EXERCISE 19.1

1. Evaluate the following integrals:

(i) $\int x^4 dx$

Solution:

Given

$$\int x^4 dx$$

Now we have integrate the given function

$$= \frac{x^{4+1}}{4+1} + C$$

$$= \frac{x^5}{5} + C$$

(ii) $\int x^{5/4} dx$

Solution:

Given

$$\int x^{5/4} dx$$

Now we have to integrate the given function

$$= \frac{x^{\frac{5}{4}+1}}{\frac{5}{4}+1} + C$$

On simplifying, we get

$$= \frac{4}{9} x^{\frac{9}{4}} + C$$

(iii) $\int \frac{1}{x^5} dx$

Solution:

Given

$$\int \frac{1}{x^5} dx$$

We can write given question as

$$\int x^{-5} dx$$

Now by integrating, we get

$$= \frac{x^{-5+1}}{-5+1} + C$$

$$= -\frac{1}{4} x^{-4} + C$$

On simplifying we get

$$= -\frac{1}{4x^4} + C$$

(iv) $\int \frac{1}{x^{\frac{3}{2}}} dx$

Solution:

Given

$$\int \frac{1}{x^{\frac{3}{2}}} dx$$

Given equation can be written as

$$\int x^{\frac{-3}{2}} dx$$

Now by integrating the above equation we get

$$= \left[\frac{x^{-\frac{3}{2}+1}}{\frac{-3}{2}+1}\right] + C$$

$$= \left[\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}\right] + C$$

On simplifying we get

$$= -\frac{2}{\sqrt{x}} + C$$

(v) $\int 3^x dx$

Solution:
Given

$$\int 3^x dx$$

We know that

$$\int a^x dx = \frac{a^x}{\log_e a} + c$$

Now by integrating the given equation by using above integration formulae we get

$$\int 3^x dx = \frac{3^x}{\log 3} + c$$

(vi) $\int \frac{1}{\sqrt[3]{x^2}} dx$

Solution:
Given

$$\int \frac{1}{\sqrt[3]{x^2}} dx$$

now above equation can be written as

$$= \int \frac{dx}{x^{2/3}}$$

$$= \int x^{-2/3} dx$$

Now by integrating the above equation we get

$$= \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + C$$

On simplifying

$$= 3x^{\frac{1}{3}} + C$$

(vii) $\int 3^{2\log_3 x} dx$

Solution:
Given

$$\int 3^{2\log_3 x} dx$$

Given equation can be written as

$$= \int 3\log_3 x^2 dx$$

On simplifying we get

$$= \int x^2 dx$$

Now by integrating the above equation we get

$$= \frac{x^3}{3} + C$$

(viii) $\int \log_x x \, dx$

Solution:

Given

$$\int \log_x x \, dx$$

Given equation can be written as

$$= \int 1 \cdot dx$$

By integrating we get

$$= x + C$$

2. Evaluate:

(i) $\int \sqrt{\frac{1 + \cos 2x}{2}} \, dx$

Solution:

Given

$$\int \sqrt{\frac{1 + \cos 2x}{2}} \, dx$$

Given equation can be written as

$$\int \sqrt{\frac{2 \cos^2 x}{2}} dx \; [\therefore 1 + \cos 2A = 2 \cos^2 A]$$

On simplifying, we get

$$= \int \cos\ x\, dx$$

On integrating

$$= \sin x + C$$

(ii) $\int \sqrt{\frac{1-\cos 2x}{2}}\, dx$

Solution:
Given

$$\int \sqrt{\frac{1-\cos 2x}{2}}\, dx$$

Given equation can be written as

$$= \int \sqrt{\frac{2\sin^2 x}{2}}\, dx \quad [\because 1-\cos 2x = 2\sin^2 x]$$

On simplifying we get

$$= \int \sin x\, dx$$

On integrating

$$= -\cos x + C$$

3. Evaluate:

$$\int \frac{e^{6\log_e x} - e^{5\log_e x}}{e^{4\log_e x} - e^{3\log_e x}}\, dx$$

Solution:
Given

$$\int \frac{e^{6\log_e x} - e^{5\log_e x}}{e^{4\log_e x} - e^{3\log_e x}} dx$$

$$= \int \left(\frac{e^{\log x^6} - e^{\log x^5}}{e^{\log x^4} - e^{\log x^3}} \right) dx$$

Above equation can be written as

$$= \int \left(\frac{x^6 - x^5}{x^4 - x^3} \right) dx$$

$$= \int \frac{x^5}{x^3} dx$$

$$= \int x^2 dx$$

Now by integrating we get

$$= \frac{x^3}{3} + C$$