NCERT Solutions for Class-XI Physics

Chapter-9

NCERT Physics Class 11

1. A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper?

1. Length of the steel wire, $L_1 = 4.7$ m

Area of cross-section of the steel wire, $A_1 = 3.0 \times 10^{-5}$ m^2

Length of the copper wire, $L_2 = 3.5$ m

Area of cross-section of the copper wire, $A_2 = 4.0 \times 10^{-5}$ m^2

Change in length $= \Delta L_1 = \Delta L_2 = \Delta L$

Force applied in both the cases = F

Young's modulus of the steel wire:

$$Y_1 = \frac{F_1}{A_1} \times \frac{L_1}{DL}$$

$$= \frac{F \times 4.7}{3.0 \times 10^{-5} \times \Delta L} \quad \text{....(i)}$$

Young's modulus of the copper wire:

$$Y_2 = \frac{F_2}{A_2} \times \frac{L_2}{\Delta L_2}$$

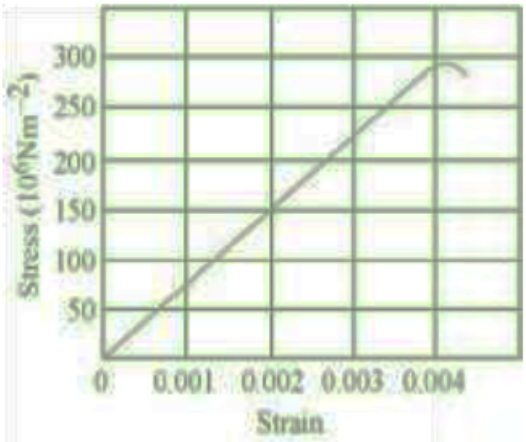
$$= \frac{F \times 3.5}{4.0 \times 10^{-5} \times \Delta L} \quad \text{...(ii)}$$

Dividing (i) by (ii), we get:

$$\frac{Y_1}{Y_2} = \frac{4.7 \times 4.0 \times 10^{-5}}{3.0 \times 10^{-5} \times 3.5} = 1.79 : 1$$

The ratio of Young's modulus of steel to that of copper is 1.79: 1.

2. Figure shows the strain-stress curve for a given material. What are (a) Young's modulus and (b) approximate yield strength for this material?



2. It is clear from the given graph that for stress 150×10^6 N/m^2, strain is 0.002.

(a) Young's modulus, Y = $= \frac{\text{stress}}{\text{strain}}$

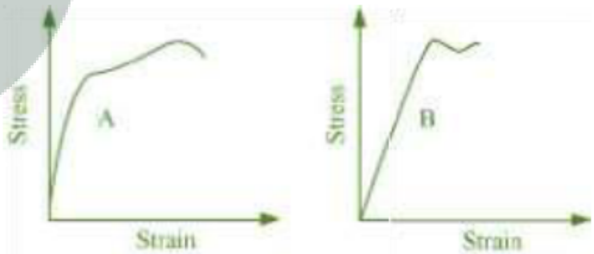
$$= \frac{150 \times 10^6}{0.002} = 7.5 \times 10^{10} \text{ N/m}^2$$

Hence, Young's modulus for the given material is 7.5×10^{10} N/m^2.

(b) The yield strength of a material is the maximum stress that the material can sustain without crossing the elastic limit.

It is clear from the given graph that the approximate yield strength of this material is 300×10^6 Nm/^{2} or 3×10^8 N/m^2.

3. The stress-strain graphs for materials A and B are shown in figure.



The graphs are drawn to the same scale.

(a) Which of the materials has the greater Young's modulus?

(b) Which of the two is the stronger material?

3. (a) A

(b) A

For a given strain, the stress for material A is more than it is for material B, as shown in the two graphs.

Young's modulus $= \frac{\text{Stress}}{\text{Strain}}$

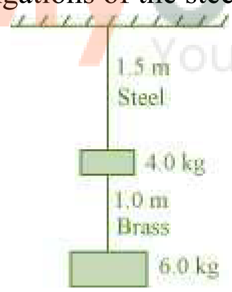
For a given strain, if the stress for a material is more, then Young's modulus is also greater for that material. Therefore, Young's modulus for material A is greater than it is for material B.

The amount of stress required for fracturing a material, corresponding to its fracture point, gives the strength of that material. Fracture point is the extreme point in a stress-strain curve. It can be observed that material A can withstand more strain than material B. Hence, material A is stronger than material B.

4. Read the following two statements below carefully and state, with reasons, if it is true or false.
(a) The Young's modulus of rubber is greater than that of steel;
(b) The stretching of a coil is determined by its shear modulus.

