

## EXERCISE 18.1

Using binomial theorem, write down the expressions of the following:

(i)  $(2x + 3y)^5$

(ii)  $(2x - 3y)^4$

(iii)  $\left(x - \frac{1}{x}\right)^6$

(iv)  $(1 - 3x)^7$

(v)  $\left(ax - \frac{b}{x}\right)^6$

(vi)  $\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6$

(vii)  $\left(\sqrt[3]{x} - \sqrt[3]{a}\right)^6$

(viii)  $(1 + 2x - 3x^2)^5$

(ix)  $\left(x + 1 - \frac{1}{x}\right)^3$

(x)  $(1 - 2x + 3x^2)^3$

**Solution:**

(i)  $(2x + 3y)^5$

Let us solve the given expression:

$$(2x + 3y)^5 = {}^5C_0 (2x)^5 (3y)^0 + {}^5C_1 (2x)^4 (3y)^1 + {}^5C_2 (2x)^3 (3y)^2 + {}^5C_3 (2x)^2 (3y)^3 + {}^5C_4$$

$$(2x)^1 (3y)^4 + {}^5C_5 (2x)^0 (3y)^5$$

$$= 32x^5 + 5 (16x^4) (3y) + 10 (8x^3) (9y)^2 + 10 (4x)^2 (27y)^3 + 5 (2x) (81y^4) + 243y^5$$

$$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$$

(ii)  $(2x - 3y)^4$

Let us solve the given expression:

$$(2x - 3y)^4 = {}^4C_0 (2x)^4 (3y)^0 - {}^4C_1 (2x)^3 (3y)^1 + {}^4C_2 (2x)^2 (3y)^2 - {}^4C_3 (2x)^1 (3y)^3 + {}^4C_4 (2x)^0$$

$$(3y)^4$$

$$= 16x^4 - 4 (8x^3) (3y) + 6 (4x^2) (9y^2) - 4 (2x) (27y^3) + 81y^4$$

$$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$$

**(iii)**  $\left(x - \frac{1}{x}\right)^6$

Let us solve the given expression:

$$\begin{aligned} & \left(x - \frac{1}{x}\right)^6 \\ &= {}^6C_0 x^6 \left(\frac{1}{x}\right)^0 - {}^6C_1 x^5 \left(\frac{1}{x}\right)^1 + {}^6C_2 x^4 \left(\frac{1}{x}\right)^2 - {}^6C_3 x^3 \left(\frac{1}{x}\right)^3 \\ &+ {}^6C_4 x^2 \left(\frac{1}{x}\right)^4 - {}^6C_5 x^1 \left(\frac{1}{x}\right)^5 + {}^6C_6 x^0 \left(\frac{1}{x}\right)^6 \\ &= x^6 - 6x^5 \times \frac{1}{x} + 15x^4 \times \frac{1}{x^2} - 20x^3 \times \frac{1}{x^3} + 15x^2 \times \frac{1}{x^4} - 6x \times \frac{1}{x^5} + \frac{1}{x^6} \\ &= x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6} \end{aligned}$$

**(iv)**  $(1 - 3x)^7$

Let us solve the given expression:

$$\begin{aligned} (1 - 3x)^7 &= {}^7C_0 (3x)^0 - {}^7C_1 (3x)^1 + {}^7C_2 (3x)^2 - {}^7C_3 (3x)^3 + {}^7C_4 (3x)^4 - {}^7C_5 (3x)^5 + {}^7C_6 (3x)^6 - {}^7C_7 (3x)^7 \\ &= 1 - 7(3x) + 21(9x^2) - 35(27x^3) + 35(81x^4) - 21(243x^5) + 7(729x^6) - 2187(x^7) \\ &= 1 - 21x + 189x^2 - 945x^3 + 2835x^4 - 5103x^5 + 5103x^6 - 2187x^7 \end{aligned}$$

