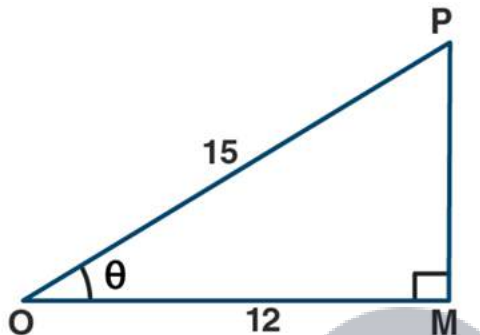
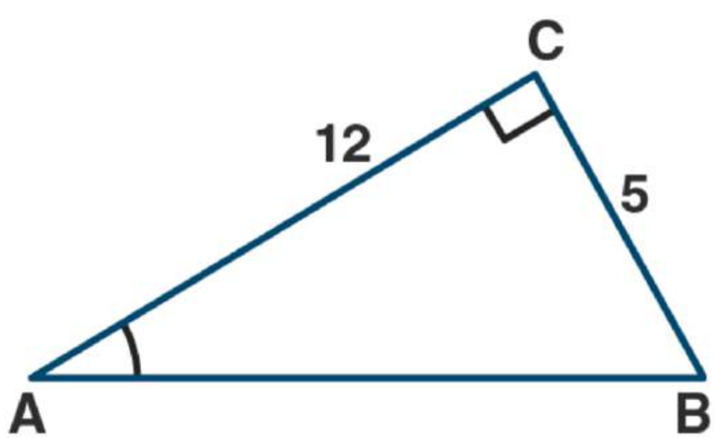
EXERCISE 17

1. (a) From the figure (1) given below, find the values of:
(i) sin θ
(ii) cos θ
(iii) tan θ
(iv) cot θ
(v) sec θ
(vi) cosec θ



(b) From the figure (2) given below, find the values of:
(i) sin A
(ii) cos A
(iii) $\sin^2 A + \cos^2 A$
(iv) $\sec^2 A - \tan^2 A$.



Solution:
(a) From right angled triangle OMP,
By Pythagoras theorem, we get
$OP^2 = OM^2 + MP^2$

$MP^2 = OP^2 + OM^2$
$MP^2 = (15)^2 – (12)^2$
$MP^2 = 225 – 144$
$MP^2 = 81$
$MP^2 = 9^2$
MP = 9
(i) sin θ = MP/OP
= 9/15
= 3/5

(ii) cos θ = OM/OP
= 12/15
= 4/5

(iii) tan θ = MP/OP
= 9/12
= ¾

(iv) cot θ = OM/MP
= 12/9
= 4/3

(v) sec θ = OP/OM
= 15/12
= 5/4

(vi) cosec θ = OP/MP
= 15/9
= /3

(b) From right angled triangle ABC,
By Pythagoras theorem, we get
$AB^2 = AC^2 + BC^2$
$AB^2 = (12)^2 + (5)^2$
$AB^2 = 144 + 25$
$AB^2 = 169$
$AB^2 = 13^2$

AB = 13

(i) sin A = BC/AB

= 5/13

(ii) cos A = AC/AB

= 12/13

(iii) $\text{Sin}^2 A + \cos^2 A = (BC/AB)^2 + (AC/AB)^2$

$= (5/13)^2 + (12/13)^2$

= (25/169) + (144/169)

= (25 + 144)/ 169

= 169/169

= 1

$\text{Sin}^2 A + \cos^2 A = 1$

(iv) $\text{Sec}^2 A - \tan^2 A = (AB/AC)^2 - (BC/AC)^2$

$= (13/12)^2 - (5/12)^2$

= (169/144) - (25/144)

= (169 - 25)/ 144

= 144/144

= 1

$\text{Sec}^2 A - \tan^2 A = 1$

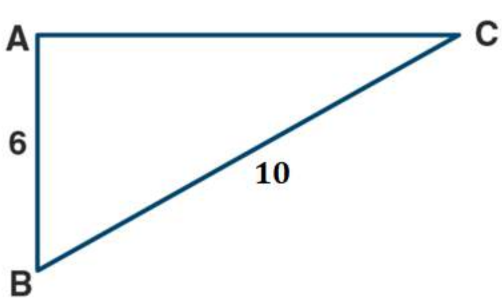
2. (a) From the figure (1) given below, find the values of:

(i) sin B

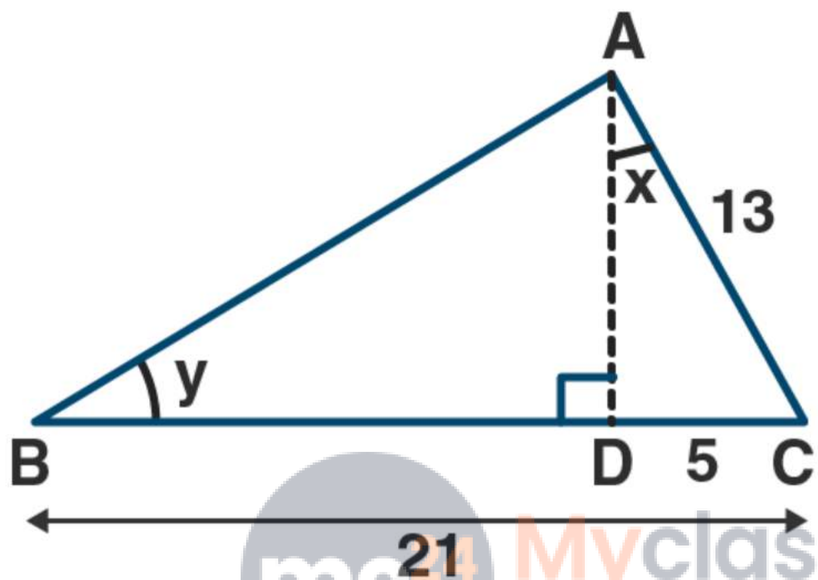
(i) cos C

(iii) sin B + sin C

(iv) sin B cos C + sin C cos B.



