Exercise 21(A)

Prove the following identities:

1. sec A – 1/ sec A + 1 = 1 – cos A/ 1 + cos A

Solution:

$$LHS = \frac{\sec A - 1}{\sec A + 1} = \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1}$$

$$= \frac{1 - \cos A}{1 + \cos A} = RHS$$

- Hence Proved

2. 1 + sin A/ 1 – sin A = cosec A + 1/ cosec A – 1

Solution:

$$LHS = \frac{1 + \sin A}{1 - \sin A}$$

$$RHS = \frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1} = \frac{\frac{1}{\sin A} + 1}{\frac{1}{\sin A} - 1}$$

$$= \frac{1 + \sin A}{1 - \sin A}$$

- Hence Proved

3. 1/ tan A + cot A = cos A sin A

Solution:

Taking L.H.S,

$$\frac{1}{\tan A + \cot A} = \sin A \cos A$$

$$LHS = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{1}{\frac{1}{\sin A \cos A}} \left(\because \sin^2 A + \cos^2 A = 1\right)$$

$$= \sin A \cos A = RHS$$

- Hence Proved

4. tan A – cot A = 1 – 2 cos^2 A/ sin A cos A

Solution:

Taking LHS,

$$\tan A - \cot A = \frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}$$
$$= \frac{\sin^2 A - \cos^2 A}{\sin A \cos A}$$
$$= \frac{1 - \cos^2 A - \cos^2 A}{\sin A \cos A} \left(\because \sin^2 A = 1 - \cos^2 A\right)$$
$$= \frac{1 - 2\cos^2 A}{\sin A \cos A}$$

- Hence Proved

5. $\sin^4 A - \cos^4 A = 2\sin^2 A - 1$
Solution:

Taking L.H.S,
$\sin^4 A - \cos^4 A$
$= (\sin^2 A)^2 - (\cos^2 A)^2$
$= (\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A)$
$= \sin^2 A - \cos^2 A$
$= \sin^2 A - (1 - \sin^2 A)$ [Since, $\cos^2 A = 1 - \sin^2 A$]
$= 2\sin^2 A - 1$
- Hence Proved

6. $(1 - \tan A)^2 + (1 + \tan A)^2 = 2\sec^2 A$
Solution:

Taking L.H.S,
$(1 - \tan A)^2 + (1 + \tan A)^2$
$= (1 + \tan^2 A + 2\tan A) + (1 + \tan^2 A - 2\tan A)$
$= 2(1 + \tan^2 A)$
$= 2\sec^2 A$ [Since, $1 + \tan^2 A = \sec^2 A$]
- Hence Proved

7. $\text{cosec}^4 A - \text{cosec}^2 A = \cot^4 A + \cot^2 A$
Solution:

$\text{cosec}^4 A - \text{cosec}^2 A$
$= \text{cosec}^2 A(\text{cosec}^2 A - 1)$
$= (1 + \cot^2 A)(1 + \cot^2 A - 1)$
$= (1 + \cot^2 A)\cot^2 A$
$= \cot^4 A + \cot^2 A =$ R.H.S
- Hence Proved

8. $\sec A(1 - \sin A)(\sec A + \tan A) = 1$

Solution:

Taking L.H.S,

sec A (1 – sin A) (sec A + tan A)

$$= \frac{1}{\cos A}(1-\sin A)\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)$$

$$= \frac{(1-\sin A)}{\cos A}\left(\frac{1+\sin A}{\cos A}\right) = \left(\frac{1-\sin^2 A}{\cos^2 A}\right)$$

$$= \left(\frac{\cos^2 A}{\cos^2 A}\right) = 1 = \text{RHS}$$

- Hence Proved

9. cosec A (1 + cos A) (cosec A – cot A) = 1

Solution:

Taking L.H.S,

$$= \frac{1}{\sin A}(1+\cos A)\left(\frac{1}{\sin A}-\frac{\cos A}{\sin A}\right)$$

$$= \frac{(1+\cos A)}{\sin A}\left(\frac{1-\cos A}{\sin A}\right)$$

$$= \frac{1-\cos^2 A}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1 = \text{RHS}$$

- Hence Proved

10. $\sec^2 A + \mathrm{cosec}^2 A = \sec^2 A \cdot \mathrm{cosec}^2 A$

Solution:

Taking L.H.S,

$$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\cos^2 A . \sin^2 A}$$

$$= \frac{1}{\cos^2 A . \sin^2 A} = \sec^2 A \,\mathrm{cosec}^2 A = \text{RHS}$$

- Hence Proved

11. $(1 + \tan^2 A) \cot A / \mathrm{cosec}^2 A = \tan A$

Solution:

Taking L.H.S,

$$\frac{\left(1+\tan^2 A\right)\cot A}{\operatorname{cosec}^2 A}$$
$$= \frac{\sec^2 A \cot A}{\operatorname{cosec}^2 A}\left(\because \sec^2 A = 1+\tan^2 A\right)$$
$$= \frac{\frac{1}{\cos^2 A}\times\frac{\cos A}{\sin A}}{\frac{1}{\sin^2 A}} = \frac{\frac{1}{\cos A \sin A}}{\frac{1}{\sin^2 A}}$$
$$= \frac{\sin A}{\cos A} = \tan A$$
= RHS

- Hence Proved

12. $\tan^2 A - \sin^2 A = \tan^2 A. \sin^2 A$
Solution:

Taking L.H.S,
$\tan^2 A - \sin^2 A$
$$= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \frac{\sin^2 A(1-\cos^2 A)}{\cos^2 A}$$
$$= \frac{\sin^2 A}{\cos^2 A}.\sin^2 A = \tan^2 A.\sin^2 A = \text{RHS}$$

- Hence Proved

13. $\cot^2 A - \cos^2 A = \cos^2 A. \cot^2 A$
Solution:

Taking L.H.S,
$\cot^2 A - \cos^2 A$
$$= \frac{\cos^2 A}{\sin^2 A} - \cos^2 A = \frac{\cos^2 A(1-\sin^2 A)}{\sin^2 A}$$
$$= \cos^2 A\frac{\cos^2 A}{\sin^2 A} = \cos^2 A.\cot^2 A = \text{RHS}$$

- Hence Proved

14. (cosec A + sin A) (cosec A – sin A) = $\cot^2 A + \cos^2 A$
Solution:

Taking L.H.S,
(cosec A + sin A) (cosec A – sin A)
$= \operatorname{cosec}^2 A - \sin^2 A$
$= (1 + \cot^2 A) - (1 - \cos^2 A)$
$= \cot^2 A + \cos^2 A$ = R.H.S

- Hence Proved

15. (sec A – cos A)(sec A + cos A) = $\sin^2$ A + $\tan^2$ A
Solution:

Taking L.H.S,
(sec A – cos A)(sec A + cos A)
= ($\sec^2$ A – $\cos^2$ A)
= (1 + $\tan^2$ A) – (1 – $\sin^2$ A)
= $\sin^2$ A + $\tan^2$ A = RHS
- Hence Proved

16. (cos A + sin A)^2 + (cosA – sin A)^2 = 2
Solution:

Taking L.H.S,
(cos A + sin A)^2 + (cosA – sin A)^2
= cos2 A + sin2 A + 2cos A sin A + cos2 A – 2cosA.sinA
= 2 ($\cos^2$ A + $\sin^2$ A) = 2 = R.H.S
- Hence Proved

17. (cosec A – sin A)(sec A – cos A)(tan A + cot A) = 1
Solution:

Taking LHS,
(cosec A – sin A)(sec A – cos A)(tan A + cot A)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\tan A + \frac{1}{\tan A}\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin A.\cos A}\right)$$

