$\Rightarrow\left(\frac{1}{2}\right)[\int dt - \int t^6 dt - \int 3t^2 dt + \int 3t^4 dt]$

$\Rightarrow\left(\frac{1}{2}\right)\left[t - \frac{t^7}{7} - \frac{3t^3}{3} + \frac{3t^5}{5}\right] + c$

$\Rightarrow\left(\frac{1}{2}\right)\left[t - \frac{t^7}{7} - t^3 + \frac{3t^5}{5}\right] + c$

Resubstituting the value of t=cosu and u=2x-3 we get

$\Rightarrow\left(\frac{1}{2}\right)\left[\cos(2x-3) - \frac{cos^7(2x-3)}{7} - cos^3(2x-3) + \frac{3cos^5(2x-3)}{5}\right] + c$

$\Rightarrow\frac{\cos(2x-3)}{2} - \frac{cos^7(2x-3)}{14} - \frac{cos^3(2x-3)}{2} + \frac{3cos^5(2x-3)}{10} + c$

Now as we know cos(-x)=cosx

$\Rightarrow\frac{\cos(2x-3)}{2} - \frac{cos^7(2x-3)}{14} - \frac{cos^3(2x-3)}{2} + \frac{3cos^5(2x-3)}{10} + c$

$=\frac{\cos(3-2x)}{2} - \frac{cos^7(3-2x)}{14} - \frac{cos^3(3-2x)}{2} + \frac{3cos^5(3-2x)}{10} + c$

Ans: $\frac{\cos(3-2x)}{2} - \frac{cos^7(3-2x)}{14} - \frac{cos^3(3-2x)}{2} + \frac{3cos^5(3-2x)}{10} + c$

6. Question

Evaluate the following integrals:

(i) $\left(\frac{1-\cos 2x}{1+\cos 2x}\right)dx$

(ii) $\left(\frac{1+\cos 2x}{1-\cos 2x}\right)dx$

Answer

(i) $\left(\frac{1-\cos 2x}{1+\cos 2x}\right)dx$

$\Rightarrow\int \frac{1-cos2x}{1+cos2x}dx$

1-cos2x=2sin^{2}x and 1+cos2x=2cos^{2}x

$\Rightarrow\int \frac{1-cos2x}{1+cos2x}dx = \int \frac{2sin^2 x}{2cos^2 x}dx$

$\Rightarrow\int tan^2 x\, dx$

Now sec^{2}x-1=tan^{2}x

$\Rightarrow\int (sec^2 x - 1)dx$

$\Rightarrow\int sec^2 x dx - \int dx$

⇒tanx-x+c

Ans: tanx-x+c

(ii) $\left(\frac{1+\cos 2x}{1-\cos 2x}\right)dx$

$\Rightarrow\int \frac{1+cos2x}{1-cos2x}dx$

$1-\cos2x=2\sin^2x$ and $1+\cos2x=2\cos^2x$

$\Rightarrow\int \frac{1+\cos2x}{1-\cos2x}dx = \int \frac{2\cos^2x}{2\sin^2x}dx$

$\Rightarrow\int \cot^2 x\, dx$

Now $\operatorname{cosec}^2x-1=\cot^2x$

$\Rightarrow\int (\operatorname{cosec}^2 x - 1)dx$

$\Rightarrow\int \operatorname{cosec}^2 x dx - \int dx$

$\Rightarrow -\cot x - x + c$

Ans: $-\cot x-x+c$

7. Question

Evaluate the following integrals:

(i) $\int \frac{1-\cos x}{1+\cos x}dx$

(ii) $\int \frac{1+\cos x}{1-\cos x}dx$

Answer

i) $\int \frac{1-\cos x}{1+\cos x}dx$

$\Rightarrow\int \frac{1-\cos x}{1+\cos x}dx$

$1-\cos x=2\sin^2x/2$ and $1+\cos x=2\cos^2x/2$

$\Rightarrow\int \frac{1-\cos x}{1+\cos x}dx = \int \frac{2\sin^2(\frac{x}{2})}{2\cos^2(\frac{x}{2})}dx$

$\Rightarrow\int \tan^2(\frac{x}{2})dx$

Now $\sec^2(x/2)-1=\tan2(x/2)$

$\Rightarrow\int \left(\sec^2(\frac{x}{2}) - 1\right)dx$

$\Rightarrow\int \sec^2(\frac{x}{2})dx - \int dx$

$\Rightarrow 2\tan(x/2)-x+c$

Ans: $2\tan(x/2)-x+c$

(ii) $\int \frac{1+\cos x}{1-\cos x}dx$

$\Rightarrow\int \frac{1+\cos x}{1-\cos x}dx$

$1-\cos x=2\sin^2x/2$ and $1+\cos x=2\cos^2x/2$

$\Rightarrow\int \frac{1+\cos x}{1-\cos x}dx = \int \frac{2\cos^2(\frac{x}{2})}{2\sin^2(\frac{x}{2})}dx$

$\Rightarrow\int \cot^2(\frac{x}{2})dx$

Now cosec^2 $(x/2)-1=\cot^2$ $(x/2)$

$\Rightarrow\int \left(\operatorname{cosec}^2(\frac{x}{2}) - 1\right)dx$

$\Rightarrow\int \operatorname{cosec}^2(\frac{x}{2})dx - \int dx$

$\Rightarrow -2\cot(x/2)-x+c$

Ans: $\Rightarrow -2\cot(x/2)-x+c$

8. Question

Evaluate the following integrals:

$$\int \sin 3x \cos 4x \, dx$$

Answer

$\Rightarrow \int sin3x \; cos4x \; dx$

Applying the formula: sinx×cosy=1/2(sin(x+y)-sin(y-x))

$\Rightarrow \frac{1}{2}\int (sin7x - sinx)dx$

$\Rightarrow \frac{1}{2}\int sin7x \; dx - \frac{1}{2}\int sinx \; dx$

$\Rightarrow \frac{-cos7x}{14} + \frac{cosx}{2} + c$

Ans: $\frac{-cos7x}{14} + \frac{cosx}{2} + c$

9. Question

Evaluate the following integrals:

$$\int \cos 4x \cos 3x \, dx$$

Answer

$\Rightarrow \int cos4x \; cos3x \; dx$

Applying the formula: cosx×cosy=1/2(cos(x+y)+cos(x-y))

$\Rightarrow \frac{1}{2}\int (cos7x + cosx)dx$

$\Rightarrow \frac{1}{2}\int cos7x \; dx + \frac{1}{2}\int cosx \; dx$

$\Rightarrow \frac{sin7x}{14} + \frac{sinx}{2} + c$

Ans: $\frac{sin7x}{14} + \frac{sinx}{2} + c$

10. Question

Evaluate the following integrals:

$$\int \sin 4x \sin 8x \, dx$$

Answer

$\Rightarrow \int sin4x \; sin8x \; dx$

Applying the formula: sinx×siny=1/2(cos(y-x)-cos(y+x))