(b) From the figure (2) given below, find the values of:
(i) tan x
(ii) cos y
(iii) $cosec^2 y - cot^2 y$
(iv) 5/sin x + 3/sin y – 3 cot y.



Solution:
From right angled triangle ABC,
By Pythagoras theorem, we get
$BC^2 = AC^2 + AB^2$
$AC^2 = BC^2 - AB^2$
$AC^2 = (10)^2 - (6)^2$
$AC^2 = 100 - 36$
$AC^2 = 64$
$AC^2 = 8^2$
AC = 8

(i) sin B = perpendicular/ hypotenuse
= AC/BC
= 8/10
= 4/5

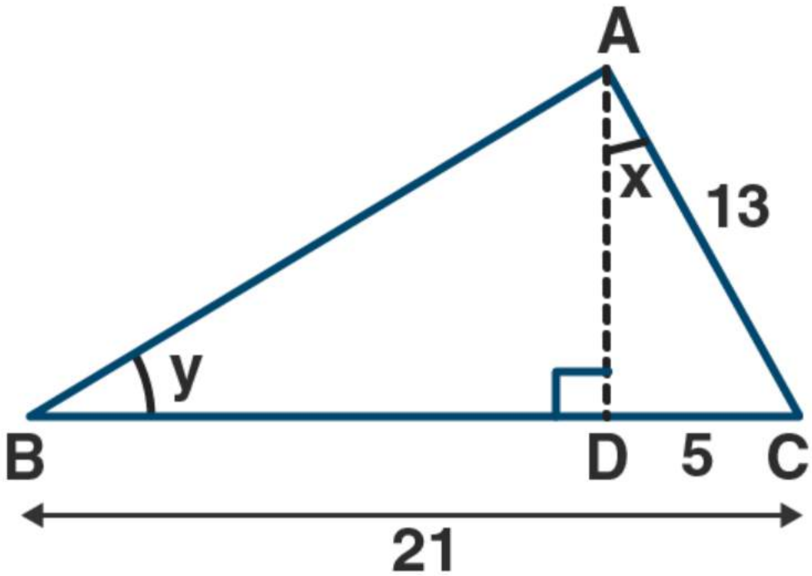
(ii) cos C = Base/hypotenuse

= AC/BC
= 8/10
= 4/5

(iii) sin B = Perpendicular/hypotenuse
= AC/BC
= 8/10
= 4/5
Sin C = perpendicular/hypotenuse
= AB/BC
= 6/10
= 3/5
Now,
Sin B + sin C = (4/5) + (3/5)
= (4 + 3)/5
= 7/5

(iv) sin B = 4/5
Cos C = 4/5
Sin C = perpendicular/ hypotenuse
= AB/BC
= 6/10
= 3/5
Cos B = Base/Hypotenuse
= AB/BC
= 6/10
= 3/5
sin B cos C + sin C cos B
= (4/5) × (4/5) × (3/5) × (3/5)
= (26/25) × (9/25)
= (16+9)/25
= 25/25
= 1
From Figure
AC = 13, CD = 5, BC =21,
BD = BC – CD
= 21 – 5

= 16



From right angled ΔACD,
By Pythagoras theorem we get
$AC = AD^2 + CD^2$
$AD^2 = AC^2 – CD^2$
$AD^2 = (13)^2 – (5)^2$
$AD^2 = 169 – 25$
$AD^2 = 144)$
$AD^2 = (12)^2$
AD = 12
From right angled ΔABD,
By Pythagoras angled ΔABD
By Pythagoras theorem we get
$AB^2 = AD^2 + BD^2$
$AB^2 = 400$
$AB^2 = (20)^2$
AB = 20

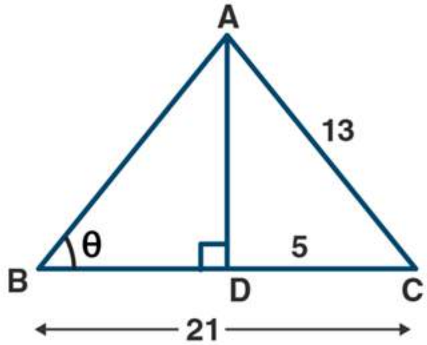
(i) tan x = perpendicular/Base (in right angled ΔACD)
=CD/AD
= 5/12

(ii) cos y = Base/Hypotenuse (in right angled ΔABD)
= BD/AB
= (20)/12 – (5/3)
Cot y = Base/Perpendicular (in right ΔABD)
=BD/AB
= 16/20 = 4/5

(iii) cos y = Hypotenuse/ perpendicular (in right angled ΔABD)
BD/AB
= 20/12
= 5/3
Cot y = Base/Perpendicular (in right ΔABD)
AB/AD
= 16/12
= 4/3
Cosec^2 y - cot^2 y = $(5/3)^2 – (4/3)^2$
= (25/9) – (16/9)
= (25-16)/9
= 9/9
= 1
Hence, cosec^2 y – cot^2 y = 1

(iv) sin x = Perpendicular/Hypotenuse (in right angled ΔACD)
= AD/AB
= 12/20
= 3/5
Cot y = Base/Perpendicular (in right angled ΔABD)
= BD/AD
= 16/12
= 4/3
(5/sin x) + (3/sin y) – 3cot y
= 5/(5/13) + 3/(3/5) – 3 × 4/3
= 5 × 13/5 + 3 × 5/3 – 3 × 4/3
= 1 × 13/1 + 1 × 5/1 – 1 × 4/1
= 13 + 5 – 4 = 18 – 4
= 14
Hence 5/sin x + 3/sin y – 3cot y = 14

3. (a) **From the figure (1) given below, find the value of sec θ.**



(b) From the figure (2) given below, find the values of:

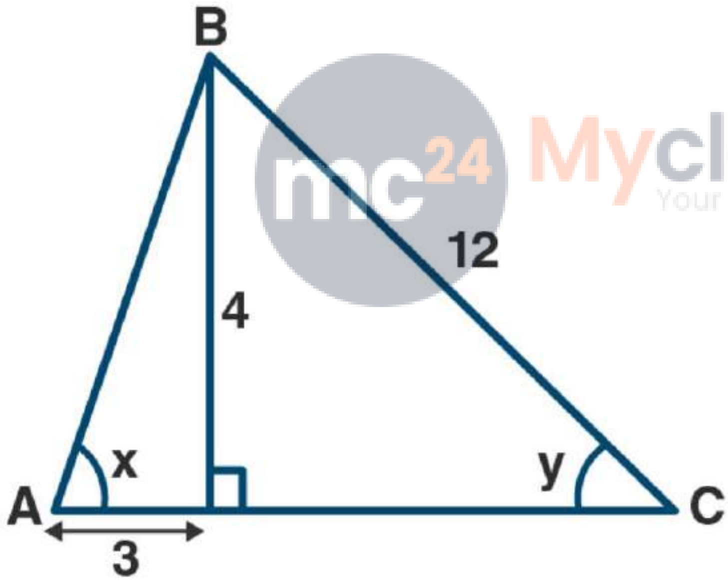
(i) sin x

(ii) cot x

(iii) $\cot^2$ x- cosec^2 x

(iv) sec y

(v) $\tan^2$ y – 1/$\cos^2$ y.



Solution:

(a) From the figure, Sec θ = AB / BD

But in ΔADC, $\angle D = 90^o$

$AC^2 = AD^2 + DC^2$ (Pythagoras Theorem)

$(13)^2 = AD^2 + 25$

$AD^2 = 169 - 25$

= 144

= $(12)^2$

AD = 12

(in right ΔABD)

$AB^2 = AD^2 + BD^2$

$= (12)^2 + (16)^2$

= 144 + 256

= 400

$= (20)^2$

AB = 20

Now, Sec θ = AB / BD

= 20/16

= 5/4

(b) let given ΔABC

BD = 3, AC = 12, AD = 4

In right angled ΔABD

By Pythagoras theorem

$AB^2 = AD^2 + BD^2$

$AB^2 = (4)^2 + (3)^2$

$AB^2 = 16 + 9$

$AB^2 = 25$

$AB^2 = (5)^2$

AB = 5

In right angled triangle ACD

By Pythagoras theorem,

$AC^2 = AD^2 + CD^2$

$CD^2 = AC^2 – AD^2$

$CD^2 = (12)^2 – (4)^2$

$CD^2 = 144 - 16$

$CD^2 = 128$

CD = √128

CD = √64 × 2 CD

= 8√2

(i) sin x = perpendicular/Hypotenuse

= AD/AB

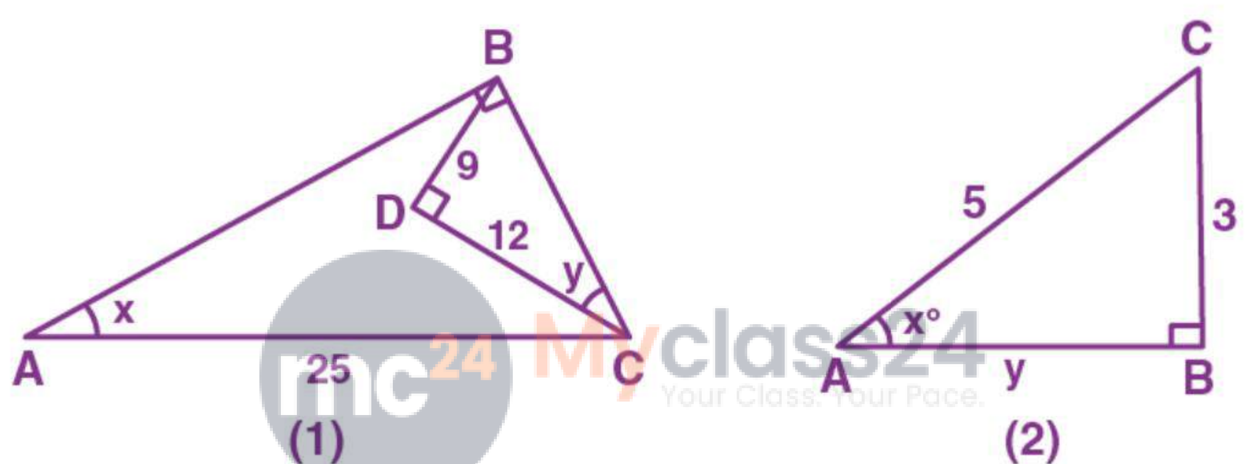
= 4/5

(ii) cot x = Base/Perpendicular
= BD/AD
= ¾

(iii) cot x = Base/ Perpendicular
BD/AD
= 3/4

(iv) cosec x = Hypotenuse / Perpendicular
AB/BD
= 5/4
$\text{Cot}^2 x - \text{cosec}^2 x$
$= (3/4)^2 - (5/4)^2$
= 9/16 – 25/16
(9 -25)/16
= -16/16
= -1
Perpendicular = Hypotenuse/Base (in right angled ΔACD)
= AD/CD
= 12/(8 √2)
= 3/(2 √2)
Cot y = Base/ Hypotenuse
= AD/CD
= 4/8 √ 2
= 1/2 √2
Cot y = Base / Hypotenuse (in right angled ΔACD)
= CD/AC
= 8√2/12
= 2√/3
Now $\tan^2 y = 1/\cos^2 y$
$= (1/2\sqrt{2})^2 - 1/(2\sqrt{2}/3)^2$
= ¼ × - ¼ × 2
= (1/8) – (9/8)
= (1-9)/8
= -8/8
= -1
$\tan^2 y - 1/\cos^2 y = -1.$

4. (a) From the figure (1) given below, find the values of:
(i) 2 sin y – cos y
(ii) 2 sin x – cos x
(iii) 1 – sin x + cos y
(iv) 2 cos x – 3 sin y + 4 tan x
(b) In the figure (2) given below, ΔABC is right-angled at B. If AB = y units, BC = 3 units and CA = 5 units, find
(i) sin x^0
(ii) y.