$= 1$
= RHS
- Hence Proved

18. 1/ sec A + tan A = sec A – tan A
Solution:

Taking LHS,

$$\frac{1}{\sec A + \tan A}$$
$$= \frac{1}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}$$
$$= \frac{\sec A - \tan A}{\sec^2 A - \tan^2 A}$$
$$= \sec A - \tan A$$
= RHS

- Hence Proved

19. cosec A + cot A = 1/ cosec A – cot A
Solution:

Taking LHS,
cosec A + cot A
$$= \frac{\operatorname{cosec} A + \cot A}{1} \times \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A - \cot A}$$
$$= \frac{\operatorname{cosec}^2 A - \cot^2 A}{\operatorname{cosec} A - \cot A} = \frac{1 + \cot^2 A - \cot^2 A}{\operatorname{cosec} A - \cot A}$$
$$= \frac{1}{\operatorname{cosec} A - \cot A}$$
= RHS

- Hence Proved

20. sec A – tan A/ sec A + tan A = 1 – 2 secA tanA + 2 tan² A
Solution:
Taking LHS,
$$\frac{\sec A - \tan A}{\sec A + \tan A}$$
$$= \frac{\sec A - \tan A}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}$$
$$= \frac{(\sec A - \tan A)^2}{\sec^2 A - \tan^2 A}$$
$$= \frac{\sec^2 A + \tan^2 A - 2\sec A \tan A}{1}$$
$= 1 + \tan^2 A + \tan^2 A – 2 \sec A \tan A$
$= 1 – 2 \sec A \tan A + 2 \tan^2 A$ = RHS

- Hence Proved

21. (sin A + cosec A)² + (cos A + sec A)² = 7 + tan² A + cot² A
Solution:

Taking LHS,

$(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2$
$= \sin^2 A + \text{cosec}^2 A + 2 \sin A \text{ cosec } A + \cos^2 A + \sec^2 A + 2\cos A \sec A$
$= (\sin^2 A + \cos^2 A) + \text{cosec}^2 A + \sec^2 A + 2 + 2$
$= 1 + \text{cosec}^2 A + \sec^2 A + 4$
$= 5 + (1 + \cot^2 A) + (1 + \tan^2 A)$
$= 7 + \tan^2 A + \cot^2 A =$ RHS

- Hence Proved

22. $\sec^2 A. \text{cosec}^2 A = \tan^2 A + \cot^2 A + 2$
Solution:

Taking,
RHS $= \tan^2 A + \cot^2 A + 2 = \tan^2 A + \cot^2 A + 2 \tan A. \cot A$
$= (\tan A + \cot A)^2 = (\sin A/\cos A + \cos A/ \sin A)^2$
$= (\sin 2 A + \cos 2 A/ \sin A.\cos A)^2 = 1/ \cos^2 A. \sin^2 A$
$= \sec^2 A. \text{cosec}^2 A =$ LHS

- Hence Proved

23. $1/ 1 + \cos A + 1/ 1 - \cos A = 2 \text{ cosec}^2 A$
Solution:

Taking LHS,

$$\frac{1}{1+\cos A} + \frac{1}{1-\cos A}$$

$$= \frac{1-\cos A + 1 + \cos A}{(1+\cos A)(1-\cos A)}$$

$$= \frac{2}{1-\cos^2 A}$$

$$= \frac{2}{\sin^2 A}$$

$$= 2\text{cosec}^2 A$$

= RHS

- Hence Proved

24. $1/ 1 - \sin A + 1/ 1 + \sin A = 2 \sec^2 A$
Solution:

Taking LHS,

$\frac{1}{1-\sin A}+\frac{1}{1+\sin A}$

$=\frac{1+\sin A+1-\sin A}{(1-\sin A)(1+\sin A)}$

$=\frac{2}{1-\sin^2 A}$

$=\frac{2}{\cos^2 A}$

$=2\sec^2 A$

= RHS

- Hence Proved