circ = 110^\circ$

16. Compute the value of x in each of the following figures:

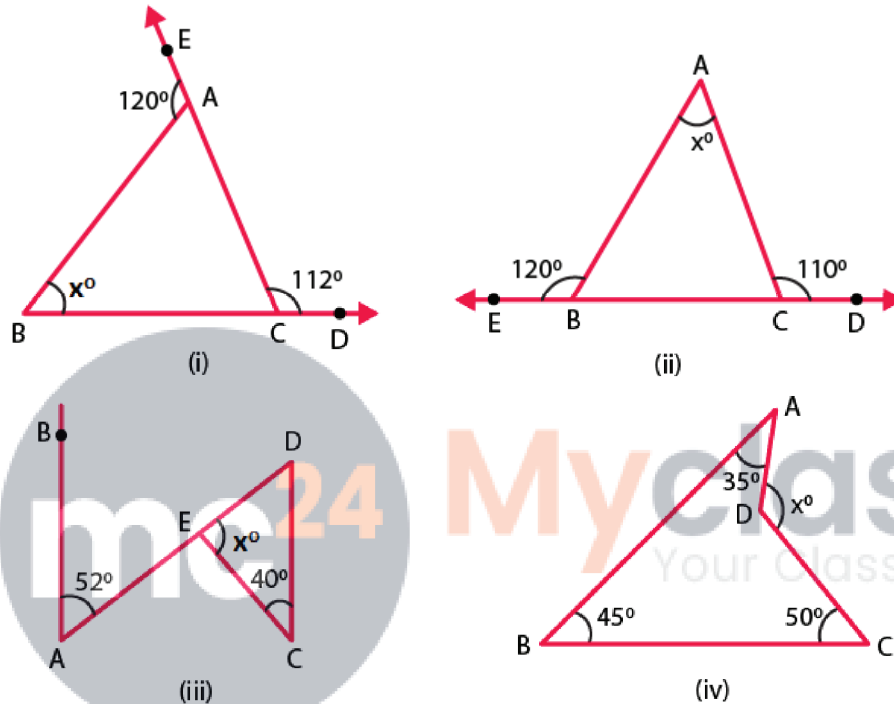


Fig. 43

Solution:

(i) From the given figure, we can write as

$\angle ACD + \angle ACB = 180^\circ$ is a linear pair

On rearranging we get

$$\angle ACB = 180^\circ - 112^\circ = 68^\circ$$

Again from the figure we have,

$\angle BAE + \angle BAC = 180^\circ$ is a linear pair

On rearranging we get,

$$\angle BAC = 180^\circ - 120^\circ = 60^\circ$$

We know that the sum of all angles of a triangle is 180° .

Therefore, for $\triangle ABC$:

$$x + \angle BAC + \angle ACB = 180^\circ$$

$$x = 180^\circ - 60^\circ - 68^\circ = 52^\circ$$

$$x = 52^\circ$$

(ii) From the given figure, we can write as

$$\angle ABC + 120^\circ = 180^\circ \text{ is a linear pair}$$

$$\angle ABC = 60^\circ$$

Again from the figure we can write as

$$\angle ACB + 110^\circ = 180^\circ \text{ is a linear pair}$$

$$\angle ACB = 70^\circ$$

We know that the sum of all angles of a triangle is 180° .

Therefore, consider $\triangle ABC$, we get

$$x + \angle ABC + \angle ACB = 180^\circ$$

$$x = 50^\circ$$

(iii) From the given figure, we can write as

$$\angle BAD = \angle ADC = 52^\circ \text{ are alternate angles}$$

We know that the sum of all the angles of a triangle is 180° .

Therefore, consider $\triangle DEC$, we have

$$x + 40^\circ + 52^\circ = 180^\circ$$

$$x = 88^\circ$$

(iv) In the given figure, we have a quadrilateral and also we know that sum of all angles in a quadrilateral is 360° .

Thus,

$$35^\circ + 45^\circ + 50^\circ + \text{reflex } \angle ADC = 360^\circ$$

On rearranging we get,

$$\text{Reflex } \angle ADC = 230^\circ$$

$$230^\circ + x = 360^\circ \text{ (A complete angle)}$$

$$x = 130^\circ$$