4. **The given statement is false.**
As there is more strain in rubber than steel and modulus of elasticity is inversely proportional to strain. Therefore, the Young's modulus of steel is greater than that of rubber.
(b) The stretching of a coil is determined by its shear modulus.
Ans: The given statement is true.
As the shear modulus of a coil relates with the change in shape of the coil and the stretching of coil changes its shape without any change in the length. Therefore, the shear modulus of elasticity is involved. Hence the stretching of a coil is determined by its shear modulus.

5. Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5 m and that of brass wire is 1.0 m. Compute the elongations of the steel and the brass wires.



5. In the above question it is given that:
Diameter of the wires is d = 0.25m.
Hence r = 0.125m.
Length of the steel wire is L_1 = 1.5m.
Length of the brass wire is L_2 = 1.0m.
Total force exerted on the steel wire is $F_1 = (4 + 6)\ g = 10g$
$\therefore\ F_1 = 10 \times 9.8 = 98N$.
Young's modulus for steel is given by

$$Y_1 = \frac{F_1}{A_1} \times \frac{L_1}{\Delta L_1}$$

Where,
ΔL_1 is the change in the length of the steel wire.
And A_1 is the area of cross-section of the steel wire.

$$\therefore A_1 = \pi_1^2$$

Young's modulus of steel is $Y_1 = 2.0 \times 10^{11}$ Pa.

$$\Rightarrow \Delta L_1 = \frac{F_1 \times L_1}{A_1 \times Y_1}$$

$$\Rightarrow \Delta L_1 = \frac{98 \times 1.5}{\pi(0.125)^2 \times 2.0 \times 10^{11}} = 1.49 \times 10^{-4} \text{ m.}$$

Total force on the brass wire is $F_2 = 6 \times 9.8 = 58.8$N.

Young's modulus for brass is given by

$$_2 = \frac{F_2}{A_2} \times \frac{L_2}{\Delta L_2}$$

Where,

ΔL_2 is the change in length of the brass wire.

And A_2 is the area of cross-section of the brass wire.

$$\therefore A_2 = \pi r_2^2$$

We have,

Young's modulus of brass is $Y_2 = 0.91 \times 10^{11}$ Pa.

$$\Rightarrow \Delta L_2 = \frac{F_2 \times L_2}{A_2 \times Y_2}$$

$$\Rightarrow \Delta L_2 = \frac{58.8 \times 1}{\pi(0.125)^2 \times 0.91 \times 10^{11}} = 1.3 \times 10^{-4} \text{ m.}$$

Clearly, the elongation of the steel wire is 1.49 × 10–4m and that of the brass wire is 1.3×10^{-4} m.

6. The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

6. Edge of the aluminium cube, L = 10 cm = 0.1 m

The mass attached to the cube, m = 100 kg

Shear modulus (η) of aluminium = 25 GPa = 25×10^9 Pa

$$\text{Shear modulus, } \eta = \frac{\text{Shear stress}}{\text{Shear strain}} == \frac{\frac{F}{A}}{\frac{L}{\Delta L}}$$

Where,

F = Applied force = mg = 100 × 9.8 = 980 N

A = Area of one of the faces of = 0.1 × 0.1 = 0.01 m2

ΔL = Vertical deflection of the cube

$$\therefore \Delta L = \frac{FL}{A\eta}$$

$$= \frac{980 \times 0.1}{10^{-2} \times (25 \times 10^9)}$$

$= 3.92 \times 10^{-7}$ m

The vertical deflection of this face of the cube is 3.92×10^{-7} m.

7. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.

7. In the above question it is given that:
Mass of the big structure is M = 50000Kg.
Inner radius of the column is r = 30 cm = 0.3m.
Outer radius of the column is R = 60 cm = 0.6m
Young's modulus of steel is $Y = 2 \times 10^{11}$ Pa
The total force exerted is F = Mg = 50000 × 9.8N.
Stress = Force exerted on as single column

$$\Rightarrow \text{Stress} = \frac{50000 \times 9.8}{4} = 122500\text{N}$$

Young's modulus is given by

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow \text{Strain} = \frac{\left(\frac{F}{A}\right)}{Y}$$

Where,
Area is given by

$$A = \pi\left(R^2 - r^2\right) = \pi\left((0.6)^2 - (0.3)^2\right).$$

$$\Rightarrow \text{Strain} = \frac{\left(\frac{50000 \times 9.8}{\pi\left((0.6)^2 - (0.3)^2\right)}\right)}{2 \times 10^{11}} = 7.22 \times 10^{-7}$$

Therefore, the compressional strain of each column is 7.22×10^{-7}.