$\Rightarrow \frac{1}{2}\int (cos4x - cos12x)dx$

$\Rightarrow \frac{1}{2}\int cos4x \; dx - \frac{1}{2}\int cos12x \; dx$

$\Rightarrow \frac{sin4x}{8} - \frac{sin12x}{24} + c$

Ans: $\frac{sin4x}{8} - \frac{sin12x}{24} + c$

11. Question

Evaluate the following integrals:

$$\int \sin 6x \cos x \, dx$$

Answer

$\Rightarrow \int sin6x\ cosx\ dx$

Applying the formula: sinx×cosy=1/2(sin(y+x)-sin(y-x))

$\Rightarrow \frac{1}{2}\int (sin7x - \sin(-5x))dx$

$\Rightarrow \frac{1}{2}\int sin7x\ dx + \frac{1}{2}\int sin5x\ dx$

$\Rightarrow \frac{-cos7x}{14} - \frac{cosx}{10} + c$

Ans: $\frac{-cos7x}{14} - \frac{cosx}{10} + c$

12. Question

Evaluate the following integrals:

$$\int \sin x \sqrt{1+\cos 2x}\, dx$$

Answer

we know that $1+\cos2x=2\cos^2x$

So, applying this identity in the given integral we get,

$\Rightarrow \int sinx\sqrt{1+cos2x}dx$

$\Rightarrow \int sinx\sqrt{(2cos^2x)}dx$

$\Rightarrow \sqrt{2}\int sinxcosxdx$

Let sinx =t

⇒ cosx dx=dt

$\Rightarrow \sqrt{2}\int tdt$

$\Rightarrow \sqrt{2}\frac{t^2}{2} + c = \frac{t^2}{\sqrt{2}} + c$

Resubstituting the value of t=sinx we get

$\Rightarrow \frac{sin^2x}{\sqrt{2}} + c$

Ans: $\frac{sin^2x}{\sqrt{2}} + c$

13. Question

Evaluate the following integrals:

$$\int \cos^4 x\, dx$$

Answer

$\Rightarrow \int cos^2xcos^2xdx$

$\Rightarrow \int (\frac{1+cos2x}{2})(\frac{1+cos2x}{2})dx \ldots (\frac{1+cos2x}{2} = cos^2x)$

$\Rightarrow \frac{1}{4}\int (1+cos2x)^2 dx$

$\Rightarrow \frac{1}{4}\int (1+cos^2 2x+2cos2x)dx$

$\Rightarrow \frac{1}{4}[\int 1dx\ +\int cos^2 2xdx+\int 2cos2x\ dx$

$\Rightarrow \frac{1}{4}[x+\int \frac{(1+cos4x)dx}{2}+2\frac{sin2x}{2}]$ ...(1+cos4x=2cos^{2}x)

$\Rightarrow \frac{1}{4}[x+\frac{1}{2}(\int dx+\int cos4xdx)+sin2x]+c$

$\Rightarrow [\frac{x}{4}+\frac{1}{2}\times\frac{1}{4}\left(\int dx+\int cos4xdx\right)+\frac{sin2x}{4}\right]+c$

$\Rightarrow [\frac{x}{4}+\left(\frac{x}{8}+\frac{sin4x}{32}\right)+\frac{sin2x}{4}\right]+c$

$\Rightarrow \frac{3x}{8}+\frac{sin4x}{32}+\frac{sin2x}{4}+c$

Ans: $\frac{3x}{8}+\frac{sin4x}{32}+\frac{sin2x}{4}+c$

14. Question

Evaluate the following integrals:

$$\int \cos 2x \cos 4x \cos 6x\ dx$$

Answer

$\Rightarrow \int cos2x\ cos4x\ cos6xdx$

$\Rightarrow \frac{1}{2}\int (cos6x+cos2x)cos6xdx$

$\Rightarrow \frac{1}{2}\int cos^2 6xdx+\frac{1}{2}\int cos2xcos6xdx$

$\Rightarrow \frac{1}{2}\int cos^2 6xdx+\frac{1}{4}\int (cos8x+cos4x)dx$

$\Rightarrow \frac{1}{2}\int cos^2 6xdx+\frac{1}{4}\int cos8x\ dx+\frac{1}{4}\int cos4x\ dx$

$\Rightarrow \frac{1}{2}\int \frac{(1+cos12x)dx}{2}+\frac{1}{4}\frac{sin8x}{8}+\frac{1}{4}\frac{sin4x}{4}+c$

$\Rightarrow \frac{1}{4}\left(x+\frac{sin12x}{12}\right)+\frac{sin8x}{32}+\frac{sin4x}{16}+c$

$\Rightarrow \frac{x}{4}+\frac{sin12x}{48}+\frac{sin8x}{32}+\frac{sin4x}{16}+c$

Ans: $\frac{x}{4}+\frac{sin12x}{48}+\frac{sin8x}{32}+\frac{sin4x}{16}+c$

15. Question

Evaluate the following integrals:

$$\int \sin^3 x \cos x\ dx$$

Answer

Let sinx =t

⇒ cosx dx =dt

$\Rightarrow \int sin^3 x\ cosx\ dx=\int t^3 dt$

$\Rightarrow \frac{t^4}{4} + c$

Resubstituting the value of t=sinx we get

$\Rightarrow \frac{sin^4 x}{4} + c$

Ans: $\frac{sin^4 x}{4} + c$

16. Question

Evaluate the following integrals:

$\int \sec^4 x \, dx$

Answer

$\Rightarrow \int sec^4 dx = \int sec^2 x sec^2 x dx$

$\Rightarrow \int sec^2 x(1 + tan^2 x)dx$

⇒Put tanx=t ⇒sec^2dx=dt

$\Rightarrow \int (1 + t^2)dt$

$\Rightarrow t + \frac{t^3}{3} + c$

Resubstituting the value of t=tanx we get

$\Rightarrow tanx + \frac{tan^3 x}{3} + c$

Ans: $tanx + \frac{tan^3 x}{3} + c$

17. Question

Evaluate the following integrals:

$\int \cos^3 x \sin^4 x \, dx$

Answer

$\Rightarrow \int cos^3 x sin^4 x \, dx$

$\Rightarrow \int cosx \, sin^4 x cos^2 x dx$

$\Rightarrow \int cosx \, sin^4 x(1 - sin^2 x)dx$

Put sinx=t

⇒cosxdx=dt

$\Rightarrow \int t^4(1 - t^2)dt$

$\Rightarrow \int t^4 dt - \int t^6 \, dt$

$\Rightarrow \frac{t^5}{5} - \frac{t^7}{7} + c$

Resubstituting the value of t=sinx we get,

$\Rightarrow \frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$

Ans: $\frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$

18. Question

Evaluate the following integrals:

$$\int \cos^4 x \sin^3 x \, dx$$

Answer

$\Rightarrow \int cos^4 x sin^3 x \, dx$

$\Rightarrow \int sinx \, sin^2 x cos^4 x dx$

$\Rightarrow \int sinx \, cos^4 x(1 - cos^2 x)dx$

Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int t^4 (t^2 - 1)dt$

$\Rightarrow \int t^6 dt - \int t^4 \, dt$

$\Rightarrow \frac{t^7}{7} - \frac{t^5}{5} + c$

Resubstituting the value of t=sinx we get,

$\Rightarrow \frac{cos^7 x}{7} - \frac{cos^5 x}{5} + c$

Ans: $\frac{cos^7 x}{7} - \frac{cos^5 x}{5} + c$

19. Question

Evaluate the following integrals:

$$\int \sin^{2/3} x \cos^3 x \, dx$$

Answer

$\Rightarrow \int cos^3 x sin^{\frac{2}{3}} x \, dx$

$\Rightarrow \int cosx \, cos^2 x sin^{\frac{2}{3}} x dx$

$\Rightarrow \int cosx \, (1 - sin^2 x) sin^{\frac{2}{3}} x dx$

Put sinx=t

⇒cosxdx=dt

$\Rightarrow \int t^{\frac{2}{3}} (1 - t^2)dt$

$\Rightarrow \int t^{\frac{2}{3}} dt - \int t^{\frac{8}{3}} \, dt$

$\Rightarrow \frac{t^{\frac{5}{3}}}{\frac{5}{3}} - \frac{t^{\frac{11}{3}}}{\frac{11}{3}} + c$

Resubstituting the value of t=sinx we get

$\Rightarrow \frac{3sin^{\frac{5}{3}} x}{5} - \frac{3sin^{\frac{11}{3}} x}{11} + c$

Ans: $\frac{3sin^{\frac{5}{3}} x}{5} - \frac{3sin^{\frac{11}{3}} x}{11} + c$

20. Question

Evaluate the following integrals:

$$\int \cos^{3/5} x \sin^3 x \, dx$$

Answer

$\Rightarrow \int sin^3 x cos^{\frac{3}{5}} x \, dx$

$\Rightarrow \int sinx \, sin^2 x cos^{\frac{3}{5}} x dx$

$\Rightarrow \int sinx \, (1 - cos^2 x) cos^{\frac{3}{5}} x dx$

Put cosx=t

⇒-sinxdx=dt

$\Rightarrow \int t^{\frac{3}{5}} (t^2 - 1) dt$

$\Rightarrow \int t^{\frac{13}{5}} dt - \int t^{\frac{3}{5}} dt$

$\Rightarrow \frac{t^{\frac{18}{5}}}{\frac{18}{5}} - \frac{t^{\frac{8}{5}}}{\frac{8}{5}} + c$

Resubstituting the value of t=cosx we get

$\Rightarrow \frac{5cos^{\frac{18}{5}} x}{18} - \frac{5cos^{\frac{8}{5}} x}{8} + c$

Ans: $\frac{5cos^{\frac{18}{5}} x}{18} - \frac{5cos^{\frac{8}{5}} x}{8} + c$

21. Question

Evaluate the following integrals:

$$\int \text{cosec}^4 2x \, dx$$

Answer

$\Rightarrow \int cosec^4 2x dx$

$\Rightarrow \int cosec^2 2x cosec^2 2x dx$

$\Rightarrow \int cosec^2 2x \, (1 + cot^2 2x) dx$

⇒cot2x=t ⇒-2cosec^2 2xdx=dt

$\Rightarrow -1/2 \int (1 + t^2) dt$

$\Rightarrow -1/2 \int dt - 1/2 \int t^2 dt$

$\Rightarrow -(\frac{1}{2})t - \frac{t^3}{6} + c$

Resubstituting the value of t=cotx we get

$\Rightarrow -\frac{\cot x}{2} - \frac{\cot^3 x}{6} + c$

Ans: $-\frac{\cot x}{2} - \frac{\cot^3 x}{6} + c$

22. Question

Evaluate the following integrals:

$$\int \frac{\cos 2x}{\cos x} dx$$

Answer

$\Rightarrow \int \frac{\cos 2x}{\cos x} dx = \int \frac{2\cos^2 x - 1}{\cos x} dx$

$\Rightarrow \int \frac{2\cos^2 x}{\cos x} dx - \int \frac{1}{\cos x} dx$

$\Rightarrow \int 2\cos x dx - \int \sec x dx$

$\Rightarrow 2\sin x - \log|\sec x + \tan x| + c$

Ans: 2sinx-log|secx+tanx|+c

23. Question

Evaluate the following integrals:

$$\int \frac{\cos x}{\cos(x+\alpha)} dx$$

Answer

$\Rightarrow \int \frac{\cos x}{\cos(x+\alpha)} dx = \int \frac{\cos((x+\alpha)-\alpha)}{\cos(x+\alpha)} dx$

$\Rightarrow \int \frac{\cos(x+\alpha)\cos\alpha + \sin(x+\alpha)\sin\alpha}{\cos(x+\alpha)} dx$

$\Rightarrow \int \cos\alpha dx + \int \tan(x+\alpha)\sin\alpha\, dx$

Now α is a constant

$\Rightarrow x\cos\alpha - \sin\alpha \log|\cos(x+\alpha)| + c$

Ans:xcos α-sin αlog|cos(x+ α)|+c

24. Question

Evaluate the following integrals:

$$\int \cos^3 x \sin 2x\, dx$$

Answer

$\Rightarrow \int \sin 2x \cos^3 x dx$

$\Rightarrow \int 2\sin x\cos x\cos^3 x dx$

$$\Rightarrow \int 2sinx\cos^4 xdx$$

Now put cosx=t

⇒-sinxdx=dt

$$\Rightarrow -2\int t^4 dt$$

$$\Rightarrow -2\times\frac{t^5}{5}+c$$

Resubstituting the value of t= cosx we get,

$$\Rightarrow \frac{-2\cos^5 x}{5}+c$$

Ans: $\frac{-2\cos^5 x}{5}+c$

25. Question

Evaluate the following integrals:

$$\int\frac{\cos^9 x}{\sin x}dx$$

Answer

$$\Rightarrow \int\frac{\cos^9 x}{sinx}dx$$

$$\Rightarrow \int\frac{\cos^9 x}{\sin^2 x}sinxdx$$

$$\Rightarrow \int\frac{\cos^9 x}{1-\cos^2 x}sinxdx$$

Put cosx =t

⇒ -sinxdx=dt

$$\Rightarrow \int\frac{t^9}{t^2-1}dt$$

Now put t^2-1=a

⇒2tdt=da

And $t^8=(a+1)^4$

$$\Rightarrow \frac{1}{2}\int\frac{(a+1)^4}{a}da$$

$$\Rightarrow \frac{1}{2}\int(a^3+4a^2+6a+\frac{1}{a}+4)da$$

$$\Rightarrow \frac{1}{2}\left(\frac{a^4}{4}+\frac{4a^3}{3}+\frac{6a^2}{2}+lna+4a\right)+c$$

$$\Rightarrow \left(\frac{a^4}{8}+\frac{2a^3}{3}+\frac{3a^2}{2}+\frac{lna}{2}+2a\right)+c$$

