

## Applications of Conic Sections

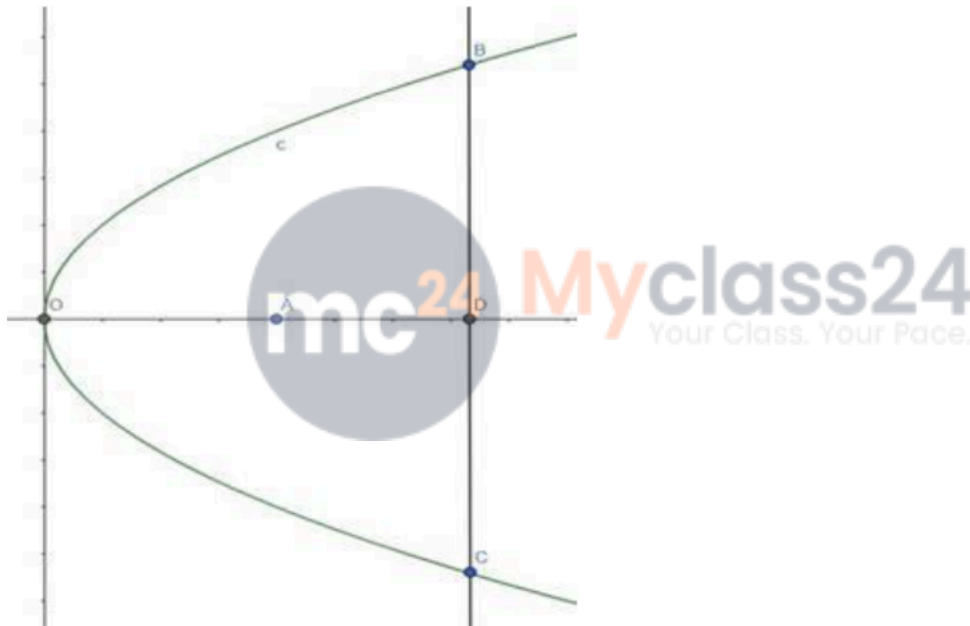
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### Exercise 25

**Q. 1.** The focus of a parabolic mirror is at a distance of 6 cm from its vertex. If the mirror is 20 cm deep, find its diameter.

**Answer :** Given: The focus of a parabolic mirror is at a distance of 6 cm from its vertex. And the mirror is 20 cm deep.

**Need to find:** Diameter of the mirror.



Here O is the vertex and A is the Focus. So,  $OA = a = 6$  cm.

OD is the deep of the mirror = 20 cm

BC is the diameter of the mirror.

Equation of the parabola is,  $y^2 = 4ax$

$$\Rightarrow y^2 = 24x$$

The mirror is 20 cm deep. That means the x-coordinate of the points B, C and D is 20

Both the points, B and C are on the parabola. Hence, the points satisfies the equation of the parabola.

$$\text{Therefore, } y^2 = 24 \times 20 = 480$$

$$\Rightarrow y = \pm 21.9$$

So, the coordinate of B is (20, 21.9) and the coordinate of C is (20, -21.9).

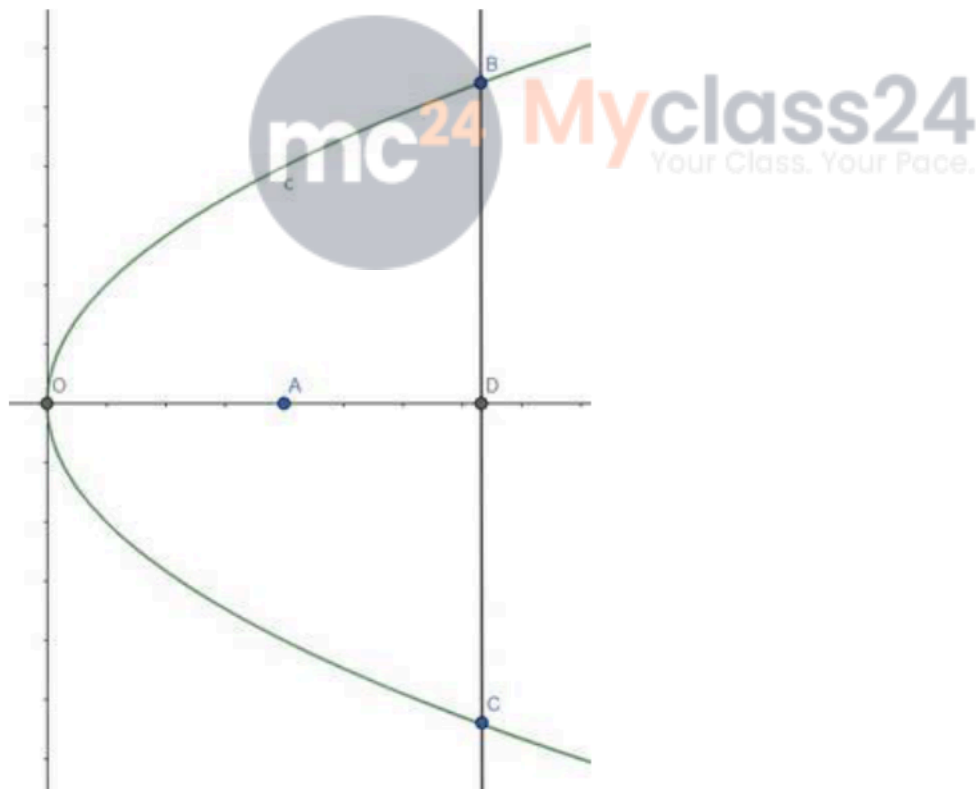
Therefore, the diameter of the mirror is = (21.9 + 21.9) cm

