NCERT Solutions for Class-XII Maths

Chapter-11.1

NCERT Maths Class 12

1. If a line makes angles 90°, 135°, 45° with x, y and z-axes respectively, find its direction cosines.

1. Let direction cosines of the line be l, m and n.

$l = \cos 90^\circ = 0$

$m = \cos 135^\circ = -\frac{1}{\sqrt{2}}$

$n = \cos 45^\circ = \frac{1}{\sqrt{2}}$

Therefore, the direction cosines of the line are $0, -\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$.

2. Find the direction cosines of a line which makes equal angles with the coordinate axes.

2. Let the direction cosines of the line making $\angle\alpha$ with x-axis, β – with y axis and γ – with z axis are l, m and n

$\Rightarrow l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$

Here given $\alpha = \beta = \gamma$ (line makes equal angles with the coordinate axes)……….1

Direction Cosines are

$\Rightarrow l = \cos\alpha$, $m = \cos\beta$ and $n = \cos\gamma$

We have

$l^2 + m^2 + n^2 = 1$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

From 1 we have

$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$3\cos^2\alpha = 1$

$\cos\alpha = \cos\alpha = \pm\sqrt{\frac{1}{3}}$

The direction cosines are

$l = \pm\sqrt{\frac{1}{3}}, m = \pm\sqrt{\frac{1}{3}}, n = \pm\sqrt{\frac{1}{3}}$

3. If a line has the direction ratios −18, 12, −4, then what are its direction cosines?

3. If a line has direction ratios of −18, 12, and −4, then its direction cosines are

$$\frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$$

i.e., $\frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$

$\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$

Thus, the direction cosines are $\frac{-9}{11}, \frac{6}{11}$ and $\frac{-2}{11}$

4. Show that the points (2, 3, 4), (−1, −2, 1), (5, 8, 7) are collinear.

4. If the direction ratios of two lines segments are proportional, then the lines are collinear.
Given A(2, 3, 4), B(−1, −2, 1), C(5, 8, 7)
Direction ratio of line joining A (2, 3, 4) and B (−1, −2, 1), are
(−1−2), (−2−3), (1−4)
= (−3, −5, −3)
So $a_1 = -3$, $b_1 = -5$, $c_1 = -3$
Direction ratio of line joining B (−1, −2, 1) and C (5, 8, 7) are
(5− (−1)), (8−(−2)), (7−1)
= (6, 10, 6)
So $a_2 = 6$, $b_2 = 10$ and $c_2 = 6$
It is clear that the direction ratios of AB and BC are of same proportions
As

$\frac{a_1}{a_2} = \frac{-3}{6} = -2$

$\frac{b_1}{b_2} = \frac{-5}{10} = -2$

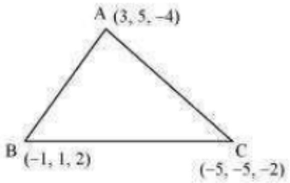
and

$\frac{c_1}{c_2} = \frac{-3}{6} = -2$

Therefore A, B, C are collinear.

5. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, −4), (−1, 1, 2) and (−5, −5, −2)

5. The vertices of ΔABC are A (3, 5, −4), B (−1, 1, 2), and C (−5, −5, −2).



The direction ratios of side AB are (−1 − 3), (1 − 5), and (2 − (−4)) i.e., −4, −4, and 6.

Then, $\sqrt{(-4)^2+(-4)^2+(6)^2} = \sqrt{16+16+36}$

$= \sqrt{68} = 2\sqrt{17}$

Therefore, the direction cosines of AB are

$$\frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}}, \frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}}, \frac{6}{\sqrt{(-4)^2+(-4)^2+(6)^2}}$$

$$\frac{-4}{2\sqrt{17}}, \frac{4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}}$$

$$\frac{-2}{\sqrt{7}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$$

The direction ratios of BC are (−5 − (−1)), (−5 − 1), and (−2 − 2) i.e., −4, −6, and −4.

Therefore, the direction cosines of BC are

$$\frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}, \frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$

$$\frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}$$

The direction ratios of CA are (−5 − 3), (−5 − 5), and (−2 − (−4)) i.e., −8, −10, and 2.

Therefore, the direction cosines of AC are

$$\frac{-8}{\sqrt{(-8)^2+(10)^2+(2)^2}}, \frac{-5}{\sqrt{(-8)^2+(10)^2+(2)^2}}, \frac{2}{\sqrt{(-8)^2+(10)^2+(2)^2}}$$

$$\frac{-8}{2\sqrt{42}}, \frac{-10}{2\sqrt{42}}, \frac{2}{2\sqrt{42}}$$