Resubstituting the value of $a=t^2-1$ and t=cosx ⇒$a=\cos^2 x-1=-\sin^2 x$ we get

$\Rightarrow \left(\frac{(-sin^2x)^4}{8} + \frac{2(-sin^2x)^3}{3} + \frac{3(-sin^2x)^2}{2} + \frac{\ln|(-sin^2x)|}{2} + 2(-sin^2x)\right) + c$

$\Rightarrow \left(\frac{sin^8x}{8} - \frac{2sin^6x}{3} + \frac{3sin^4x}{2} + \frac{2\ln|(-sinx)|}{2} - 2sin^2x\right) + c$

$\Rightarrow \left(\frac{sin^8x}{8} - \frac{2sin^6x}{3} + \frac{3sin^4x}{2} + \ln(sinx) - 2sin^2x\right) + c$

Ans: $\left(\frac{sin^8x}{8} - \frac{2sin^6x}{3} + \frac{3sin^4x}{2} + \ln(sinx) - 2sin^2x\right) + c$

26. Question

Evaluate the following integrals:

$\int \cos^4 2x\, dx$

Answer

$\Rightarrow \int cos^2 2x cos^2 2x dx$

$\Rightarrow \int (\frac{1+cos4x}{2})(\frac{1+cos4x}{2}) dx \ldots (\frac{1+cos4x}{2} = cos^2 2x)$

$\Rightarrow \frac{1}{4}\int (1 + cos4x)^2 dx$

$\Rightarrow \frac{1}{4}\int (1 + cos^2 4x + 2cos4x) dx$

$\Rightarrow \frac{1}{4}[\int 1dx + \int cos^2 4xdx + \int 2cos4x\, dx$

$\Rightarrow \frac{1}{4}[x + \int \frac{(1+cos8x)dx}{2} + 2\frac{sin4x}{4}]$ ...(1+cos8x=2cos^2 4x)

$\Rightarrow \frac{1}{4}[x + \frac{1}{2}\left(\int dx + \int cos8xdx\right) + (\frac{sin4x}{2})\right] + c$

$\Rightarrow [\frac{x}{4} + \frac{1}{2} \times \frac{1}{4}\left(\int dx + \int cos8xdx\right) + \frac{sin4x}{8}\right] + c$

$\Rightarrow [\frac{x}{4} + \left(\frac{x}{8} + \frac{sin8x}{64}\right) + \frac{sin4x}{8}\right] + c$

$\Rightarrow \frac{3x}{8} + \frac{sin8x}{64} + \frac{sin4x}{8} + c$

Ans: $\frac{3x}{8} + \frac{sin8x}{64} + \frac{sin4x}{8} + c$

27. Question

Evaluate the following integrals:

$\int \frac{\sin^2 x}{(1+\cos x)^2} dx$

Answer

Doing tangent half angle substitution we get,

$\Rightarrow \int \frac{sin^2x}{(1+cos^2x)} dx = \int \frac{(\frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}})^2}{[1+(\frac{1-tan^2\frac{x}{2}}{1-tan^2\frac{x}{2}})]^2}$

Substitute u=tan(x/2)

⇒2du=sec^2(x/2)dx

⇒dx=$\frac{2du}{u^2+1}$

$\Rightarrow 2\int \frac{u^2}{1+u^2}du$

$\Rightarrow 2\int \frac{1+u^2}{1+u^2}du - 2\int \frac{1}{1+u^2}du$

$\Rightarrow 2\int du - \tan^{-1}u + c$

$\Rightarrow 2u - \tan^{-1}u + c$

Resubstituting the values we get,

$$\Rightarrow 2\tan\frac{x}{2} - \tan^{-1}\tan\frac{x}{2} + c$$

$$\Rightarrow 2\tan\frac{x}{2} - \frac{x}{2} + c$$

Ans: $2\tan\frac{x}{2} - \frac{x}{2} + c$

28. Question

Evaluate the following integrals:

$$\int \frac{dx}{(3\cos x + 4\sin x)}$$

Answer

$$\int \frac{dx}{3cosx + 4sinx} = \int \frac{dx}{3\left(\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}\right) + 4\left(\frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{3 + 8tan\frac{x}{2} - 3tan^2\frac{x}{2}}$$

Let tan$\frac{x}{2}$=t

$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$

$$\Rightarrow \int \frac{2dt}{3+8t-3t^2} = \frac{2}{3}\int \frac{dt}{1+\frac{8}{3}t-t^2} = \frac{2}{3}\int \frac{dt}{1-\left(t-\frac{4}{3}\right)^2+\frac{16}{9}}$$

$$\Rightarrow \frac{2}{3}\int \frac{dt}{\frac{25}{9}-\left(t-\frac{4}{3}\right)^2} = \frac{2}{3}\int \frac{dt}{\left(\frac{5}{3}\right)^2-\left(t-\frac{4}{3}\right)^2}$$

$$\Rightarrow \frac{2}{3}\times\frac{1}{2\times\frac{5}{3}}\ln\left|\frac{\frac{5}{3}+\left(t-\frac{4}{3}\right)}{\frac{5}{3}-\left(t-\frac{4}{3}\right)}\right| + c = \frac{1}{5}\ln\left|\frac{1+3t}{9-3t}\right| + c$$

Resubstituting the value of t we get

$$\Rightarrow \frac{1}{5}\ln\left|\frac{1+3tan\frac{x}{2}}{9-3tan\frac{x}{2}}\right| + c$$

Ans: $\frac{1}{5}\ln\left|\frac{1+3tan\frac{x}{2}}{9-3tan\frac{x}{2}}\right| + c$

29. Question

Evaluate the following integrals:

$$\int \frac{dx}{(a\cos x + b\sin x)^2},\ a > 0 \text{ and } b > 0$$

Answer

$$\int \frac{dx}{(acosx + bsinx)^2}$$

Taking bcosx common from the denominator we get,

$$\int \frac{dx}{b^2cos^2x(\frac{a}{b} + tanx)^2}$$

$$\Rightarrow \frac{1}{b^2}\int \frac{sec^2xdx}{(\frac{a}{b} + tanx)^2}$$

Let (a/b)+tanx=t

$$\therefore sec^2xdx = dt$$

$$\Rightarrow \frac{1}{b^2}\int \frac{dt}{t^2} = \frac{-1}{b^2} \times \frac{1}{t} = \frac{-1}{b^2t} + c$$

Resubstituting the value of t = (a/b)+tanx we get

$$\Rightarrow \frac{-1}{b^2(\frac{a}{b} + tanx)} + c = \frac{-1}{ab + b^2tanx} + c$$