Solution:

(a) In a right angled ΔBCD,
Using Pythagoras theorem

$BC^2 = BD^2 + CD^2$
Substituting the values
$BC^2 = 9^2 + 12^2$
By further calculation
$BC^2 = 81 + 144 = 225$
$BC^2 = 15^2$
BC = 15

In a right angled ΔABC,
Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$
We can write it as
$AB^2 = AC^2 – BC^2$
Substituting the values
$AB^2 = 25^2 – 15^2$
By further calculation
$AB^2 = 625 – 225 = 400$
So we get
$AB^2 = 20^2$
AB = 20

(i) We know that
In right angled ΔBCD
sin y = perpendicular/ hypotenuse
sin y = BD/ BC
Substituting the values
sin y = 9/15 = 3/5

In right angled ΔBCD
cos y = base/hypotenuse
cos y = CD/BC
Substituting the values
cos y = 12/15 = 4/5

Here
2sin y – cos y = 2 × 3/5 – 4/5
We can write it as
= 6/5 – 4/5
= 2/5

Therefore, 2 sin y – cos y = 2/5

(ii) In right angled ΔABC
sin x = perpendicular/ hypotenuse
sin x = BC/AC
Substituting the values
sin x = 15/25 = 3/5

In right angled ΔABC
cos x = base/hypotenuse
cos x = AB/AC
Substituting the values
cos x = 20/25 = 4/5

Here
2 sin x – cos x = 2 × 3/5 – 4/5
We can write it as
= 6/5 – 4/5
= 2/5
Therefore, 2 sin x – cos x = 2/5.

(iii) In right angled ΔABC
sin x = perpendicular/hypotenuse
sin x = BC/AC
Substituting the values
sin x = 12/25 = 3/5

In right angled ΔBCD
cos y = base/hypotenuse
cos y = CD/BC
Substituting the values
cos y = 12/15 = 4/5

Here
1 – sin x + cos y = 1 – 3/5 + 4/5
By further calculation
= (5 – 3 + 4)/ 5
So we get
= (9 – 3)/ 5
= 6/5

Therefore, 1 – sin x + cos y = 6/5.

(iv) In right angled ΔBCD
cos x = base/hypotenuse

cos x = AB/AC
Substituting the values
cos x = 20/25 = 4/5

In right angled ΔBCD
sin y = perpendicular/hypotenuse
sin y = BD/BC
Substituting the values
sin y = 9/15 = 3/5

In right angled ΔABC
tan x = perpendicular/base
tan x = BC/AB
Substituting the values
tan x = 15/20 = ¾

Here
2 cos x – 3 sin y + 4 tan x = 2 × 4/5 – 3 × 3/5 + 4 × ¾
By further calculation
= 8/5 – 9/5 3/1
Taking LCM
= (8 – 9 + 15)/5
So we get
= (23 – 9)/ 5
= 14/5

(b) It is given that
AB = y units, BC = 3 units, CA = 5 units

(i) In right angled ΔABC
sin x = perpendicular/hypotenuse
sin x = BC/AC
Substituting the values
sin x = 3/5

(ii) In right angled ΔABC
Using Pythagoras theorem

$AC^2 = BC^2 + AB^2$
We can write it as
$AB^2 = AC^2 – BC^2$
Substituting the values
$AB^2 = 5^2 – 3^2$
By further calculation
$AB^2 = 25 – 9 = 16$
So we get
$AB^2 = 4^2$
AB = 4
y = 4 units
Therefore, y = 4 units.

5. In a right-angled triangle, it is given that angle A is an acute angle and that tan A=5/12. Find the values of:
(i) cos A
(ii) cosec A- cot A.

Solution:
Here, ABC is right angled triangle
∠A is an acute angle and ∠C = 90^o
tan A = 5/12
BC/AC =5/12
Let BC = 5x and AC = 12x
From right angled ΔABC
By Pythagoras theorem, we get
$AB^2 = (5x)^2 + (12x)^2$
$AB^2 = 25x^2 + 144x^2$
$AB^2 = 169x^2$

(i) cos A = Base/ Hypotenuse
= AC / AB
= 12x/13x
=12/13

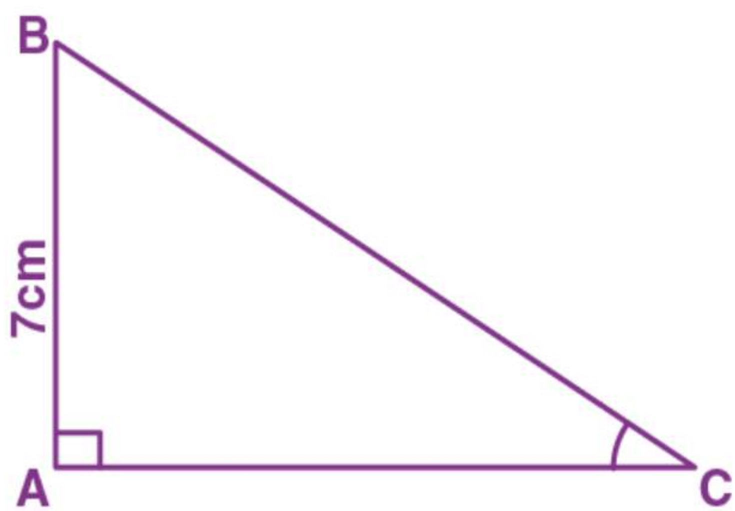
(ii) cosec A = Hypotenuse/perpendicular
= AC / BC