**(v)**  $\left(ax - \frac{b}{x}\right)^6$

Let us solve the given expression:

$$\begin{aligned} & \left(ax - \frac{b}{x}\right)^6 \\ &= {}^6C_0 (ax)^6 \left(\frac{b}{x}\right)^0 - {}^6C_1 (ax)^5 \left(\frac{b}{x}\right)^1 + {}^6C_2 (ax)^4 \left(\frac{b}{x}\right)^2 - {}^6C_3 (ax)^3 \left(\frac{b}{x}\right)^3 \\ &+ {}^6C_4 (ax)^2 \left(\frac{b}{x}\right)^4 - {}^6C_5 (ax)^1 \left(\frac{b}{x}\right)^5 + {}^6C_6 (ax)^0 \left(\frac{b}{x}\right)^6 \\ &= a^6 x^6 - 6a^5 x^5 \times \frac{b}{x} + 15a^4 x^4 \times \frac{b^2}{x^2} - 20a^3 b^3 \times \frac{b^3}{x^3} + 15a^2 x^2 \times \frac{b^4}{x^4} - 6ax \times \frac{b^5}{x^5} + \frac{b^6}{x^6} \\ &= a^6 x^6 - 6a^5 x^4 b + 15a^4 x^2 b^2 - 20a^3 b^3 + 15 \frac{a^2 b^4}{x^2} - 6 \frac{ab^5}{x^4} + \frac{b^6}{x^6} \end{aligned}$$

$$(vi) \left( \sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}} \right)^6$$

Let us solve the given expression:

$$\begin{aligned} &= {}^6C_0 \left( \sqrt{\frac{x}{a}} \right)^6 \left( \sqrt{\frac{a}{x}} \right)^0 - {}^6C_1 \left( \sqrt{\frac{x}{a}} \right)^5 \left( \sqrt{\frac{a}{x}} \right)^1 + {}^6C_2 \left( \sqrt{\frac{x}{a}} \right)^4 \left( \sqrt{\frac{a}{x}} \right)^2 - {}^6C_3 \left( \sqrt{\frac{x}{a}} \right)^3 \left( \sqrt{\frac{a}{x}} \right)^3 \\ &+ {}^6C_4 \left( \sqrt{\frac{x}{a}} \right)^2 \left( \sqrt{\frac{a}{x}} \right)^4 - {}^6C_5 \left( \sqrt{\frac{x}{a}} \right)^1 \left( \sqrt{\frac{a}{x}} \right)^5 + {}^6C_6 \left( \sqrt{\frac{x}{a}} \right)^0 \left( \sqrt{\frac{a}{x}} \right)^6 \\ &= \frac{x^3}{a^3} - 6 \frac{x^2}{a^2} + 15 \frac{x}{a} - 20 + 15 \frac{a}{x} - 6 \frac{a^2}{x^2} + \frac{a^3}{x^3} \end{aligned}$$

$$(vii) \left( \sqrt[3]{x} - \sqrt[3]{a} \right)^6$$

Let us solve the given expression:

$$\begin{aligned} &= {}^6C_0 (\sqrt[3]{x})^6 (\sqrt[3]{a})^0 - {}^6C_1 (\sqrt[3]{x})^5 (\sqrt[3]{a})^1 + {}^6C_2 (\sqrt[3]{x})^4 (\sqrt[3]{a})^2 - {}^6C_3 (\sqrt[3]{x})^3 (\sqrt[3]{a})^3 \\ &+ {}^6C_4 (\sqrt[3]{x})^2 (\sqrt[3]{a})^4 - {}^6C_5 (\sqrt[3]{x})^1 (\sqrt[3]{a})^5 + {}^6C_6 (\sqrt[3]{x})^0 (\sqrt[3]{a})^6 \\ &= x^2 - 6x^{5/3}a^{1/3} + 15x^{4/3}a^{2/3} - 20xa + 15x^{2/3}a^{4/3} - 6x^{1/3}a^{5/3} + a^2 \end{aligned}$$

$$(viii) (1 + 2x - 3x^2)^5$$

Let us solve the given expression:

Let us consider  $(1 + 2x)$  and  $3x^2$  as two different entities and apply the binomial theorem.

$$(1 + 2x - 3x^2)^5 = {}^5C_0 (1 + 2x)^5 (3x^2)^0 - {}^5C_1 (1 + 2x)^4 (3x^2)^1 + {}^5C_2 (1 + 2x)^3 (3x^2)^2 - {}^5C_3 (1 + 2x)^2 (3x^2)^3 + {}^5C_4 (1 + 2x)^1 (3x^2)^4 - {}^5C_5 (1 + 2x)^0 (3x^2)^5$$

$$\begin{aligned} &= (1 + 2x)^5 - 5(1 + 2x)^4 (3x^2) + 10(1 + 2x)^3 (9x^4) - 10(1 + 2x)^2 (27x^6) + 5(1 + 2x)(81x^8) - 243x^{10} \end{aligned}$$