Ans: $\frac{-1}{ab+b^2tanx} + c$

30. Question

Evaluate the following integrals:

$$\int \frac{dx}{(\cos x - \sin x)}$$

Answer

$$\int \frac{dx}{cosx - sinx} = \int \frac{dx}{\left(\frac{1 - tan^2\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right) - \left(\frac{2tan\frac{x}{2}}{1 + tan^2\frac{x}{2}}\right)}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{1 - 2tan\frac{x}{2} - tan^2\frac{x}{2}}$$

Let $tan\frac{x}{2}$=t

$$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$$

$$\Rightarrow \int \frac{2dt}{1-2t-t^2} = -2\int \frac{dt}{t^2+2t-1} = -2\int \frac{dt}{(t+1)^2-2}$$
$$= -2\int \frac{dt}{(t+1)^2-(\sqrt{2})^2}$$

$\Rightarrow -2 \times \frac{1}{2\times\sqrt{2}} \ln\left|\frac{t+1-\sqrt{2}}{t+1+\sqrt{2}}\right| + c$ resubstituting the value of t we get

$$\Rightarrow \frac{-1}{\sqrt{2}} \ln\left|\frac{tan\frac{x}{2}+1-\sqrt{2}}{tan\frac{x}{2}+1+\sqrt{2}}\right| + c = \frac{-1}{\sqrt{2}} \ln\left|\tan\left(\frac{\pi}{8}-\frac{x}{2}\right)\right| + c$$

Ans: $\frac{-1}{\sqrt{2}} \ln\left|\tan\left(\frac{\pi}{8}-\frac{x}{2}\right)\right| + c$

31. Question

Evaluate the following integrals:

$$\int (2\tan x - 3\cot x)^2 dx$$

Answer

$$\int (2tanx - 3cotx)^2 dx$$

$$\Rightarrow \int (4tan^2x + 9cot^2x - 12tanxcotx)dx$$

$$\Rightarrow \int (4(sec^2x - 1) + 9(cosec^2x - 1) - 12)dx$$

$$\Rightarrow \int 4sec^2x\,dx + \int 9cosec^2x dx - \int 25dx$$

$$\Rightarrow 4\tan x - 9\cot x - 25x + c$$

Ans: 4tanx-9cotx-25x+c

32. Question

Evaluate the following integrals:

$$\int \sin x \sin 2x \sin 3x\, dx$$

Answer

$\Rightarrow \int sinx\ sin2xsin3x\ dx$

Applying the formula: sinx×siny=1/2(cos(y-x)-cos(y+x))

$\Rightarrow \frac{1}{2}\int (cos2x - cos4x)sin2xdx$

$\Rightarrow \frac{1}{2}\int sin2xcos2x\ dx - \frac{1}{2}\int sin2xcos4x\ dx$

$\Rightarrow \frac{1}{4}\int sin4x\ dx - \frac{1}{4}\int (sin6x - sin2x)dx$

$\Rightarrow \frac{-cos4x}{16} + \frac{cos6x}{24} - \frac{cos2x}{8} + c$

Ans: $\frac{-cos4x}{16} + \frac{cos6x}{24} - \frac{cos2x}{8} + c$

33. Question

Evaluate the following integrals:

$$\int\left(\frac{1-\cot x}{1+\cot x}\right)dx$$

Answer

$$\Rightarrow \int \frac{1-cotx}{1+cotx}dx = \int \frac{1-\frac{cosx}{sinx}}{1+\frac{cosx}{sinx}}dx$$

$$\Rightarrow \int \frac{sinx-cosx}{sinx+cosx}dx = -\int \frac{cosx-sinx}{sinx+cosx}dx$$

$$\Rightarrow -\int \frac{d(sinx+cosx)}{sinx+cosx}$$

$$\Rightarrow -\log|\sin x+\cos x|+c$$

Ans: -log(sinx+cosx)+c

34. Question

Evaluate the following integrals:

$$\int\frac{dx}{(2\sin x+\cos x+3)}$$

Answer

$$\int \frac{dx}{cosx+2sinx+3} = \int \frac{dx}{\left(\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)+2\left(\frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}\right)+3}$$

$$\Rightarrow \int \frac{sec^2\frac{x}{2}dx}{3+1+3tan^2\frac{x}{2}+4tan\frac{x}{2}-tan^2\frac{x}{2}}$$

Let tan$\frac{x}{2}$=t

$$\therefore \frac{1}{2}sec^2\frac{x}{2}dx = dt$$

$$\Rightarrow \int \frac{2dt}{4+4t+2t^2} = \int \frac{dt}{2+2t+t^2} = \frac{2}{3}\int \frac{dt}{(t+1)^2+2-1}$$

$$\Rightarrow \int \frac{dt}{(t+1)^2+1} = \int \frac{dt}{(1)^2+(t+1)^2}$$

$$\Rightarrow \tan^{-1}(t+1)+c$$

Resubstituting the value of t we get

$$\Rightarrow \tan^{-1}(tan\frac{x}{2}+1)+c$$

Ans: $\tan^{-1}(tan\frac{x}{2}+1)+c$

Exercise 13C

1. Question

Evaluate the following integrals:

$$\int x e^x dx$$

Answer

Using BY PART METHOD.

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x.e^x dx = x\int e^x - \int\frac{dx}{dx}.\int e^x dx$$

$$= xe^x - \int 1.e^x dx$$

$$= xe^x - e^x + c$$

$$= e^x(x-1) + c$$

2. Question

Evaluate the following integrals:

$$\int x \cos x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and cos x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\cos xdx = x\int \cos x - \int\left[\frac{dx}{dx}.\int \cos xdx\right]dx$$

$$= x\sin x - \int 1.\sin xdx$$

$$= x\sin x + \cos x + c$$

3. Question

Evaluate the following integrals:

$$\int xe^{2x}dx$$

Answer

Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function and e^{2x} is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int xe^{2x}dx = x\int e^{2x}dx - \int\left[\frac{dx}{dx}.\int e^{2x}dx\right]dx$$

$$= x\frac{e^{2x}}{2} - \int 1.\frac{e^{2x}}{2}dx$$

$$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{2\times 2} + c$$

$$= x\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + c$$

4. Question

Evaluate the following integrals:

$$\int x \sin 3x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Sin 3x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\sin 3xdx = x\int \sin 3xdx - \int\left[\frac{dx}{dx}.\int \sin 3xdx\right]dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) - \int 1.\left(\frac{-\cos 3x}{3}\right)dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) + \left(\frac{\sin 3x}{3\times 3}\right) + c$$

$$= x\left(\frac{-\cos 3x}{3}\right) + \left(\frac{\sin 3x}{9}\right) + c$$

5. Question

Evaluate the following integrals:

$$\int x\cos 2x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\cos 2x dx = x\int \cos 2x dx - \int\left[\frac{dx}{dx}.\int \cos 2x dx\right]dx$$

$$= x\left(\frac{\sin 2x}{2}\right) - \int 1.\left(\frac{\sin 2x}{2}\right)dx$$

$$= x\left(\frac{\sin 2x}{2}\right) + \left(\frac{\cos 2x}{2\times 2}\right) + c$$

