EXERCISE 4.5

1. Find the principal values of each of the following:
(i) $\text{cosec}^{-1}(-\sqrt{2})$
(ii) $\text{cosec}^{-1}(-2)$
(iii) $\text{cosec}^{-1}(2/\sqrt{3})$
(iv) $\text{cosec}^{-1}(2\cos(2\pi/3))$

Solution:
(i) Given $\text{cosec}^{-1}(-\sqrt{2})$
Let $y = \text{cosec}^{-1}(-\sqrt{2})$
$\text{Cosec } y = -\sqrt{2}$
$-\text{Cosec } y = \sqrt{2}$
$-\text{Cosec}(\pi/4) = \sqrt{2}$
$-\text{Cosec}(\pi/4) = \text{cosec}(-\pi/4)$ [since $-\text{cosec}\,\theta = \text{cosec}(-\theta)$]
The range of principal value of cosec^{-1} $[-\pi/2, \pi/2] - \{0\}$ and $\text{cosec}(-\pi/4) = -\sqrt{2}$
$\text{Cosec}(-\pi/4) = -\sqrt{2}$
Therefore the principal value of $\text{cosec}^{-1}(-\sqrt{2})$ is $-\pi/4$

(ii) Given $\text{cosec}^{-1}(-2)$
Let $y = \text{cosec}^{-1}(-2)$
$\text{Cosec } y = -2$
$-\text{Cosec } y = 2$
$-\text{Cosec}(\pi/6) = 2$
$-\text{Cosec}(\pi/6) = \text{cosec}(-\pi/6)$ [since $-\text{cosec}\,\theta = \text{cosec}(-\theta)$]
The range of principal value of cosec^{-1} $[-\pi/2, \pi/2] - \{0\}$ and $\text{cosec}(-\pi/6) = -2$
$\text{Cosec}(-\pi/6) = -2$
Therefore the principal value of $\text{cosec}^{-1}(-2)$ is $-\pi/6$

(iii) Given $\text{cosec}^{-1}(2/\sqrt{3})$
Let $y = \text{cosec}^{-1}(2/\sqrt{3})$
$\text{Cosec } y = (2/\sqrt{3})$
$\text{Cosec}(\pi/3) = (2/\sqrt{3})$
Therefore range of principal value of cosec^{-1} is $[-\pi/2, \pi/2] - \{0\}$ and $\text{cosec}(\pi/3) = (2/\sqrt{3})$
Thus, the principal value of $\text{cosec}^{-1}(2/\sqrt{3})$ is $\pi/3$

(iv) Given $\text{cosec}^{-1}(2\cos(2\pi/3))$
But we know that $\cos(2\pi/3) = -½$
Therefore $2\cos(2\pi/3) = 2 \times -½$
$2\cos(2\pi/3) = -1$
By substituting these values in $\text{cosec}^{-1}(2\cos(2\pi/3))$ we get,
$\text{Cosec}^{-1}(-1)$
Let $y = \text{cosec}^{-1}(-1)$
$-\text{Cosec } y = 1$
$-\text{Cosec}(\pi/2) = \text{cosec}(-\pi/2)$ [since $-\text{cosec}\,\theta = \text{cosec}(-\theta)$]
The range of principal value of cosec^{-1} $[-\pi/2, \pi/2] - \{0\}$ and $\text{cosec}(-\pi/2) = -1$
$\text{Cosec}(-\pi/2) = -1$
Therefore the principal value of $\text{cosec}^{-1}(2\cos(2\pi/3))$ is $-\pi/2$