= 13x /5x
= 13/5

cosec A - cot A = 13/5 – 12/5
= (13-12)/5
= 1/5

6. (a) In ΔABC, ∠A = 90^0. If AB = 7 cm and BC – AC = 1 cm, find:
(i) sin C
(ii) tan B
(b) In ΔPQR, ∠Q = 90^0. If PQ = 40 cm and PR + QR = 50 cm, find:
(i) sin P
(ii) cos P
(iii) tan R.
Solution:

(a) In right ΔABC
∠A = 90^0
AB = 7 cm
BC – AC = 1 cm
BC = 1 + AC



We know that
$BC^2 = AB^2 + AC^2$
Substituting the value of BC
$(1 + AC)^2 = AB^2 + AC^2$

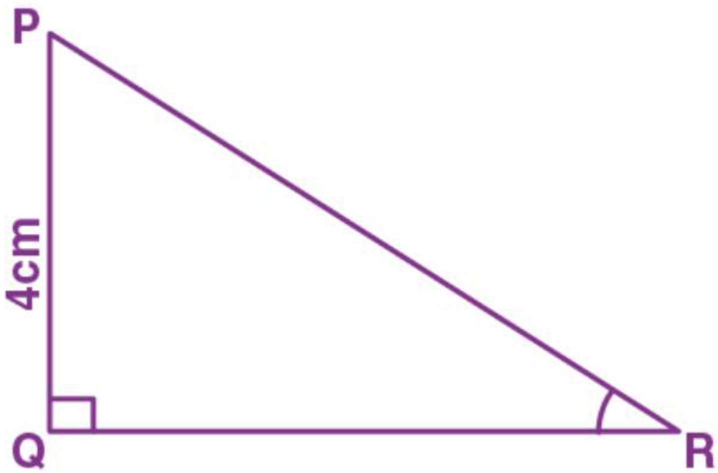
$1 + AC^2 + 2AC = 7^2 + AC^2$
By further calculation
$1 + AC^2 + 2AC = 49\ AC^2$
$2AC = 49 – 1 – 48$
So we get
AC = 48/2 = 24 cm

Here
BC = 1 + AC
Substituting the value
BC = 1 + 24 = 25 cm

(i) sin C = AB/BC = 7/25

(ii) tan B = AC/AB = 24/7

(b) In right ΔPQR
$\angle Q = 90^0$
PQ = 40 cm
PQ + QR = 50 cm
We can write it as
PQ = 50 – QR



Using Pythagoras theorem
$PR^2 = PQ^2 + QR^2$
$(50 – QR)^2 = (40)^2 + QR^2$

By further calculation

$2500 + QR^2 - 100QR = 1600 + QR^2$

So we get

$2500 - 1600 = 100QR$

$100QR = 900$

By division

QR = 900/100 = 9

We get

PR = 50 – 9 = 41

(i) sin P = QR/PR = 9/41

(ii) cos P = PQ/PR = 40/41

(iii) tan R = PQ/QR = 40/9

7. In triangle ABC, AB = 15 cm, AC = 15 cm and BC = 18 cm. Find

(i) cos ∠ABC

(ii) sin ∠ACB.

Solution:

Here ABC is a triangle in which

AB = 15 cm, AC = 15 cm and BC = 18 cm

Draw AD perpendicular to BC , D is mid-point of BC.

Then, BD – DC = 9 cm

in right angled triangle ABD,

By Pythagoras theorem, we get

$AB^2 = AD^2 + BD^2$

$AD^2 = AB^2 - BD^2$

$AD^2 = (15)^2 - (9)^2$

$AD^2 = 225 - 81$

$AD^2 = 144$

AD – 12 cm

(i) cos ∠ABC = Base/ / Hypotenuse

(In right angled ΔABD, ∠ABC = ∠ABD)