$$= x\left(\frac{\sin 2x}{2}\right) + \left(\frac{\cos 2x}{4}\right) + c$$

6. Question

Evaluate the following integrals:

$$\int x \log 2x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log 2x is the first function, and x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x\log 2x dx = \log 2x\int xdx - \int\left[\frac{d\log 2x}{dx}.\int xdx\right]dx$$

$$= \log 2x.\frac{x^2}{2} - \int\left[\frac{1\times 2}{2x}\frac{x^2}{2}\right]dx$$

$$= \frac{x^2}{2}\log 2x - \int\frac{x}{2}dx$$

$$= \frac{x^2}{2}\log 2x - \frac{x^2}{2\times 2} + c$$

$$= \frac{x^2}{2}\log 2x - \frac{x^2}{4} + c$$

7. Question

Evaluate the following integrals:

$$\int x \operatorname{cosec}^2 x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is the first function, and $\operatorname{cosec}^2 x$ is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x \operatorname{cosec}^2 x dx = x\int \operatorname{cosec}^2 x - \int\left[\frac{dx}{dx}.\int \operatorname{cosec}^2 x dx\right]dx$$

$$= x(-\cot x) - \int 1.(-\cot x)dx$$

$$= -x\cot x + \int \cot x dx$$

$$= -x\cot x + \ln|\sin x| + c$$

8. Question

Evaluate the following integrals:

$$\int x^2 \cos x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and cos x is the second function.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x^2 \cos x dx = x^2\int \cos x dx - \int\left[\frac{dx^2}{dx}.\int \cos x dx\right]dx$$

$$= x^2 \sin x - \int[2x \times \sin x]dx$$

$$= x^2 \sin x - 2\left[\int x \sin x dx\right]$$

Again applying by the part method in the second half, we get

$$x^2 \sin x - 2\int x \sin x dx$$

$$= x^2 \sin x - 2\left[x\int \sin x dx - \int\left(\frac{dx}{dx}.\int \sin x dx\right)dx\right]$$

$$= x^2 \sin x - 2\left[x(-\cos x) - \int 1.(-\cos x)dx\right]$$

$$= x^2 \sin x - 2[-x\cos x + \sin x] + c$$

$$= x^2 \sin x + 2x\cos x - 2\sin x + c$$

9. Question

Evaluate the following integrals:

$$\int x \sin^2 x \, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

Writing $\sin^2 x = \frac{1+\cos 2x}{2}$

We have

$$\int x\sin^2 x dx = \int x\left(\frac{1-\cos 2x}{2}\right)dx$$

$$= \int\left(\frac{x}{2}-\frac{x\cos 2x}{2}\right)dx$$

$$= \int\frac{x}{2}dx - \int\frac{x\cos 2x}{2}dx$$

$$= \frac{x^2}{2\times 2} - \frac{1}{2}\int x\cos 2x dx$$

Taking X as first function and Cos 2x as the second function.

$$= \frac{x^2}{4} - \frac{1}{2}\left\{x\int\cos 2x dx - \int\left(\frac{dx}{dx}.\int\cos 2x dx\right)dx\right\}$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{x.\frac{\sin 2x}{2} - \int\left(1.\frac{\sin 2x}{2}\right)dx\right\}$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{\frac{x\sin 2x}{2} - \left(\frac{-\cos 2x}{2\times 2}\right)\right\} + c$$

$$= \frac{x^2}{4} - \frac{1}{2}\left\{\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right\} + c$$

$$= \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c$$

10. Question

Evaluate the following integrals:

$$\int x \tan^2 x\, dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Using Integration by part

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

Writing $\tan^2 x = \sec^2 x - 1$

We have

$$\int x\tan^2 x dx = \int x\left(\sec^2 x - 1\right)dx$$

$$= \int x\sec^2 x dx - \int x dx$$

Using x as the first function and $\sec^2 x$ as the second function

$$\int x\sec^2 x dx - \int x dx$$

$$= \left\{x\int \sec^2 x dx - \int\left(\frac{dx}{dx}.\int \sec^2 x dx\right)dx\right\} - \frac{x^2}{2}$$

$$= \left\{x.\tan x - \int 1.\tan x dx\right\} - \frac{x^2}{2}$$

$$= x\tan x - \ln|\sec x| - \frac{x^2}{2} + c$$

$$= x\tan x - \ln\left|\frac{1}{\cos x}\right| - \frac{x^2}{2} + c$$

$$x\tan x + \ln|\cos x| - \frac{x^2}{2} + c$$

11. Question

Evaluate the following integrals:

$$\int x^2 e^x dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^x is the second function.

Using Integration by part

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\int x^2 e^x dx = \left[x^2\int e^x dx - \int\left(\frac{dx^2}{dx}.\int e^x dx\right)dx\right]$$

$$= x^2 e^x - \int 2x.e^x dx$$

$$= x^2 e^x - 2\int xe^x dx$$

$$= x^2 e^x - 2\left[x\int e^x dx - \int\left(\frac{dx}{dx}.\int e^x dx\right)dx\right]$$

$$= x^2 e^x - 2\left[xe^x - \int 1.e^x dx\right]$$

$$= x^2 e^x - 2\left[xe^x - e^x\right] + c$$

$$= x^2 e^x - 2xe^x + 2e^x + c$$

$$= e^x\left(x^2 - 2x + 2\right) + c$$

12. Question

Evaluate the following integrals:

$$\int x^2 \cos^3 x\, dx$$

Answer

We know that $\cos 3x = 4\cos^3 x - 3\cos x$

$$\cos^3 x = \frac{\cos 3x + 3\cos x}{4}$$

$$\int x^2 \cos^3 x dx = \int x^2 \left(\frac{\cos 3x + 3\cos x}{4}\right) dx$$

$$= \frac{1}{4}\left(\int x^2 \cos 3x dx + 3\int x^2 \cos x dx\right)$$

Taking X^2 as the first function and cos 3x and cos x as the second function and applying By part method.