$$\begin{aligned} &= {}^5C_0 (2x)^0 + {}^5C_1 (2x)^1 + {}^5C_2 (2x)^2 + {}^5C_3 (2x)^3 + {}^5C_4 (2x)^4 + {}^5C_5 (2x)^5 - \\ &15x^2 [{}^4C_0 (2x)^0 + {}^4C_1 (2x)^1 + {}^4C_2 (2x)^2 + {}^4C_3 (2x)^3 + {}^4C_4 (2x)^4] + 90x^4 [1 + 8x^3 + 6x + 12x^2] - 270x^6(1 + 4x^2 + 4x) + 405x^8 + 810x^9 - 243x^{10} \end{aligned}$$

$$\begin{aligned} &= 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5 - 15x^2 - 120x^3 - 360x^4 - 480x^5 - \\ &240x^6 + 90x^4 + 720x^7 + 540x^5 + 1080x^6 - 270x^6 - 1080x^8 - 1080x^7 + 405x^8 + 810x^9 - \\ &243x^{10} \end{aligned}$$

$$\begin{aligned} &= 1 + 10x + 25x^2 - 40x^3 - 190x^4 + 92x^5 + 570x^6 - 360x^7 - 675x^8 + 810x^9 - \\ &243x^{10} \end{aligned}$$

$$(ix) \left(x + 1 - \frac{1}{x}\right)^3$$

Let us solve the given expression:

$$\begin{aligned} &= {}^3C_0(x+1)^3\left(\frac{1}{x}\right)^0 - {}^3C_1(x+1)^2\left(\frac{1}{x}\right)^1 + {}^3C_2(x+1)^1\left(\frac{1}{x}\right)^2 - {}^3C_3(x+1)^0\left(\frac{1}{x}\right)^3 \\ &= (x+1)^3 - 3(x+1)^2 \times \frac{1}{x} + 3 \frac{x+1}{x^2} - \frac{1}{x^3} \\ &= x^3 + 1 + 3x + 3x^2 - \frac{3x^2 + 3 + 6x}{x} + 3 \frac{x+1}{x^2} - \frac{1}{x^3} \\ &= x^3 + 1 + 3x + 3x^2 - 3x - \frac{3}{x} - 6 + \frac{3}{x} + \frac{3}{x^2} - \frac{1}{x^3} \\ &= x^3 + 3x^2 - 5 + \frac{3}{x^2} - \frac{1}{x^3} \end{aligned}$$

$$(x) (1 - 2x + 3x^2)^3$$

Let us solve the given expression:

$$\begin{aligned} &= {}^3C_0(1-2x)^3 + {}^3C_1(1-2x)^2(3x^2) + {}^3C_2(1-2x)(3x^2)^2 + {}^3C_3(3x^2)^3 \\ &= (1-2x)^3 + 9x^2(1-2x)^2 + 27x^4(1-2x) + 27x^6 \\ &= 1 - 8x^3 + 12x^2 - 6x + 9x^2(1 + 4x^2 - 4x) + 27x^4 - 54x^5 + 27x^6 \\ &= 1 - 8x^3 + 12x^2 - 6x + 9x^2 + 36x^4 - 36x^3 + 27x^4 - 54x^5 + 27x^6 \\ &= 1 - 6x + 21x^2 - 44x^3 + 63x^4 - 54x^5 + 27x^6 \end{aligned}$$

2. Evaluate the following:

$$(i) (\sqrt{x+1} + \sqrt{x-1})^6 + (\sqrt{x+1} - \sqrt{x-1})^6$$

$$(ii) (x + \sqrt{x^2 - 1})^6 + (x - \sqrt{x^2 - 1})^6$$

$$(iii) (1 + 2\sqrt{x})^5 + (1 - 2\sqrt{x})^5$$

$$(iv) (\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$$



$$(v) (3 + \sqrt{2})^5 - (3 - \sqrt{2})^5$$

$$(vi) (2 + \sqrt{3})^7 + (2 - \sqrt{3})^7$$

$$(vii) (\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$$

$$(viii) (0.99)^5 + (1.01)^5$$

$$(ix) (\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$$

$$(x) \left\{ a^2 + \sqrt{a^2 - 1} \right\}^4 + \left\{ a^2 - \sqrt{a^2 - 1} \right\}^4$$

