Exercise 13(B)

Solution1:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔABD and applying Pythagoras theorem we get,

$$AB^2 = AD^2 + BD^2$$
$$c^2 = h^2 + (a-x)^2$$
$$h^2 = c^2 - (a-x)^2 \quad \ldots\ldots(i)$$

First, we consider the ΔACD and applying Pythagoras theorem we get,

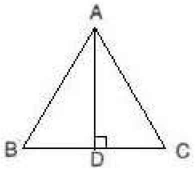
$$AC^2 = AD^2 + CD^2$$
$$b^2 = h^2 + x^2$$
$$h^2 = b^2 - x^2 \quad \ldots\ldots(ii)$$

From (*i*) and (*ii*) we get,

$$c^2 - (a-x)^2 = b^2 - x^2$$
$$c^2 - a^2 - x^2 + 2ax = b^2 - x^2$$
$$c^2 = a^2 + b^2 - 2ax$$

Hence Proved.

Solution 2:



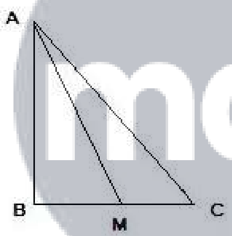
In equilateral ΔABC, $AD \perp BC$.

Therefore, BD = DC = x/2 cm.

$$\text{In right-angled } \Delta ADC$$
$$AC^2 = AD^2 + DC^2$$
$$\Rightarrow (x)^2 = AD^2 + \left(\frac{x}{2}\right)^2$$
$$\Rightarrow AD^2 = (x^2) - \left(\frac{x}{2}\right)^2$$
$$\Rightarrow AD^2 = \left(\frac{x}{2}\right)^2$$
$$\Rightarrow AD = \left(\frac{x}{2}\right) cm$$

Solution 3:

The pictorial form of the given problem is as follows,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔABM and applying Pythagoras theorem we get,

$$AM^2 = AB^2 + BM^2$$
$$AB^2 = AM^2 - BM^2 \quad \ldots\ldots (i)$$

Now, we consider the ΔABC and applying Pythagoras theorem we get,

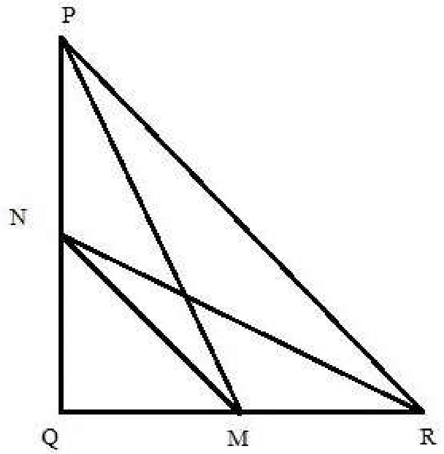
$$AC^2 = AB^2 + BC^2$$
$$AB^2 = AC^2 - BC^2 \quad \ldots\ldots (ii)$$

From (*i*) and (*ii*) we get,

$$AM^2 - BM^2 = AC^2 - BC^2$$
$$AM^2 + BC^2 = AC^2 + BM^2$$

Hence Proved

Solution 4:



We draw , PM,MN,NR

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Since, M andN are the mid-points of the sides QR and PQ respectively, therefore, PN=NQ,QM=RM

(i)

First, we consider the ΔPQM, and applying Pythagoras theorem we get,

$$\begin{aligned} PM^2 &= PQ^2 + MQ^2 \\ &= (PN+NQ)^2 + MQ^2 \\ &= PN^2 + NQ^2 + 2PN.NQ + MQ^2 \\ &= MN^2 + PN^2 + 2PN.NQ \quad \begin{bmatrix} \text{From, } \Delta MNQ, \\ MN^2 = NQ^2 + MQ^2 \end{bmatrix} \ldots\ldots(i) \end{aligned}$$