$$= 43.8 \text{ cm}$$

**Q. 2. A parabolic reflector is 5 cm deep and its diameter is 20 cm. How far is its focus from the vertex?**

**Answer :** Given: Parabolic reflector is 5 cm deep & its diameter is 20 cm

**Need to find:** Distance of its focus from the vertex.



Reflector is 5 cm deep, i.e.,  $OD = 5 \text{ cm}$

Diameter of the mirror is 20 cm, i.e.,  $BC = 20 \text{ cm}$

Let, the equation of the parabola is  $y^2 = 4ax$ , where  $a$  is the distance of the focus from the vertex.

The x-coordinate of the points B and C is 5.

D is the middle point of BC which is upon the x-axis.

So, we can say that  $BD = CD = 10$  cm.

So, the coordinate of the point B is (5, 10)

Putting the values of the equation,

$$y^2 = 4ax$$

$$\Rightarrow 100 = 4a \times 5$$

$$\Rightarrow 20a = 100$$

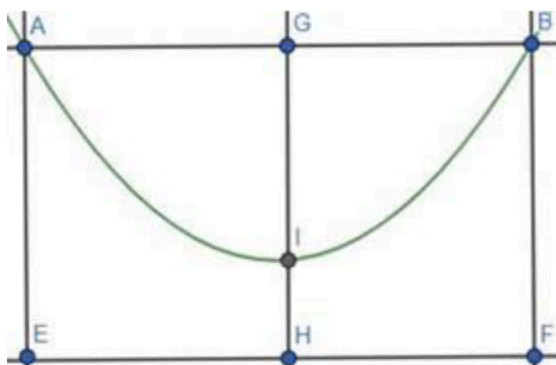
$$\Rightarrow a = 5$$

That means, the focus is 5 cm far from the vertex.

**Q. 3. The towers of bridge, hung in the form of a parabola, have their tops 30 m above the roadway, and are 200 m apart. If the cable is 5 m above the roadway at the center of the bridge, find the length of the vertical supporting cable, 30 m from the center.**

**Answer :** Given: Top of the towers are 30 m above the roadway and are 200 m apart. Cable is 5 m above the roadway at center.

**Need to find:** Length of the vertical supporting cable, 30 m from the center.



A and B are the top of the towers. AE and BF are the height of the towers. H is the center of the bridge. HI is the 5 m above from the roadway.

Let, the equation of the parabola be:  $x^2 = 4a(y - b)$

Here  $b = 5$ . So,  $x^2 = 4a(y - 5)$

Here,  $AB = 200$  m and  $BF = 30$  m.

So, the coordinate of the point B is (100, 30)

The point is on the parabola.

Hence,  $x^2 = 4a(y - 5)$

$$\Rightarrow 10000 = 4a(30 - 5)$$

$$\Rightarrow 10000 = 4a \times 25$$

$$\Rightarrow a = 100$$

Now we need to find, the length of the vertical supporting cable, 30 m from the center.

The x-coordinate of the point, 30 m from the center, is 30.