**Solution:**

$$(i) (\sqrt{x+1} + \sqrt{x-1})^6 + (\sqrt{x+1} - \sqrt{x-1})^6$$

Let us solve the given expression:

$$\begin{aligned} &= 2 \left[ {}^6C_0 (\sqrt{x+1})^6 (\sqrt{x-1})^0 + {}^6C_2 (\sqrt{x+1})^4 (\sqrt{x-1})^2 \right. \\ &+ {}^6C_4 (\sqrt{x+1})^2 (\sqrt{x-1})^4 + {}^6C_6 (\sqrt{x+1})^0 (\sqrt{x-1})^6 \left. \right] \\ &= 2 \left[ (x+1)^3 + 15(x+1)^2(x-1) + 15(x+1)(x-1)^2 + (x-1)^3 \right] \\ &= 2 \left[ x^3 + 1 + 3x + 3x^2 + 15(x^2 + 2x + 1)(x-1) + 15(x+1)(x^2 + 1 - 2x) + x^3 - 1 + 3x - 3x^2 \right] \\ &= 2 \left[ 2x^3 + 6x + 15x^3 - 15x^2 + 30x^2 - 30x + 15x - 15 + 15x^3 + 15x^2 - 30x^2 - 30x + 15x + 15 \right] \\ &= 2 \left[ 32x^3 - 24x \right] \\ &= 16x \left[ 4x^2 - 3 \right] \end{aligned}$$

$$(ii) \left( x + \sqrt{x^2 - 1} \right)^6 + \left( x - \sqrt{x^2 - 1} \right)^6$$

Let us solve the given expression:

$$\begin{aligned} &= 2 \left[ {}^6C_0 x^6 (\sqrt{x^2 - 1})^0 + {}^6C_2 x^4 (\sqrt{x^2 - 1})^2 + {}^6C_4 x^2 (\sqrt{x^2 - 1})^4 + \right. \\ &{}^6C_6 x^0 (\sqrt{x^2 - 1})^6 \left. \right] \\ &= 2 \left[ x^6 + 15x^4(x^2 - 1) + 15x^2(x^2 - 1)^2 + (x^2 - 1)^3 \right] \end{aligned}$$

$$\begin{aligned}
 &= 2 \left[ x^6 + 15x^6 - 15x^4 + 15x^2 (x^4 - 2x^2 + 1) + (x^6 - 1 + 3x^2 - 3x^4) \right] \\
 &= 2 \left[ x^6 + 15x^6 - 15x^4 + 15x^6 - 30x^4 + 15x^2 + x^6 - 1 + 3x^2 - 3x^4 \right] \\
 &= 64x^6 - 96x^4 + 36x^2 - 2
 \end{aligned}$$

$$(iii) (1 + 2\sqrt{x})^5 + (1 - 2\sqrt{x})^5$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 \left[ {}^5C_0 (2\sqrt{x})^0 + {}^5C_2 (2\sqrt{x})^2 + {}^5C_4 (2\sqrt{x})^4 \right] \\
 &= 2 \left[ 1 + 10(4x) + 5(16x^2) \right] \\
 &= 2 \left[ 1 + 40x + 80x^2 \right]
 \end{aligned}$$

$$(iv) (\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 \left[ {}^6C_0 (\sqrt{2})^6 + {}^6C_2 (\sqrt{2})^4 + {}^6C_4 (\sqrt{2})^2 + {}^6C_6 (\sqrt{2})^0 \right] \\
 &= 2 \left[ 8 + 15(4) + 15(2) + 1 \right] \\
 &= 2 \left[ 99 \right] \\
 &= 198
 \end{aligned}$$

$$(v) (3 + \sqrt{2})^5 - (3 - \sqrt{2})^5$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 \left[ {}^5C_1 (3^4) (\sqrt{2})^1 + {}^5C_3 (3^2) (\sqrt{2})^3 + {}^5C_5 (3^0) (\sqrt{2})^5 \right] \\
 &= 2 \left[ 5(81)(\sqrt{2}) + 10(9)(2\sqrt{2}) + 4\sqrt{2} \right] \\
 &= 2\sqrt{2} (405 + 180 + 4) \\
 &= 1178\sqrt{2}
 \end{aligned}$$

$$(vi) (2 + \sqrt{3})^7 + (2 - \sqrt{3})^7$$

Let us solve the given expression:

$$\begin{aligned}
 &= 2 \left[ {}^7C_0 (2^7) (\sqrt{3})^0 + {}^7C_2 (2^5) (\sqrt{3})^2 + {}^7C_4 (2^3) (\sqrt{3})^4 + {}^7C_6 (2^1) (\sqrt{3})^6 \right] \\
 &= 2 \left[ 128 + 21(32)(3) + 35(8)(9) + 7(2)(27) \right] \\
 &= 2 \left[ 128 + 2016 + 2520 + 378 \right] \\
 &= 2 \left[ 5042 \right] \\
 &= 10084
 \end{aligned}$$

