Exercise 5(E)

Factorize:

1. $x^2 + 1/4x^2 + 1 – 7x – 7/2x$

Solution:

We have, $x^2 + \frac{1}{4x^2} + 1 - 7x - \frac{7}{2x}$

$= [x^2 + 1/(2x)^2 + 2 \times x \times 1/(2x)] – 7 [x + 1/(2x)]$

$= (x + 1/2x)^2 – 7(x + 1/x)$

Taking out (x + 1/2x) as common,

$= (x + 1/2x) (x + 1/2x - 7)$

2. $9a^2 + 1/9a^2 – 2 – 12a + 4/3a$

Solution:

We have, $(9a)^2 + \frac{1}{(9a)^2} - 2 - 12a + \frac{4}{3a}$

$= (3a)^2 + \frac{1}{(3a)^2} - 2 \times 3a \times \frac{1}{3a} - 4\left(3a - \frac{1}{3a}\right)$

$= \left(3a - \frac{1}{3a}\right)^2 - 4\left(3a - \frac{1}{3a}\right)$

Taking (3a - 1/3a) as common,

$= \left(3a - \frac{1}{3a}\right)\left[\left(3a - \frac{1}{3a}\right) - 4\right]$

$= \left(3a - \frac{1}{3a}\right)\left(3a - 4 - \frac{1}{3a}\right)$

3. $x^2 + (a^2 + 1) x/a + 1$

Solution:

We have, $x^2 + \frac{a^2+1}{a}x + 1$

$= x^2 + ax + \frac{1}{a}x + 1$

$= x(x+a) + \frac{1}{a}(x+a)$

$= (x+a)\left(x + \frac{1}{a}\right)$

4. $x^4 + y^4 - 27x^2y^2$
Solution:

We have, $x^4 + y^4 - 27x^2y^2$
$= x^4 + y^4 - 2x^2y^2 - 25x^2y^2$
$= [(x^2) + (y^2) - 2x^2y^2] - 25x^2y^2$
$= (x^2 - y^2) - (5xy)^2$
$= (x^2 - y^2 - 5xy)(x^2 - y^2 + 5xy)$ [As $x^2 - y^2 = (x + y)(x - y)$]

5. $4x^4 + 9y^4 + 11x^2y^2$
Solution:

We have, $4x^4 + 9y^4 + 11x^2y^2$
$= 4x^4 + 9y^4 + 12x^2y^2 - x^2y^2$
$= (2x^2)^2 + (3y^2)^2 + 2(2x^2)(3y^2) - (xy)^2$
$= (2x^2 + 3y^2)^2 - (xy)^2$
$= (2x^2 + 3y^2 - xy)(2x^2 + 3y^2 + xy)$ [As $x^2 - y^2 = (x + y)(x - y)$]

6. $x^2 + 1/x^2 - 3$
Solution:

We have, $x^2 + 1/x^2 - 3$
$= x^2 + 1/x^2 - 2 - 1$
$= [x^2 + 1/x^2 - (2 \times x \times 1/x)] - 1^2$
$= (x - 1/x)^2 - 1^2$
$= (x - 1/x - 1)(x - 1/x + 1)$ [As $x^2 - y^2 = (x + y)(x - y)$]

7. $a - b - 4a^2 + 4b^2$
Solution:

We have, $a - b - 4a^2 + 4b^2$
$= (a - b) - 4(a^2 - b^2)$

= (a - b) – 4(a - b)(a + b) [As $x^2 - y^2 = (x + y)(x - y)$]
Taking (a - b) common,
= (a - b) [1 – 4(a + b)]
= (a - b) [1 – 4a – 4b]

8. $(2a - 3)^2 - 2(2a - 3)(a - 1) + (a - 1)^2$
Solution:

We have, $(2a - 3)^2 - 2(2a - 3)(a - 1) + (a - 1)^2$
Comparing with the identity, $(a - b)^2 = a^2 - 2ab + b^2$
$= [(2a - 3) - (a - 1)]^2$
$= (2a - a - 3 + 1)^2$
$= (a - 2)^2$

9. $(a^2 - 3a)(a^2 - 3a + 7) + 10$
Solution:

Let's substitute $(a^2 - 3a) = x$
Then the given expression becomes,
= x(x + 7) + 10
$= x^2 + 7x + 10$
$= x^2 + 5x + 2x + 10$ [By splitting the middle term]
= x(x + 5) + 2(x + 5)
= (x + 2) (x + 5)
Resubstituting the value of x,
$= (a^2 - 3a + 2)(a^2 - 3a + 5)$
$= (a^2 - 3a + 5)(a^2 - 3a + 2)$
$= (a^2 - 3a + 5)[a^2 - 2a - a + 2]$ [By splitting the middle term]
$= (a^2 - 3a + 5)[a(a - 2) - 1(a - 5)]$
$= (a^2 - 3a + 5)[(a - 1)(a - 2)]$
$= (a^2 - 3a + 5)(a - 1)(a - 2)$

10. $(a^2 - a)(4a^2 - 4a - 5) - 6$
Solution:

Let's $a^2 - a = x$
Then the expression becomes,
= x(4x - 5) – 6
$= 4x^2 - 5x - 6$
$= 4x^2 - 8x + 3x - 6$
= 4x(x - 2) + 3(x - 2)
= (4x + 3) (x - 2)
Resubstituting the value of x,
$= (4a^2 - 4a + 3)(a^2 - a - 2)$
$= (4a^2 - 4a + 3)(a^2 - 2a + a - 2)$
$= (4a^2 - 4a + 3)[a(a - 2) + 1(a - 2)]$

$= (4a^2 – 4a + 3) [(a + 1) (a – 2)]$
$= (4a^2 – 4a + 3) (a + 1) (a – 2)$

11. $x^4 + y^4 - 3x^2y^2$
Solution:

We have, $x^4 + y^4 - 3x^2y^2$
$= (x^4 + y^4 - 2x^2y^2) - x^2y^2$
$= (x^2 - y^2) – (xy)^2$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= (x^2 – y^2 – xy) (x^2 – y^2 + xy)$

12. $5a^2 - b^2 - 4ab + 7a - 7b$
Solution:

We have, $5a^2 - b^2 - 4ab + 7a - 7b$
$= 4a^2 + a^2 - b^2 - 4ab + 7a - 7b$
$= a^2 - b^2 + 4a^2 - 4ab + 7a - 7b$
$= (a^2 - b^2) + 4a(a – b) + 7(a – b)$
$= (a + b)(a - b) + 4a(a - b) + 7(a - b)$ [As $x^2 – y^2 = (x + y)(x - y)$]
$= (a - b) [(a + b) + 4a + 7]$
$= (a - b) (5a + b + 7)$

13. $12(3x - 2y)^2 - 3x + 2y - 1$
Solution:

We have, $12(3x - 2y)^2 - 3x + 2y – 1$
$= 12(3x - 2y)^2 – (3x - 2y) – 1$
Let's substitute $(3x – 2y) = a$
Then, the expression becomes
$= 12a^2 – a – 1$
$= 12a^2 – 4a + 3a – 1$
$= 4a (3a - 1) + 1(3a - 1)$
$= (4a + 1) (3a - 1)$
Now, resubstituting the value of 'a' in the above
$= [4(3x – 2y) + 1] [3(3x – 2y) - 1]$
$= (12x – 8y + 1) (9x – 6y – 1)$

14. $4(2x - 3y)^2 - 8x+12y - 3$
Solution:

We have, $4(2x - 3y)^2 - 8x+12y – 3$
$= 4(2x - 3y)^2 – 4(2x + 3y) – 3$
Let's substitute $(2x – 3y) = a$
$= 4(a^2) – 4a – 3$
$= 4a^2 – 6a + 2a – 3$ [By splitting the middle term]
$= 2a(2a - 3) + 1(2a - 3)$

= (2a - 3) (2a + 1)
Now, resubstituting the value of 'a' in the above
= [2(2x – 3y) – 3] [2(2x – 3y) + 1]
= (4x – 6y - 3) (4x – 6y + 1)

15. $3 - 5x + 5y - 12(x - y)^2$
Solution:

We have, $3 - 5x + 5y - 12(x - y)^2$
$= 3 - 5(x - y) - 12(x - y)^2$
Let's substitute (x – y) = a
$= 3 - 5a - 12a^2$
$= 3 - 9a + 4a - 12a^2$ [By splitting the middle term]
= 3(1 – 3a) + 4a(1 – 3a)
= (1 – 3a) (4a + 3)
Now, resubstituting the value of 'a' in the above
= [1 – 3(x - y)] [4(x - y) + 3]
= (1 – 3x + 3y) (4x – 4y + 3)