$$\frac{1}{4}\left(\int x^2 \cos 3x dx + 3\int x^2 \cos x dx\right)$$

$$= \frac{1}{4}\left\{\left(x^2 \int \cos 3x dx - \int\left[\frac{dx^2}{dx}.\int \cos 3x dx\right]dx\right) + 3\left(x^2 \int \cos x dx - \int\left[\frac{dx^2}{dx}.\int \cos x dx\right]dx\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{x^2.\sin 3x}{3} - \int 2x.\frac{\sin 3x}{3}dx\right) + 3\left(x^2 \sin x - \int 2x.\sin x dx\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\int x \sin 3x dx\right) + 3\left(x^2 \sin x - 2\int x \sin x dx\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[x\int \sin 3x dx - \int\left(\frac{dx}{dx}.\int \sin 3x dx\right)dx\right]\right) + 3\left(x^2 \sin x - 2\left[x\int \sin x dx - \int\left(\frac{dx}{dx}.\int \sin x dx\right)dx\right]\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[x\frac{-\cos 3x}{3} - \int 1.\frac{-\cos 3x}{3}dx\right]\right) + 3\left(x^2 \sin x - 2\left[-x\cos x - \int -\cos x dx\right]\right)\right\}$$

$$= \frac{1}{4}\left\{\left(\frac{x^2 \sin 3x}{3} - \frac{2}{3}\left[\frac{-x\cos 3x}{3} + \frac{\sin 3x}{9}\right]\right) + 3\left(x^2 \sin x + 2x\cos x - 2\sin x\right)\right\} + c$$

$$= \frac{1}{4}\left\{\frac{x^2 \sin 3x}{3} + \frac{2x\cos 3x}{9} - \frac{2\sin 3x}{27} + 3x^2 \sin x + 6x\cos x - 6\sin x\right\} + c$$

$$= \frac{x^2 \sin 3x}{12} + \frac{x\cos 3x}{18} - \frac{\sin 3x}{54} + \frac{3x^2 \sin x}{4} + \frac{3x\cos x}{2} - \frac{3}{2}\sin x + c$$

13. Question

Evaluate the following integrals:

$$\int x^2 e^{3x} dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and e^{3x} is the second function.

Using Integration by part

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right]dx$$

$$\int x^2 e^{3x} dx = x^2 \int e^{3x} dx - \int\left(\frac{dx^2}{dx}.\int e^{3x} dx\right) dx$$

$$= x^2 \frac{e^{3x}}{3} - \int 2x.\frac{e^{3x}}{3} dx$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3}\int x e^{3x} dx$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3}\left(x \int e^{3x} dx - \int\left[\frac{dx}{dx}.\int e^{3x} dx\right] dx\right)$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3}\left(x \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} dx\right)$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2}{3}\left(x \frac{e^{3x}}{3} - \frac{e^{3x}}{9}\right) + c$$

$$= x^2 \frac{e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2e^{3x}}{27} + c$$

$$= e^{3x}\left(\frac{x^2}{3} - \frac{2x}{9} + \frac{2}{27}\right) + c$$

14. Question

Evaluate the following integrals:

$$\int x^2 \sin^2 x\, dx$$

Answer

We can write $\sin^2 x = \frac{1-\cos 2x}{2}$

We have

$$\int x^2\left(\frac{1-\cos 2x}{2}\right) dx = \int \frac{x^2}{2} - \frac{x^2 \cos 2x}{2} dx$$

$$= \int \frac{x^2}{2} dx - \int \frac{x^2 \cos 2x}{2} dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x^2 is the first function, and Cos 2x is the second function.

Using Integration by part

$$\int a.b.dx = a\int b dx - \int\left[\frac{da}{dx}.\int b dx\right] dx$$

$$=\frac{x^3}{3\times 2}-\frac{1}{2}\int x^2\cos 2x dx$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2\int \cos 2x dx-\int\left[\frac{dx^2}{dx}.\int \cos 2x dx\right]dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\int 2x.\frac{\sin 2x}{2}dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\int x.\sin 2x dx\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\left[x\int \sin 2x dx-\int\left(\frac{dx}{dx}.\int \sin 2x dx\right)dx\right]\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}-\left[x\frac{-\cos 2x}{2}-\int 1.\frac{-\cos 2x}{2}dx\right]\right)$$

$$=\frac{x^3}{6}-\frac{1}{2}\left(x^2.\frac{\sin 2x}{2}+\frac{x\cos 2x}{2}-\frac{\sin 2x}{4}\right)+c$$

$$=\frac{x^3}{6}-\frac{x^2\sin 2x}{4}-\frac{x\cos 2x}{4}+\frac{\sin 2x}{8}+c$$

15. Question

Evaluate the following integrals:

$$\int x^3\log 2x\ dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log2x is the first function, and x^3 is the second function.

Using Integration by part

$$\int a.b.dx=a\int bdx-\int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x^3\log 2x dx=\log 2x\int x^3dx-\int\left(\frac{d\log 2x}{dx}.\int x^3dx\right)dx$$

$$=\log 2x\frac{x^4}{4}-\int\frac{1.2}{2x}.\frac{x^4}{4}dx$$

$$=\log 2x\frac{x^4}{4}-\frac{1}{4}\int x^3dx$$

$$=\log 2x\frac{x^4}{4}-\frac{1}{4}.\frac{x^4}{4}+c$$

$$=\log 2x\frac{x^4}{4}-\frac{x^4}{16}+c$$

16. Question

Evaluate the following integrals:

$\int x \cdot \log(x+1)dx$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(x + 1) is first function and x is second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int x\log(x+1) = \log(x+1)\int xdx - \int\left(\frac{d\log(x+1)}{dx}.\int xdx\right)dx$$

$$= \log(x+1)\frac{x^2}{2} - \int\frac{1}{x+1}\times\frac{x^2}{2}dx$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\int\frac{x^2-1+1}{x+1}dx$$

Adding and subtracting 1 in the numerator,

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int\frac{x^2-1}{x+1} + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int\frac{(x+1)(x-1)}{x+1} + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\left(\int(x-1) + \frac{1}{x+1}\right)dx\right]$$

$$= \log(x+1)\frac{x^2}{2} - \frac{1}{2}\left[\frac{x^2}{2} - x + \log(x+1)\right] + c$$

$$= \log(x+1)\frac{x^2}{2} - \frac{x^2}{4} + \frac{x}{2} - \frac{\log(x+1)}{2} + c$$

$$= \log(x+1)\frac{x^2-1}{2} - \frac{x^2}{4} + \frac{x}{2} + c$$

17. Question

Evaluate the following integrals:

$$\int\frac{\log x}{x^n}dx$$

Answer

We can write it as $\int x^{-n}.\log x dx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here logx is the first function, and x^{-n} is the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\Rightarrow \int x^{-n}\log x dx = \log x\int x^{-n}dx - \int\left(\frac{d\log x}{dx}.\int x^{-n}dx\right)dx$$

$$= \log x\left(\frac{x^{-n+1}}{-n+1}\right) - \int \frac{1}{x}.\frac{x^{-n+1}}{-n+1}dx$$

$$= \frac{x^{-n+1}\log x}{1-n} + \frac{1}{1-n}\int \frac{x^{-n}.x}{x}dx$$

$$= \frac{x^{-n+1}\log x}{1-n} + \frac{1}{1-n}\times\frac{x^{-n+1}}{-n+1} + c$$

$$= \frac{x^{-n+1}\log x}{1-n} - \frac{x^{-n+1}}{(1-n)^2} + c$$

18. Question

Evaluate the following integrals:

$$\int 2x^3 e^{x^2} dx$$

Answer

We can write it as $\int 2.x.x^2.e^{x^2}dx$

Let $x^2 = t$

$2xdx = dt$

Using the relation in the above condition, we get

$$\int 2x.x^2.e^{x^2}dx = \int t.e^t dt$$

Integrating with respect to t

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function, and e^t is the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int te^t dt = t\int e^t dt - \int\left(\frac{dt}{dt}.\int e^t dt\right)dt$$

$$= te^t - \int 1.e^t dt$$

$$= te^t - e^t + c$$

Replacing t with x^2,we get

$$x^2e^{x^2} - e^{x^2} + c$$

$$= e^{x^2}\left(x^2-1\right) + c$$

19. Question

Evaluate the following integrals:

$\int x\sin^3 x\, dx$

Answer

We know that Sin3x = 3Sinx - 4Sin^3x

Sin^3x = (3Sinx - Sin3x)/4

$$\int x\sin^3 x dx = \int x\left(\frac{3\sin x - \sin 3x}{4}\right)dx$$

$$= \frac{1}{4}\int 3x\sin x - x\sin 3x dx$$

$$= \frac{3}{4}\int x\sin x dx - \frac{1}{4}\int x\sin 3x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and sinx and sin3x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{3}{4}\int x\sin x dx - \frac{1}{4}\int x\sin 3x dx$$

$$= \frac{3}{4}\left(x\int \sin x dx - \int\left[\frac{dx}{dx}.\int \sin x dx\right]dx\right) - \frac{1}{4}\left(s\int \sin 3x dx - \int\left[\frac{dx}{dx}.\int \sin 3x dx\right]dx\right)$$

$$= \frac{3}{4}\left(-x\cos x + \int \cos x dx\right) - \frac{1}{4}\left(\frac{-x\cos 3x}{3} + \int \frac{\cos 3x}{3}dx\right)$$

$$= \frac{3}{4}(-x\cos x + \sin x) - \frac{1}{4}\left(\frac{-x\cos 3x}{3} + \frac{\sin 3x}{9}\right) + c$$

$$= \frac{-3x\cos x}{4} + \frac{3\sin x}{4} + \frac{x\cos 3x}{12} - \frac{\sin 3x}{36} + c$$

20. Question

Evaluate the following integrals:

$\int x\cos^3 x\, dx$

Answer

We can write $\cos^3$x = (cos3x + 3cosx)/4, we have

$$\int x\cos^3 x dx = \int x\left(\frac{\cos 3x + 3\cos x}{4}\right)dx$$

$$= \frac{1}{4}\int x\cos 3x dx + \frac{3}{4}\int x\cos x dx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and cosx and cos3x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$= \frac{1}{4}\left(x\int \cos 3xdx - \int\left[\frac{dx}{dx}.\int \cos 3xdx\right]dx\right) + \frac{3}{4}\left(x\int \cos xdx - \int\left[\frac{dx}{dx}.\int \cos xdx\right]dx\right)$$

$$= \frac{1}{4}\left(x\frac{\sin 3x}{3} - \int\frac{\sin 3x}{3}dx\right) + \frac{3}{4}\left(x\sin x - \int \sin xdx\right)$$

$$= \frac{1}{4}\left(\frac{x\sin 3x}{3} + \frac{\cos 3x}{9}\right) + \frac{3}{4}\left(x\sin x + \cos x\right) + c$$

$$= \frac{x\sin 3x}{12} + \frac{\cos 3x}{36} + \frac{3x\sin x}{4} + \frac{3\cos x}{4} + c$$

21. Question

Evaluate the following integrals:

$$\int x^3 \cos x^2 dx$$

Answer

We can write it as

$$\int x.x^2 \cos x^2 dx$$

Now let $x^2 = t$

2xdx = dt

Xdx = dt/2

Now

$$\frac{1}{2}\int t\cos tdt$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is the first function and cost as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int t\cos tdt = \frac{1}{2}\left(t\int \cos tdt - \int\left[\frac{dt}{dt}.\int \cos tdt\right]dt\right)$$

$$= \frac{1}{2}\left(t\sin t - \int \sin tdt\right)$$

$$= \frac{1}{2}\left(t\sin t + \cos t\right) + c$$

Replacing t with x^2

$$= \frac{1}{2}x^2 \sin x^2 + \frac{1}{2}\cos x^2 + c$$

22. Question

Evaluate the following integrals:

$$\int \sin x \log(\cos x)dx$$

Answer

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here log(cosx) is the first function and sinx as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\int \sin x \log(\cos x)dx = \log(\cos x)\int \sin xdx - \int\left(\frac{d\log(\cos x)}{dx}.\int \sin xdx\right)dx$$

$$= -\cos x \log(\cos x) + \int \frac{-\sin x}{\cos x}.\cos xdx$$

$$= -\cos x \log(\cos x) - \int \sin xdx$$

$$= -\cos x \log(\cos x) + \cos x + c$$

23. Question

Evaluate the following integrals:

$$\int x \sin x \cos x\, dx$$

Answer

We know that Sin2x = 2Sinxcosx

$$\int x\sin x\cos xdx = \frac{1}{2}\int x\sin 2xdx$$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here x is first function and sin2x as the second function.

$$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$$

$$\frac{1}{2}\int x\sin 2xdx = \frac{1}{2}\left(x\int \sin 2xdx - \int\left[\frac{dx}{dx}.\int \sin 2xdx\right]dx\right)$$

$$= \frac{1}{2}\left(x\frac{-\cos 2x}{2} + \int \frac{\cos 2x}{2}dx\right)$$

$$= \frac{1}{2}\left(\frac{-x\cos 2x}{2} + \frac{\sin 2x}{4}\right) + c$$

$$= \frac{-x\cos 2x}{4} + \frac{\sin 2x}{8} + c$$

24. Question

Evaluate the following integrals:

$\int \cos\sqrt{x}\ dx$

Answer

Let √x = t

$\frac{1}{2\sqrt{x}}dx = dt$

$\Rightarrow dx = 2\sqrt{x}dt$

$\Rightarrow dx = 2tdt$

We can write it as

$\int \cos\sqrt{x}dx = 2\int t\cos tdt$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here t is first function and cos t as the second function.

$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$

$\Rightarrow 2\int t\cos tdt = 2\left(t\int \cos tdt - \int\left[\frac{dt}{dt}\right]\int \cos tdt\right)dt$

$= 2\left(t\sin t - \int \sin tdt\right)$

$= 2t\sin t + 2\cos t + c$

Replacing t with √x

= 2√xsin√x + 2cos√x + c

= 2(cos√x + √xsin√x) + c

25. Question

Evaluate the following integrals:

$\int \operatorname{cosec}^3 x\, dx$

Answer

We can write it as $\int \operatorname{cosec}^3 dx = \int \operatorname{cosec}x.\operatorname{cosec}^2 xdx$

Using BY PART METHOD. Using the superiority list as ILATE (Inverse Logarithm Algebra Trigonometric Exponential). Taking the first function to the one which comes first in the list.

Here cosecx is first function and $\operatorname{cosec}^2 x$ as the second function.

$\int a.b.dx = a\int bdx - \int\left[\frac{da}{dx}.\int bdx\right]dx$