$$(vii) (\sqrt{3} + 1)^5 - (\sqrt{3} - 1)^5$$

Let us solve the given expression:

$$\begin{aligned} &= 2 [{}^5C_1 (\sqrt{3})^4 + {}^5C_3 (\sqrt{3})^2 + {}^5C_5 (\sqrt{3})^0] \\ &= 2 [5(9) + 10(3) + 1] \\ &= 2 [76] \\ &= 152 \end{aligned}$$

$$(viii) (0.99)^5 + (1.01)^5$$

Let us solve the given expression:

$$\begin{aligned} &= (1 - 0.01)^5 + (1 + 0.01)^5 \\ &= 2 [{}^5C_0 (0.01)^0 + {}^5C_2 (0.01)^2 + {}^5C_4 (0.01)^4] \\ &= 2 [1 + 10(0.0001) + 5(0.00000001)] \\ &= 2 [1.00100005] \\ &= 2.0020001 \end{aligned}$$

$$(ix) (\sqrt{3} + \sqrt{2})^6 - (\sqrt{3} - \sqrt{2})^6$$

Let us solve the given expression:

$$\begin{aligned} &= 2 [{}^6C_1 (\sqrt{3})^5 (\sqrt{2})^1 + {}^6C_3 (\sqrt{3})^3 (\sqrt{2})^3 + {}^6C_5 (\sqrt{3})^1 (\sqrt{2})^5] \\ &= 2 [6(9\sqrt{3})(\sqrt{2}) + 20(3\sqrt{3})(2\sqrt{2}) + 6(\sqrt{3})(4\sqrt{2})] \\ &= 2 [\sqrt{6}(54 + 120 + 24)] \\ &= 396\sqrt{6} \end{aligned}$$

$$(x) \left\{ a^2 + \sqrt{a^2 - 1} \right\}^4 + \left\{ a^2 - \sqrt{a^2 - 1} \right\}^4$$

Let us solve the given expression:

$$\begin{aligned} &= 2 \left[ {}^4C_0 (a^2)^4 (\sqrt{a^2 - 1})^0 + {}^4C_2 (a^2)^2 (\sqrt{a^2 - 1})^2 + {}^4C_4 (a^2)^0 (\sqrt{a^2 - 1})^4 \right] \\ &= 2 \left[ a^8 + 6a^4 (a^2 - 1) + (a^2 - 1)^2 \right] \\ &= 2 [a^8 + 6a^6 - 6a^4 + a^4 + 1 - 2a^2] \\ &= 2a^8 + 12a^6 - 10a^4 - 4a^2 + 2 \end{aligned}$$

**3. Find  $(a + b)^4 - (a - b)^4$ . Hence, evaluate  $(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4$ .**

**Solution:**

Firstly, let us solve the given expression:

$$(a + b)^4 - (a - b)^4$$

The above expression can be expressed as,

$$\begin{aligned}(a + b)^4 - (a - b)^4 &= 2 [{}^4C_1 a^3b^1 + {}^4C_3 a^1b^3] \\ &= 2 [4a^3b + 4ab^3] \\ &= 8 (a^3b + ab^3)\end{aligned}$$

Now,

Let us evaluate the expression:

$$\begin{aligned}(\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 \\ \text{So consider, } a = \sqrt{3} \text{ and } b = \sqrt{2} \text{ we get,} \\ (\sqrt{3} + \sqrt{2})^4 - (\sqrt{3} - \sqrt{2})^4 &= 8 (a^3b + ab^3) \\ &= 8 [(\sqrt{3})^3 (\sqrt{2}) + (\sqrt{3}) (\sqrt{2})^3] \\ &= 8 [(3\sqrt{6}) + (2\sqrt{6})] \\ &= 8 (5\sqrt{6}) \\ &= 40\sqrt{6}\end{aligned}$$

**4. Find  $(x + 1)^6 + (x - 1)^6$ . Hence, or otherwise evaluate  $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$ .**