16. $9x^2 + 3x - 8y - 64y^2$
Solution:

We have, $9x^2 + 3x - 8y - 64y^2$
On rearranging,
$= 9x^2 - 64y^2 + 3x - 8y$
$= [(3x)^2 - (8y)^2] + (3x - 8y)$
= (3x – 8y) (3x + 8y) + (3x – 8y) [As $x^2 - y^2 = (x + y)(x - y)$]
Taking (3x – 8y) as common,
= (3x – 8y) (3x + 8y + 1)

17. $2\sqrt{3}x^2 + x - 5\sqrt{3}$
Solution:

We have, $2\sqrt{3}x^2 + x - 5\sqrt{3}$
By splitting the middle term,
$= 2\sqrt{3}x^2 + 6x - 5x - 5\sqrt{3}$
$= 2\sqrt{3}x(x + \sqrt{3}) - 5(x + \sqrt{3})$
$= (2\sqrt{3}x - 5)(x + \sqrt{3})$

18. $\frac{1}{4}(a + b)^2 - 9/16\,(2a - b)^2$
Solution:

We have, $\frac{1}{4}(a+b)^2 - \frac{9}{16}(2a-b)^2$

$= \frac{1}{4}\left[(a+b)^2 - \frac{9}{4}(2a-b)^2\right]$

$= \frac{1}{4}\left[(a+b)^2 - \left[\frac{3}{2}(2a-b)\right]^2\right]$

$= \frac{1}{4}\left[\left(a+b+\frac{3}{2}(2a-b)\right)\left(a+b-\frac{3}{2}(2a-b)\right)\right]$ [As $x^2 - y^2 = (x + y)(x - y)$]

$= \frac{1}{4}\left[\left(a+b+3a-\frac{3b}{2}\right)\left(a+b-3a+\frac{3b}{2}\right)\right]$

$= \frac{1}{4}\left[\left(4a-\frac{b}{2}\right)\left(\frac{5b}{2}-2a\right)\right]$

$= \frac{1}{4}\left[\left(\frac{8a-b}{2}\right)\left(\frac{5b-4a}{2}\right)\right]$

$= \frac{1}{4}\left[\frac{1}{4}(8a-b)(5b-4a)\right]$

$= \frac{1}{16}(8a-b)(5b-4a)$

19. $2(ab + cd) - a^2 - b^2 + c^2 + d^2$
Solution:

We have, $2(ab + cd) - a^2 - b^2 + c^2 + d^2$
$= 2ab + 2cd - a^2 - b^2 + c^2 + d^2$
On rearranging and grouping, we get
$= (c^2 + d^2 + 2cd) - (a^2 + b^2 - 2ab)$
$= (c + d)^2 - (a - b)^2$
$= [c + d - (a - b)] [c + d + (a - b)]$ [As $x^2 - y^2 = (x + y)(x - y)$]
$= (c + d - a + b) (c + d + a - b)$

20. Find the value of:
(i) $987^2 - 13^2$
(ii) $(67.8)^2 - (32.2)^2$
(iii) $[(6.7)^2 - (3.3)^2]/ (6.7 - 3.3)$
(iv) $[(18.5)^2 - (6.5)^2]/ (18.5 - 6.5)$

Solution:

(i) We have, $987^2 - 13^2$
$= (987 + 13)(987 - 13)$
$= 1000 \times 974$
$= 974000$

(ii) We have, $(67.8)^2 - (32.2)^2$
$= (67.8 + 32.2)(67.8 - 32.2)$
$= 100 \times 35.6$
$= 3560$

(iii) We have, $\dfrac{(6.7)^2 - (3.3)^2}{6.7 - 3.3}$

$= \dfrac{(6.7 + 3.3)(6.7 - 3.3)}{(6.7 - 3.3)}$

$= 10$

(iv) We have, $\dfrac{(18.5)^2 - (6.5)^2}{18.5 + 6.5}$

$= \dfrac{(18.5 + 6.5)(18.5 - 6.5)}{(18.5 + 6.5)}$

$= 12$