8. A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain?

8. Length of the piece of copper, $l = 19.1 \text{ mm} = 19.1 \times 10^{-3}$ m
Breadth of the piece of copper, $b = 15.2 \text{ mm} = 15.2 \times 10^{-3}$ m
Area of the cooper piece:
$A = l \times b = 19.1 \times 10^{-3} \times 15.2 \times 10^{-3} \times 15.2 \times 10^{-3} = 2.9 \times 10^{-4} \text{ m}^2$
Tension force applied on the piece of copper, F = 44500 MN

Modulus of elasticity of copper, $\eta = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\text{Strain}}$

$$\therefore \text{ Strain} = \frac{F}{A\eta}$$

$$= \frac{44500}{2.9 \times 10^{-4} \times 42 \times 10^{9}}$$

$= 3.65 \times 10^{-3}$.

9. A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 108 N m–2, what is the maximum load the cable can support?

9. Radius of the steel cable, r = 1.5 cm = 0.015 m

Maximum allowable stress = 10^8N m^{-2}

Maximum stress = $\frac{\text{Maximum Force}}{\text{Area of cross-section}}$

∴ Maximum force = Maximum stress × Area of cross- section

= 108 × $\pi(0.015)^2$

= 7.065 × 104N

Hence, the cable can support the maximum load of 7.065 × 10^4 N.

10. The tension force acting on each wire is the same. Thus, the extension in each case is the same. Since the wires are of the same length, the strain will also be the same. The relation for Young's modulus is given as:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\text{Strain}} = \frac{\frac{4F}{\pi d^2}}{\text{Strain}} \qquad \text{(i)}$$

Where,

F = Tension force

A = Area of cross-section

D = Diameter of the wire

It can be inferred from equation (i) that $Y \propto \frac{1}{d^2}$

Young's modulus for iron, Y_1 = 190 × 10^9 Pa

Diameter of the iron wire = d1

Young's modulus for copper, Y^2 = 110 × 10^9 Pa

Diameter of the copper wire = d^2

Therefore, the ratio of their diameters is given as:

$$\frac{d_2}{d_1} = \sqrt{\frac{Y_1}{Y_2}} = \sqrt{\frac{190 \times 10^9}{110 \times 10^9}} = \sqrt{\frac{19}{11}} = 1.31:1$$

11. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

11. In the above question it is given that:

Mass is m = 14.5 kg.

Length of the steel wire is 1 = 1.0 m

Angular velocity is ω = 2rev/s

Cross-sectional area of the wire is a = 0.065 cm^2 = 0.065 × 10^{-4} m^2.

Consider the elongation of the wire when the mass is at the lowest point of its path to be Δl.

The total force on the mass when the mass is placed at the position of the vertical circle is given by:

$F = mg + ml\omega^2$

$\Rightarrow$ F = 14.5 × 9.8 + 14.5 × 1 × 2^4 = 200.1N

Young's modulus is given by:

$$Y = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow Y = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta l}{l}\right)}$$

$$\Rightarrow \Delta l = \frac{Fl}{AY}$$

We known that Young's modulus for steel is 2 × 10^{11} Pa

Therefore,

$$\Delta l = \frac{220.1 \times 1}{0.065 \times 10^{-4} \times 2 \times 10^{11}} = 1.53 \times 10^{-4}\,\text{m}$$

Thus, the elongation of the wire is 1.53 × 10^{-4} m.

12. Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013 × 10^5 Pa). Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

12. In the above question it is given that:

Initial volume is V_1 = 100.01 = 100 × 10^{-3} m^3.

Final volume is V_2 = 100.51 = 100.5 × 10^{-3} m^3.

Thus, the increase in volume is $V_2 - V^1$ = 0.5 × 10^{-3} m^3.

Increase in pressure is ΔP = 100 atm = 100 × 1.013 × 105 Pa.

The formula for bulk modulus is

$$\text{Bulk Modulus} = \frac{\Delta P}{\left(\frac{\Delta V}{V_1}\right)} = \frac{\Delta p V_1}{\Delta V}$$

$$\Rightarrow \text{Bulk Modulus} = \frac{100 \times 1.013 \times 10^5 \times 100 \times 10^{-3}}{0.5 \times 10^{-3}\,\text{m}^3} = 2.026 \times 10^9\,\text{Pa}$$

We know that Bulk modulus of air is 1 × 10^5 Pa.

$$\Rightarrow \frac{\text{Bulk modulus of water}}{\text{Bulk modulus of air}} = \frac{2.026 \times 10^9}{1 \times 10^5} = 2.026 \times 10^4$$

This ratio is very high because air is more compressible than water.

13. What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03 × 10^{-3} kg/m^3?

13. In the above question it is given that:

Pressure at the given depth is p = 80.0 atm = 80 × 1.01 × 10^5 Pa.