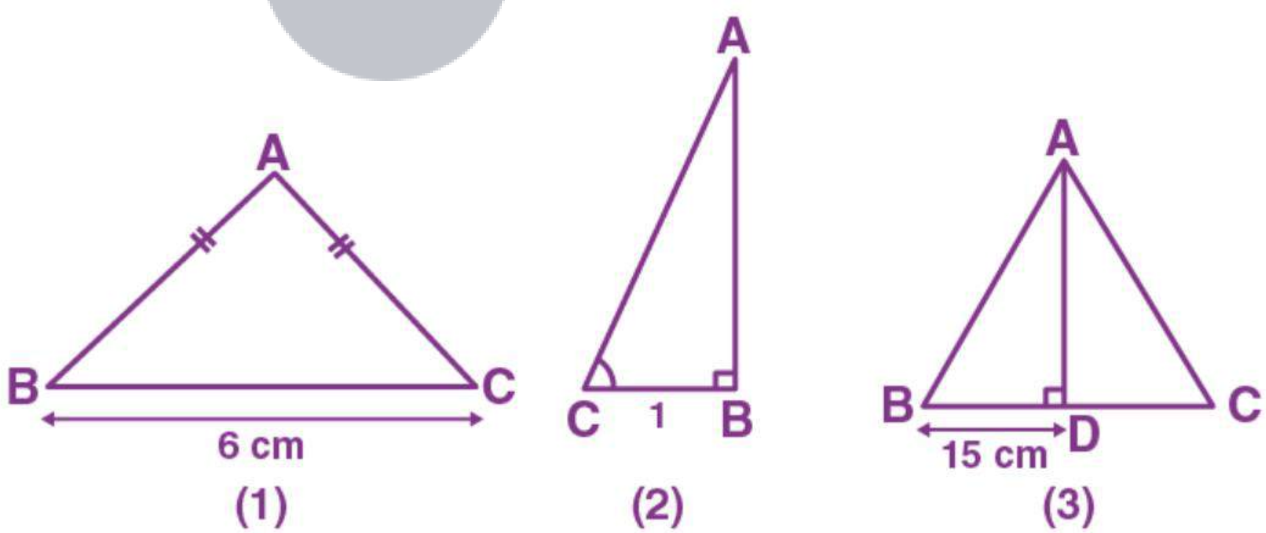
= BD / AB

= 9/15

= 3/5

(ii) sin ∠ACB = sin ∠ACD
= perpendicular/ Hypotenuse
= AD/AC
= 12/15
= 4/5

8. (a) In the figure (1) given below, ΔABC is isosceles with AB = AC = 5 cm and BC = 6 cm. Find
(i) sin C
(ii) tan B
(iii) tan C – cot B.
(b) In the figure (2) given below, ΔABC is right-angled at B. Given that ∠ACB = θ, side AB = 2 units and side BC = 1 unit, find the value of $\sin^2 \theta + \tan^2 \theta$.
(c) In the figure (3) given below, AD is perpendicular to BC, BD = 15 cm, sin B = 4/5 and tan C = 1.
(i) Calculate the lengths of AD, AB, DC and AC.
(ii) Show that $\tan^2 B - 1/\cos^2 B = -1$.



Solution:

(a) It is given that
ΔABC is isosceles with AB = AC = 5 cm and BC = 6 cm
Construct AD perpendicular to BC

D is the mid point of BC
So BD = CD
Here
BD = CD = 6/2 = 3 cm

In right angled ΔABD
Using Pythagoras theorem
$AB^2 = AD^2 + BD^2$
We can write it as
$AD^2 = AB^2 – BD^2$
Substituting the values
$AD^2 = 5^2 – 3^2$
By further calculation
$AD^2 = 25 – 9 = 16$
So we get
$AD^2 = 4^2$
AD = 4 cm

(i) In right angled ΔACD
sin C = perpendicular/hypotenuse
sin C = AD/AC = 4/5

(ii) In right angled ΔABD
tan B = perpendicular/base
tan B = AD/BD = 4/3

(iii) In right angled ΔACD
tan C = perpendicular/base
tan C = AD/CD = 4/3

In right angled ΔABD
cot B = base/perpendicular
cot B = BD/AD = ¾

Here
tan C – cot B = 4/3 – ¾
Taking LCM

tan C – cot B = (16 – 9)/ 12 = 7/12

(b) It is given that
ΔABC is right-angled at B
AB = 2 units and BC = 1 unit

In right angled ΔABC
Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
Substituting the values
$AC^2 = 2^2 + 1^2$
$AC^2 = 4 + 1 = 5$
So we get
$AC^2 = 5$
AC = √5 units

In right angled ΔABC
sin θ = perpendicular/hypotenuse
sin θ = AB/AC = 2/√5

In right angled ΔABC
tan θ = perpendicular/base
tan θ = AB/BC = 2/1

We know that
$\sin^2 \theta + \tan^2 \theta = (2/\sqrt{5})^2 + (2/1)^2$
By further calculation
= 4/5 + 4/1
Taking LCM
= (4 + 20)/ 5
= 24/5
= 4 4/5

(c) (i) In ΔABC
AD is perpendicular to BC
BD = 15 cm
sin B = 4/5

tan C = 1

In ΔABD
sin B = perpendicular/hypotenuse
sin B = AD/AB = 4/5

Consider AD = 4x and AB = 5x
Using Pythagoras theorem
In right angled ΔABD
$AB^2 = AD^2 + BD^2$
We can write it as
$BD^2 = AB^2 – AD^2$
Substituting the values
$(15)^2 = (5x)^2 – (4x)^2$
$225 = 25x^2 – 16x^2$
By further calculation
$225 = 9x^2$
$x^2 = 225/9 = 25$
So we get
$x = \sqrt{25} = 5$

Here
AD = 4 × 5 = 20
AB = 5 × 5 = 25

In right angled ΔACD
tan C = perpendicular/base
So we get
tan C = AD/CD = 1/1

Consider AD = X then CD = x
In right angled ΔADC
Using Pythagoras theorem
$AC^2 = AD^2 + CD^2$
Substituting the values
$AC^2 = x^2 + x^2$ …..(1)
So the equation becomes

$AC^2 = 20^2 + 20^2$
$AC^2 = 400 + 400 = 800$
So we get
$AC = \sqrt{800} = 20\sqrt{2}$

Length of AD = 20 cm
Length of AB = 25 cm
Length of DC = 20 cm
Length of AC = $20\sqrt{2}$ cm

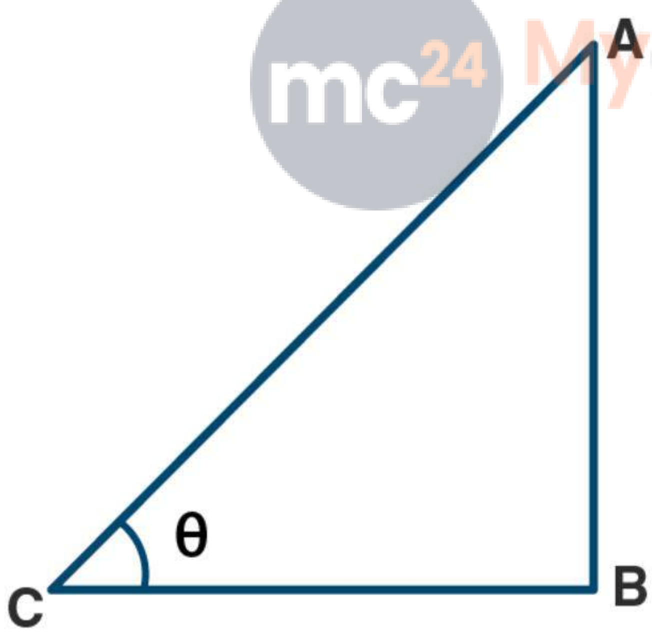
(ii) In right angled ΔABD
tan B = perpendicular/base
So we get
tan B = AD/BD
Substituting the values
tan B = 20/15 = 4/3

In right angled ΔABD
cos B = base/hypotenuse
So we get
cos B = BD/AB
Substituting the values
cos B = 15/25 = 3/5

Here
LHS = $\tan^2 B - 1/\cos^2 B$
Substituting the values
$= (4/3)^2 - 1/(3/5)^2$
By further calculation
$= (4)^2/(3)^2 - (5)^2/(3)^2$
= 16/9 – 25/9
So we get
= (16 – 25)/9
= -9/9
= - 1
= RHS

Hence, proved.