Now, we consider the ΔRNQ, and applying Pythagoras theorem we get,

$$\begin{aligned} RN^2 &= NQ^2 + RQ^2 \\ &= NQ^2 + (QM+RM)^2 \\ &= NQ^2 + QM^2 + RM^2 + 2QM.RM \\ &= MN^2 + RM^2 + 2QM.RM \end{aligned}$$

$$\ldots\ldots(ii)$$

Adding (*i*) and (*ii*) we get,

$$\begin{aligned} PM^2 + RN^2 &= MN^2 + PN^2 + 2PN.NQ + MN^2 + RM^2 + 2QM.RM \\ PM^2 + RN^2 &= 2MN^2 + PN^2 + RM^2 + 2PN.NQ + 2QM.RM \\ PM^2 + RN^2 &= 2MN^2 + NQ^2 + QM^2 + 2(QN^2) + 2(QM^2) \\ PM^2 + RN^2 &= 2MN^2 + MN^2 + 2MN^2 \\ PM^2 + RN^2 &= 5MN^2 \end{aligned}$$

Hence Proved

(ii)

We consider the ΔPQM, and applying Pythagoras theorem we get,

$$\begin{aligned} PM^2 &= PQ^2 + MQ^2 \\ 4PM^2 &= 4PQ^2 + 4MQ^2 \quad \begin{bmatrix} \text{Multiplying both} \\ \text{sides by 4} \end{bmatrix} \\ 4PM^2 &= 4PQ^2 + 4\cdot\left(\frac{1}{2}QR\right)^2 \quad \left[MQ = \frac{1}{2}QR\right] \\ 4PM^2 &= 4PQ^2 + 4\cdot\frac{1}{4}QR^2 \\ 4PM^2 &= 4PQ^2 + QR^2 \end{aligned}$$

Hence Proved

(iii)

We consider the $\Delta RQN,$ and applying Pythagoras theorem we get,

$$RN^2 = NQ^2 + RQ^2$$

$$4RN^2 = 4NQ^2 + 4QR^2 \qquad \begin{bmatrix}\text{Multiplying both} \\ \text{sides by 4}\end{bmatrix}$$

$$4RN^2 = 4QR^2 + 4\cdot\left(\frac{1}{2}PQ\right)^2 \qquad \left[NQ = \frac{1}{2}PQ\right]$$

$$4RN^2 = 4QR^2 + 4\cdot\frac{1}{4}PQ^2$$

$$4RN^2 = PQ^2 + 4QR^2$$

Hence Proved

(iv)

First, we consider the $\Delta PQM,$ and applying Pythagoras theorem we get,

$$PM^2 = PQ^2 + MQ^2$$

$$= (PN + NQ)^2 + MQ^2$$

$$= PN^2 + NQ^2 + 2PN.NQ + MQ^2$$

$$= MN^2 + PN^2 + 2PN.NQ \qquad \begin{bmatrix}\text{From, } \Delta MNQ, \\ MN^2 = NQ^2 + MQ^2\end{bmatrix} \ldots\ldots(i)$$

Now, we consider the $\Delta RNQ,$ and applying Pythagoras theorem we get,

$$RN^2 = NQ^2 + RQ^2$$

$$= NQ^2 + (QM + RM)^2$$

$$= NQ^2 + QM^2 + RM^2 + 2QM.RM$$

$$= MN^2 + RM^2 + 2QM.RM \qquad \ldots\ldots(ii)$$

Adding (*i*) and (*ii*) we get,

$$PM^2 + RN^2 = MN^2 + PN^2 + 2PN.NQ + MN^2 + RM^2 + 2QM.RM$$

$$PM^2 + RN^2 = 2MN^2 + PN^2 + RM^2 + 2PN.NQ + 2QM.RM$$

$$PM^2 + RN^2 = 2MN^2 + NQ^2 + QM^2 + 2(QN^2) + 2(QM^2)$$

$$PM^2 + RN^2 = 2MN^2 + MN^2 + 2MN^2$$

$$PM^2 + RN^2 = 5MN^2$$

$$4(PM^2 + RN^2) = 4\cdot 5\cdot(NQ^2 + MQ^2)$$

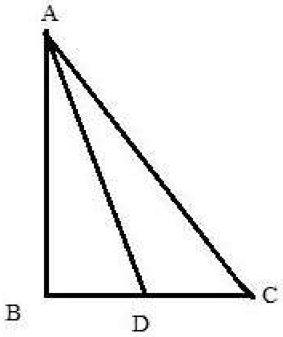
$$4(PM^2 + RN^2) = 4\cdot 5\cdot\left[\left(\frac{1}{2}PQ\right)^2 + \left(\frac{1}{2}QR\right)^2\right]$$

$$\left[\because NQ = \frac{1}{2}PQ, MQ = \frac{1}{2}QR\right]$$

$$4(PM^2 + RN^2) = 5PR^2$$

Hence Proved

Solution 5:



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

In triangle ABC, $\angle B = 90^{\circ}$ and D is the mid-point of BC. Join AD. Therefore, BD=DC

First, we consider the ΔADB, and applying Pythagoras theorem we get,

$$AD^2 = AB^2 + BD^2$$
$$AB^2 = AD^2 - BD^2 \quad ......(i)$$

Similarly, we get from rt. angle triangles ABC we get,

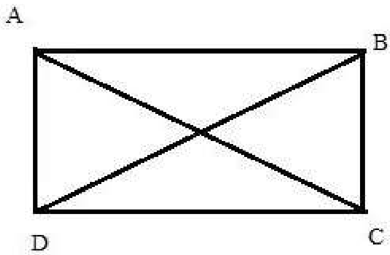
$$AC^2 = AB^2 + BC^2$$
$$AB^2 = AC^2 - BC^2 \quad ......(ii)$$

From (i) and (ii) ,

$$AC^2 - BC^2 = AD^2 - BD^2$$
$$AC^2 = AD^2 - BD^2 + BC^2$$
$$AC^2 = AD^2 - CD^2 + 4CD^2 \quad \left[BD = CD = \frac{1}{2}BC\right]$$
$$AC^2 = AD^2 + 3CD^2$$

Hence proved.

Solution 6:



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Since, ABCD is a rectangle angles A,B,C and D are rt. angles.

First, we consider the ΔACD, and applying Pythagoras theorem we get,

$$AC^2 = DA^2 + CD^2 \quad ......(i)$$

Similarly, we get from rt. angle triangle BDC we get,

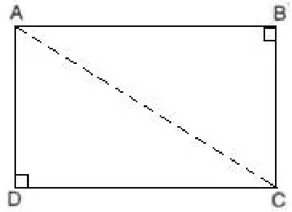
$$BD^2 = BC^2 + CD^2$$
$$= BC^2 + AB^2 \quad \left[\text{In a rectangle, opposite sides are equal, } \therefore CD = AB\right] \quad ......(ii)$$

Adding (i) and (ii) ,

$$AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$$

Hence proved.

Solution 7:



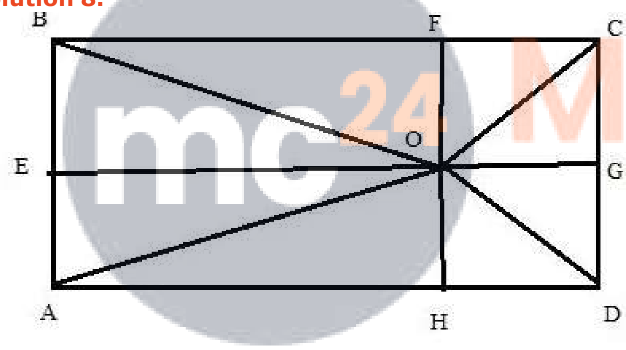
In quadrilateral ABCD, $\angle B = 90^\circ$ and $\angle D = 90^\circ$.
So, ΔABC and ΔADC are right-angled triangles.

In ΔABC using Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$\Rightarrow AB^2 = AC^2 - BC^2 \ldots\ldots\ldots(i)$

In ΔADC, using Pythagoras theorem,
$AC^2 = AD^2 + DC^2 \ldots\ldots\ldots(ii)$

$LHS = 2AC^2 - AB^2$
$= 2AC^2 - \left(AC^2 - BC^2\right)$ [from(i)]
$= 2AC^2 - AC^2 + BC^2$
$= AC^2 + BC^2$
$= AD^2 + DC^2 + BC^2$ [from(ii)]
$= RHS$

Solution 8:



Draw rectangle ABCD with arbitrary point O within it, and then draw lines OA, OB, OC, OD. Then draw lines from point O perpendicular to the sides: OE, OF, OG, OH.