So,  $30 \times 30 = 4a(y - 5)$

$$\Rightarrow 900 = 400(y - 5)$$

$$\Rightarrow y - 5 = \frac{9}{4}$$

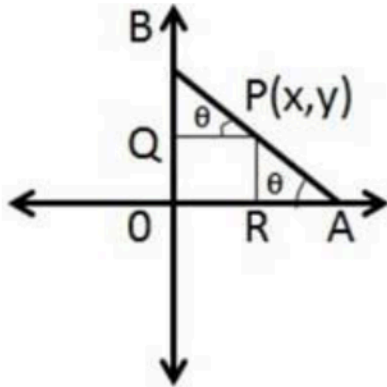
$$\Rightarrow y = \frac{9}{4} + 5 = \frac{29}{4}$$

So, the length of the vertical supporting cable is  $\frac{29}{4}$  m = 7.25 m

**Q. 4. A rod of length 15 cm moves with its ends always touching the coordinate axes. Find the equation of the locus of a point P on the rod, which is at a distance of 3 cm from the end in contact with the x-axis.**

**Answer :** Given: A rod of length 15 cm moves with its ends always touching the coordinate axes. A point P on the rod, which is at a distance of 3 cm from the end in contact with the x-axis

**Need to find:** Find the equation of the locus of a point P



Here AB is the rod making an angle  $\theta$  with the x-axis.

Here  $AP = 3$ .

$$PB = AB - AP = 12 - 3 = 9 \text{ cm}$$

Here, PQ is the perpendicular drawn from the x-axis and RP is the perpendicular drawn from y-axis.

Let, the coordinates of the point P is  $(x, y)$ .

Now, in the triangle  $\Delta BPQ$ ,

$$\cos\theta = \frac{x}{PB} = \frac{x}{9}$$

And in the triangle  $\Delta PAR$ ,

$$\sin\theta = \frac{y}{AP} = \frac{y}{3}$$

We know,  $\sin^2\theta + \cos^2\theta = 1$

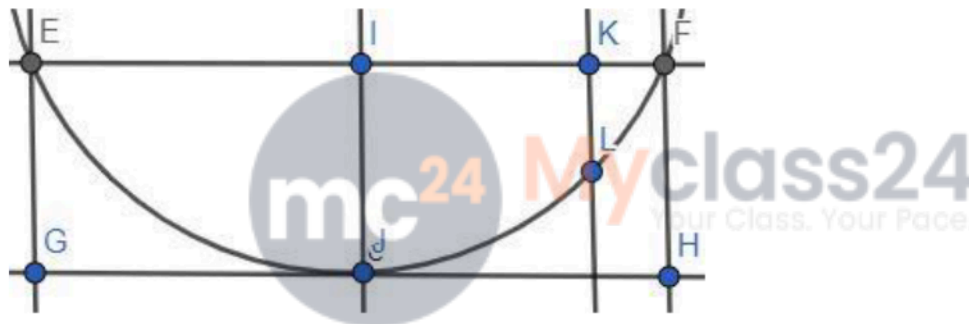
$$\Rightarrow \frac{x^2}{81} + \frac{y^2}{9} = 1$$

This is the locus of the point P.

**Q. 5. A beam is supported at its ends by supports which are 12 m apart. Since the load is concentrated at its center, there is a deflection of 3 cm at the center, and the deflected beam is in the shape of a parabola. How far from the center is the deflection 1 cm?**

**Answer :** Given: A beam is supported at its ends by supports which are 12 m apart. There is a deflection of 3 cm at the center, and the deflected beam is in the shape of a parabola.

**Need to find:** How far from the center is the deflection 1 cm



Here EF are the ends of the beam and they are 12 m apart.

IJ is the deflection of 3 cm at the center.

We know, that the distance  $IF = \frac{12}{2} = 6 \text{ m} = 600 \text{ cm}$  and the deflection  $IJ = FH = 3 \text{ cm}$ .

So, the coordinate of the point F is (600, 3)

Let, the equation of the parabola is:  $x^2 = 4ay$

F point is on the parabola. So, putting the coordinates of F in the equation we get,

$$x^2 = 4ay$$

$$\Rightarrow 3600 = 4a \times 3$$

$$\Rightarrow a = 300$$

Here KL denotes the deflection of 1 cm.

So, at the point L the value of y-coordinate is  $(3 - 1) = 2$

So, by the equation,

$$\Rightarrow x^2 = 4ay = 4 \times 300 \times 2 = 2400$$

$$\Rightarrow x = 49 \text{ cm.}$$

So, the distance of the point of 1 cm deflection from the center is 49 cm.