9. If sin θ =3/5 and θ is acute angle, find
(i) cos θ
(ii) tan θ.
Solution:
Let Δ ABC be a right angled at B
Let ∠ACB = θ
Given that, sin θ = 3/5
AB/AC = 3/5
Let AB = 3x
then AC = 5x
In right angled Δ ABC,
By Pythagoras theorem,
We get



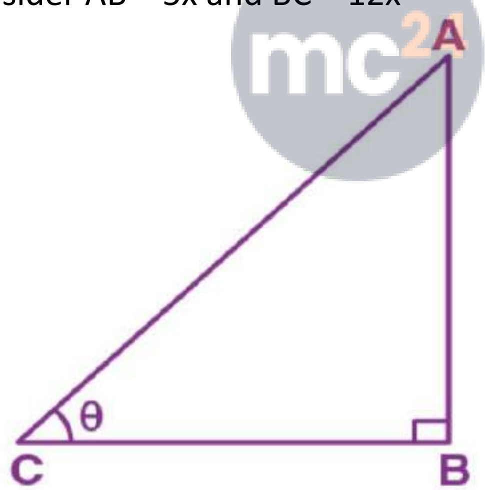
$(5x)^2 = (3x)^2 + BC^2$
$BC^2 = (5x)^2 – (3x)^2$
$BC^2 = (2x)^2$
BC = 4x

(i) cos θ = Base/ Hypotenuse
= BC / AC
= 4x /5x
= 4/5

(ii) tan θ = perpendicular/Base
= AB/BC
= 3x/4x
= ¾

10. Given that tan θ = 5/12 and θ is an acute angle, find sin θ and cos θ.
Solution:

Consider Δ ABC be right angled at B and ∠ACB = θ
It is given that
tan θ = 5/12
AB/BC = 5/12
Consider AB = 5x and BC = 12x



In right angled Δ ABC
Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
Substituting the values
$AC^2 = (5x)^2 + (12x)^2$
By further calculation
$AC^2 = 25x^2 + 144x^2 = 169x^2$
So we get

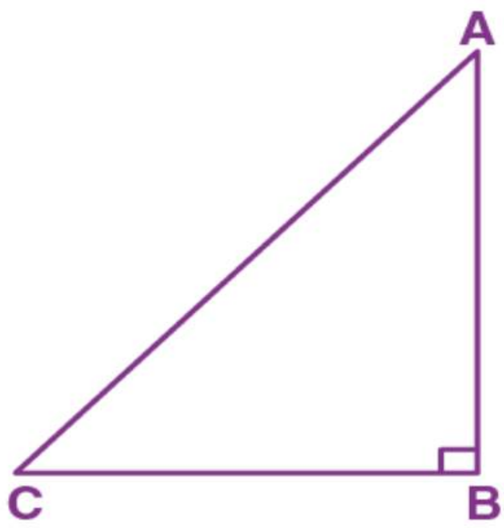
$AC^2 = (13x)^2$
AC = 13x

In right angled Δ ABC
sin θ = perpendicular/hypotenuse
So we get
sin θ = AB/AC = 5x/13x = 5/13

In right angled Δ ABC
cos θ = base/hypotenuse
So we get
cos θ = BC/AC
Substituting the values
cos θ = 12x/13x = 12/13

11. If sin θ = 6/10, find the value of cos θ + tan θ.
Solution:

Consider Δ ABC be right angled at B and ∠ACB = θ
It is given that
sin θ = AB/AC
sin θ = 6/10
Take AB = 6x then AC = 10x



In right angled Δ ABC
Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$
Substituting the values
$(10x)^2 = (6x)^2 + BC^2$
By further calculation
$BC^2 = 100x^2 – 36x^2 = 64x^2$
So we get
$BC^2 = (8x)^2$
BC = 8x

In right angled Δ ABC
cos θ = base/hypotenuse
cos θ = BC/AC
Substituting the values
cos θ = 8x/10x = 4/5

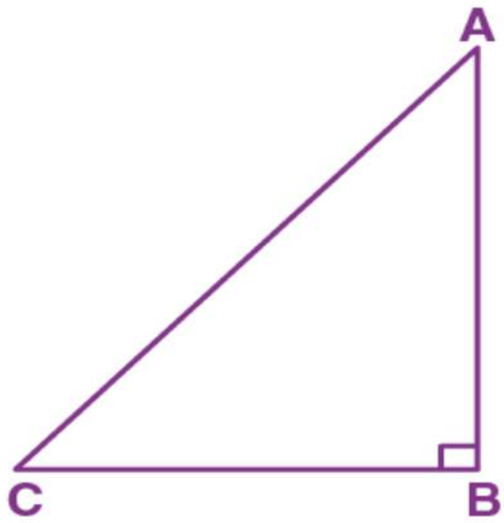
In right angled Δ ABC
tan θ = perpendicular/base
tan θ = AB/BC
Substituting the values
tan θ = 6x/8x = ¾

Here
cos θ + tan θ = 4/5 + ¾
Taking LCM
= (16 + 15)/ 20
= 31/20
= 1 11/20

12. If tan = 4/3, find the value of sin θ + cos θ (both sin θ and cos θ are positive).
Solution:

Let ΔABC be a right angled
∠ACB = θ
Given that, tan θ = 4/3
(AB/BC = 4/3)
Give that, tan θ = 4/3
(AB/BC = 4/3)