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Using Pythagorean theorem we have from the above diagram:

$OA^2 = AH^2 + OH^2 = AH^2 + AE^2$

$OC^2 = CG^2 + OG^2 = EB^2 + HD^2$

$OB^2 = EO^2 + BE^2 = AH^2 + BE^2$

$OD^2 = HD^2 + OH^2 = HD^2 + AE^2$

Adding these equalities we get:

$OA^2 + OC^2 = AH^2 + HD^2 + AE^2 + EB^2$

$OB^2 + OD^2 = AH^2 + HD^2 + AE^2 + EB^2$

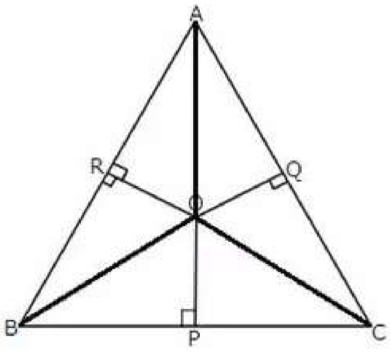
From which we prove that for any point within the rectangle there is the relation

$OA^2 + OC^2 = OB^2 + OD^2$

Hence Proved.

Solution 9:

Here, we first need to join OA, OB, and OC after which the figure becomes as follows,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides. First, we consider the ΔARO and applying Pythagoras theorem we get,

$$AO^2 = AR^2 + OR^2$$

$$AR^2 = AO^2 - OR^2 \quad \ldots\ldots(i)$$

Similarly, from triangles, BPO,COQ,AOQ,CPO and BRO we get the following results,

$$BP^2 = BO^2 - OP^2 \quad \ldots\ldots \ (ii)$$

$$CQ^2 = OC^2 - OQ^2 \quad \ldots\ldots \ (iii)$$

$$AQ^2 = AO^2 - OQ^2 \quad \ldots\ldots \ (iv)$$

$$CP^2 = OC^2 - OP^2 \quad \ldots\ldots \ (v)$$

$$BR^2 = OB^2 - OR^2 \quad \ldots\ldots \ (vi)$$

Adding (*i*), (*ii*) and (*iii*),we get

$$AR^2 + BP^2 + CQ^2 = AO^2 - OR^2 + BO^2 - OP^2 + OC^2 - OQ^2 \quad \ldots\ldots(vii)$$

Adding (*iv*), (*v*) and (*vi*),we get ,

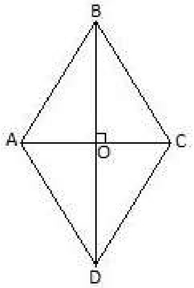
$$AQ^2 + CP^2 + BR^2 = AO^2 - OR^2 + BO^2 - OP^2 + OC^2 - OQ^2 \quad \ldots\ldots(viii)$$

From (*vii*) and (*viii*), we get,

$AR^2 + BP^2 + CQ^2 = AQ^2 + CP^2 + BR^2$

Hence proved.

Solution 10:



Diagonals of the rhombus are perpendicular to each other.

In quadrilateral ABCD, $\angle AOD = \angle COD = 90°$.
So, ΔAOD and ΔCOD are right-angled triangles.

In ΔAOD using Pythagoras theorem,

$AD^2 = OA^2 + OD^2$

$\Rightarrow OA^2 = AD^2 - OD^2 ....(i)$

In ΔCOD using Pythagoras theorem,

$CD^2 = OC^2 + OD^2$

$\Rightarrow OC^2 = CD^2 - OD^2 ....(ii)$

$LHS = OA^2 + OC^2$

$= AD^2 - OD^2 + CD^2 - OD^2 \qquad [from (i) and (ii)]$

$= AD^2 + CD^2 - 2OD^2$

$= AD^2 + AD^2 - 2\left(\frac{BD}{2}\right)^2 \qquad \left[AD = CD \text{ and } OD = \frac{BD}{2}\right]$

$= 2AD^2 - \frac{BD^2}{2}$

$= RHS$

Solution 11:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

We consider the ΔACD and applying Pythagoras theorem we get,

$AC^2 = AD^2 + DC^2$

$= (AB^2 - DB^2) + (DB + BC)^2$

$= BC^2 - DB^2 + DB^2 + BC^2 + 2DB \cdot BC \quad (\text{Given}, AB = BC)$

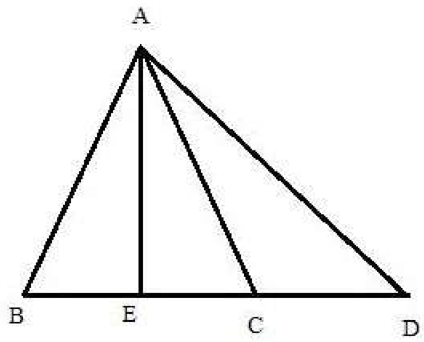
$= 2BC^2 + 2DB \cdot BC$

$= 2BC(BC + DB)$

$= 2BC \cdot DC$

Hence Proved.