**Solution:**

Firstly, let us solve the given expression:

$$\begin{aligned}(x + 1)^6 + (x - 1)^6 \\ \text{The above expression can be expressed as,} \\ (x + 1)^6 + (x - 1)^6 &= 2 [{}^6C_0 x^6 + {}^6C_2 x^4 + {}^6C_4 x^2 + {}^6C_6 x^0] \\ &= 2 [x^6 + 15x^4 + 15x^2 + 1]\end{aligned}$$

Now,

Let us evaluate the expression:

$$\begin{aligned}(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6 \\ \text{So consider, } x = \sqrt{2} \text{ then we get,} \\ (\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6 &= 2 [x^6 + 15x^4 + 15x^2 + 1] \\ &= 2 [(\sqrt{2})^6 + 15 (\sqrt{2})^4 + 15 (\sqrt{2})^2 + 1] \\ &= 2 [8 + 15 (4) + 15 (2) + 1] \\ &= 2 [8 + 60 + 30 + 1] \\ &= 198\end{aligned}$$

**5. Using binomial theorem evaluate each of the following:**

- (i)  $(96)^3$
- (ii)  $(102)^5$
- (iii)  $(101)^4$
- (iv)  $(98)^5$

**Solution:**

- (i)  $(96)^3$

We have,



$$(96)^3$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$\begin{aligned}(96)^3 &= (100 - 4)^3 \\ &= {}^3C_0 (100)^3 (4)^0 - {}^3C_1 (100)^2 (4)^1 + {}^3C_2 (100)^1 (4)^2 - {}^3C_3 (100)^0 (4)^3 \\ &= 1000000 - 120000 + 4800 - 64 \\ &= 884736\end{aligned}$$

$$\text{(ii) } (102)^5$$

We have,

$$(102)^5$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$\begin{aligned}(102)^5 &= (100 + 2)^5 \\ &= {}^5C_0 (100)^5 (2)^0 + {}^5C_1 (100)^4 (2)^1 + {}^5C_2 (100)^3 (2)^2 + {}^5C_3 (100)^2 (2)^3 + {}^5C_4 (100)^1 \\ &+ {}^5C_5 (100)^0 (2)^5 \\ &= 10000000000 + 1000000000 + 40000000 + 800000 + 8000 + 32 \\ &= 11040808032\end{aligned}$$

$$\text{(iii) } (101)^4$$

We have,

$$(101)^4$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$\begin{aligned}(101)^4 &= (100 + 1)^4 \\ &= {}^4C_0 (100)^4 + {}^4C_1 (100)^3 + {}^4C_2 (100)^2 + {}^4C_3 (100)^1 + {}^4C_4 (100)^0 \\ &= 100000000 + 4000000 + 60000 + 400 + 1 \\ &= 104060401\end{aligned}$$

$$\text{(iv) } (98)^5$$

We have,

$$(98)^5$$

Let us express the given expression as two different entities and apply the binomial theorem.

$$\begin{aligned}(98)^5 &= (100 - 2)^5 \\ &= {}^5C_0 (100)^5 (2)^0 - {}^5C_1 (100)^4 (2)^1 + {}^5C_2 (100)^3 (2)^2 - {}^5C_3 (100)^2 (2)^3 + {}^5C_4 (100)^1 (2)^4 \\ &- {}^5C_5 (100)^0 (2)^5 \\ &= 10000000000 - 1000000000 + 40000000 - 800000 + 8000 - 32 \\ &= 9039207968\end{aligned}$$

**6. Using binomial theorem, prove that  $2^{3n} - 7n - 1$  is divisible by 49, where  $n \in \mathbb{N}$ .**

**Solution:**

Given:

$$2^{3n} - 7n - 1$$

$$\text{So, } 2^{3n} - 7n - 1 = 8^n - 7n - 1$$

Now,

$$8^n - 7n - 1$$

$$8^n = 7n + 1$$

$$= (1 + 7)^n$$

$$= {}^n C_0 + {}^n C_1 (7)^1 + {}^n C_2 (7)^2 + {}^n C_3 (7)^3 + {}^n C_4 (7)^4 + {}^n C_5 (7)^5 + \dots + {}^n C_n (7)^n$$

$$8^n = 1 + 7n + 49 [{}^n C_2 + {}^n C_3 (7^1) + {}^n C_4 (7^2) + \dots + {}^n C_n (7)^{n-2}]$$

$$8^n - 1 - 7n = 49 (\text{integer})$$

So now,

$$8^n - 1 - 7n \text{ is divisible by } 49$$

Or

$$2^{3n} - 1 - 7n \text{ is divisible by } 49.$$

Hence proved.