Consider the given depth to be h.

Density of water at the surface is ρ1 = 1.03 × 103 kg/m^3

Consider ρ_2 to be the density of water at the depth h.

Consider V_1 to be the volume of water of mass m at the surface.

Consider V_2 to be the volume of water of mass m at the depth h.

Consider VΔ to be the change in volume.

$\Delta V = V_1 - V2$

$$\Rightarrow \Delta V = m\left[\left(\frac{1}{\rho_1}\right)-\left(\frac{1}{\rho_2}\right)\right]$$

Now,

Volumetric strain = $m\left[\left(\frac{1}{\rho_1}\right)-\left(\frac{1}{\rho_2}\right)\right]\times\left(\frac{\rho_1}{m}\right)$

$$\Rightarrow \frac{\Delta V}{V_1} = 1-\left(\frac{\rho_1}{\rho_2}\right) \quad \ldots. (1)$$

Bulk modulus is given by:

Bulk modulus = $\frac{pV_1}{\Delta V}$

$$\Rightarrow \frac{\Delta V}{V_1} = \frac{p}{B}$$

Compressibility of water is given by:

$\frac{1}{B} = 45.8 \times 10^{-11}\ Pa^{-1}$

$$\Rightarrow \frac{\Delta V}{V_1} = 80\times1.013\times10^5\times45.8\times10^{-11} = 3.71\times10^{-3} \quad \ldots(2)$$

From equations (1) and (2) we get:

$$1-\left(\frac{\rho_1}{\rho_2}\right) = 3.71\times10^{-3}$$

$$\Rightarrow \rho_2 = \frac{1.03\times10^3}{\left[1-\left(3.71\times10^{-3}\right)\right]} = 1.034\times10^3\ kgm^{-3}$$

Clearly, the density of water at the given depth (h) is $1.034 \times 10^3\ kgm^{-3}$.

14. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

14. In the above question it is given that:

They hydraulic pressure exerted on the glass slab is $p = 10$ atm $= 10 \times 1.013 \times 10^5$ Pa

Also, we know that bulk modulus of glass is B = 37 × 109 N/m,2.

Bulk modulus is given by the relation:

$$B = \frac{P}{\left(\frac{\Delta V}{V}\right)}$$

Where

$\frac{\Delta V}{V}$ is the fractional change in volume.

$$\Rightarrow \left(\frac{\Delta V}{V}\right) = \frac{p}{B}$$

$$\Rightarrow \left(\frac{\Delta V}{V}\right) = \frac{10\times1.013\times10^5}{37\times10^9} = 2.73\times10^{-5}$$

Clearly, the fractional change in the volume of the glass slab is 2.73×10^{-5}

15. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7×10^6 Pa.

15. In the above question it is given that:

The length of an edge of the solid copper cube is $l = 10cm = 0.1m$.

Hydraulic pressure is $p = 7 \times 10^6$ Pa.

Bulk modulus of copper is $B = 140 \times 10^9$ Pa.

Bulk modulus is given by the relation:

$$B = \frac{p}{\left(\frac{\Delta V}{V}\right)}$$

Where,

$\frac{\Delta V}{V}$ is the volumetric strain

ΔV is the change in volume.

V is the original volume.

$$\Rightarrow \Delta V = \frac{pV}{B}$$

The original volume of the cube is $V = l^3$.

$$\Rightarrow \Delta V = \frac{pl^3}{B}$$

$$\Rightarrow \Delta V = \frac{7\times10^6\times(0.1)^3}{140\times10^9} = 5\times10^{-8}\,m^3 = 5\times10^{-2}\,cm^3$$

Clearly, the volume contraction of the solid copper cube is 5×10^{-2} cm^3.

16. How much should the pressure on a liter of water be changed to compress it by 0.10%?

16. In the above question it is given that:

Volume of water is V = 1L.

The water is to be compressed by 0.10%

$$\therefore \text{ Fractional change } = \frac{\Delta V}{V} = \frac{0.1}{100\times1} = 10^{-3}$$

$$\therefore \text{ Fractional change } = \frac{\Delta V}{V} = \frac{0.1}{100\times1} = 10^{-3}$$

Bulk modulus is given by the relation:

$$B = \frac{p}{\left(\frac{\Delta V}{V}\right)}$$

$$\Rightarrow p = B\times\left(\frac{\Delta V}{V}\right)$$

We know that, bulk modulus of water is B = 2.2 × 109 N/m^2

$$\Rightarrow p = 2.2\times10^9\times10^{-3} = 2.2\times10^6\,N/m^2.$$

Clearly, the pressure on water should be 2.2×10^6 N/m^2.