Solution 12:



In an isosceles triangle ABC; AB = AC and D is point on BC produced. Construct AE perpendicular BC.

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
We consider the rt. angled ΔAED and applying Pythagoras theorem we get,

$$AD^2 = AE^2 + ED^2$$
$$AD^2 = AE^2 + (EC + CD)^2 \quad ......(i)$$
$$[\because ED = EC + CD]$$

Similarly, in ΔAEC,

$$AC^2 = AE^2 + EC^2$$
$$AE^2 = AC^2 - EC^2 \quad ......(ii)$$
$$\text{putting } AE^2 = AC^2 - EC^2 \text{ in } (i), \text{we get,}$$
$$AD^2 = AC^2 - EC^2 + (EC + CD)^2$$
$$= AC^2 + CD(CD + 2EC)$$
$$AD^2 = AC^2 + BD \cdot CD \quad [\because 2EC + CD = BD]$$

Hence Proved

Solution 13:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.
We consider the rt. angled ΔACD and applying Pythagoras theorem we get,

$$CD^2 = AC^2 + AD^2$$
$$CD^2 = AC^2 + (AB + BD)^2 \quad [\because AD = AB + BD]$$
$$CD^2 = AC^2 + AB^2 + BD^2 + 2AB \cdot BD \quad ......(i)$$

Similarly, in ΔABC.

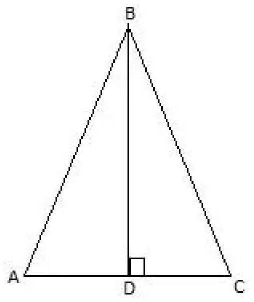
$$BC^2 = AC^2 + AB^2$$
$$BC^2 = 2AB^2 \quad [AB = AC]$$
$$AB^2 = \frac{1}{2}BC^2 \quad ......(ii)$$

Putting, AB^2 from (ii) in (i) we get,

$$CD^2 = AC^2 + \frac{1}{2}BC^2 \quad + BD^2 + 2AB \cdot BD$$
$$CD^2 - BD^2 = AB^2 + AB^2 + 2AB \cdot (AD - AB)$$
$$CD^2 - BD^2 = AB^2 + AB^2 + 2AB \cdot AD - 2AB^2$$
$$CD^2 - BD^2 = 2AB \cdot AD$$
$$DC^2 - BD^2 = 2AB \cdot AD$$

Hence Proved.

Solution 14:



In right angled ΔADB

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = AB^2 - BD^2 ....(i)$

$AC = AD + DC$

$\Rightarrow AC^2 = (AD + DC)^2$

$\Rightarrow AC^2 = AD^2 + DC^2 + 2AD \times DC$

$\Rightarrow AC^2 = AB^2 - BD^2 + DC^2 + 2AD \times DC \quad [from(i)]$

$\Rightarrow AC^2 = AC^2 - BD^2 + DC^2 + 2AD \times DC \quad [AB = AC]$

$\Rightarrow BD^2 - DC^2 = 2AD \times DC$

Solution 15:

Here,

$BD : DC = 1 : 3$

$\Rightarrow BD = \frac{1}{4} BC \text{ and } CD = \frac{3}{4} BC$

$AC^2 = AD^2 + CD^2 \text{ and } AB^2 = AD^2 + BD^2$

Therefore,

$AC^2 - AB^2 = CD^2 - BD^2$

$= \left(\frac{3}{4} BC\right)^2 - \left(\frac{1}{4} BC\right)^2$

$= \frac{9}{16} BC^2 - \frac{1}{16} BC^2$

$= \frac{1}{2} BC^2$

$\therefore 2AC^2 - 2AB^2 = BC^2$

$2AC^2 = 2AB^2 + BC^2$

Hence proved