Let AB = 4x,
then BC = 3x



In right angled ΔABC
By Pythagoras theorem, we get
$AC^2 = AB^2 + BC^2$
$AC^2 = AB^2 + BC^2$
$AC^2 = AB^2 + BC^2$
$AC^2 = AB^2 + BC^2$
$(AC^2 = (4x)^2 + (3x)^2$
$AC^2 = 16x^2 + 9x^2$
$AC^2 = 25x^2$
$AC^2 = (5x)^2$
AC = 5x
Sin θ = perpendicular/Hypotenuse
= AB/AC
= 4x/5x
= 4/5
Cos θ = Base/Hypotenuse
= BC/AC
= 3x/5x
= 3/5
Sin θ + cos θ
= 4/5 + 3/5
= (4 + 3)/5
= 7/5

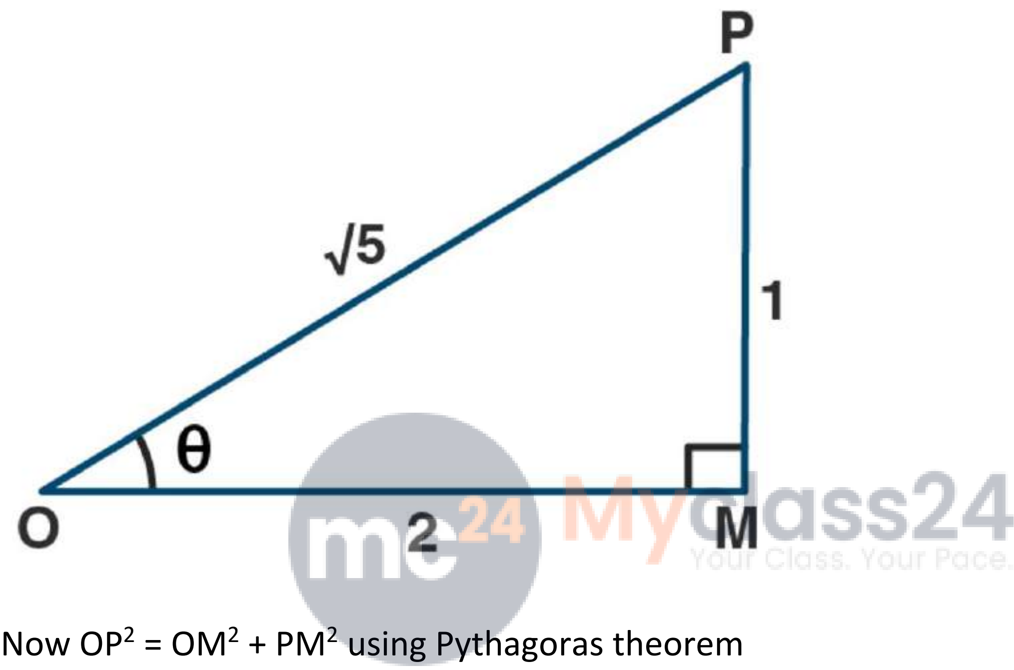
Hence, Sin θ + cos θ = 7/5 = 1 2/5

13. 1f cosec = √5 and θ is less than 90^0, find the value of cot θ – cos θ.

Solution:

Given cosec θ = √5/1 = OP/PM

OP = √5 and PM = 1



Now $OP^2 = OM^2 + PM^2$ using Pythagoras theorem

$(\sqrt{5})^2 = OM^2 + 1^2$

$5 = OM^2 + 1$

$OM^2 = 5 – 1$

$OM^2 = 4$

OM = 2

Now cot θ = OM/PM

= 2/1

= 2

Cos θ = OM/OP

= 2/√5

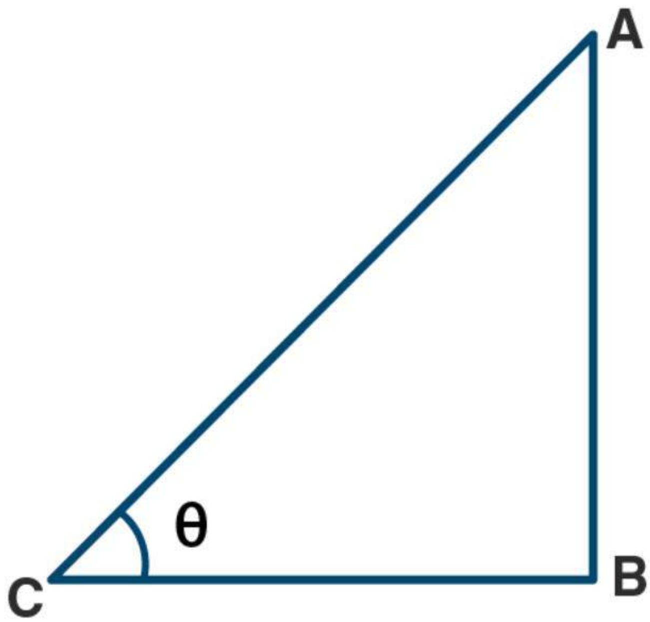
Now cot θ - Cos θ = 2 – (2/√5)

= 2 (√5 – 1)/ √5

14. Given sin θ = p/q, find cos θ + sin θ in terms of p and q.

Solution:

Given that sin θ = p/q



Which implies,

AB/AC = p/q

Let AB = px

And then AC = qx

In right angled triangle ABC

By Pythagoras theorem,

We get

$AC^2 = AB^2 + BC^2$

$BC^2 = AC^2 – AB^2$

$BC^2 = q^2x^2 – p^2x^2$

$BC^2 = (q^2 – p^2)x^2$

$BC = \sqrt{(q^2 – p^2)}x$

In right angled triangle ABC,

Cos θ = base/ hypotenuse

= BC/AC

$= \sqrt{(q^2 – p^2)}x/qx$

$= \sqrt{(q^2 – p^2)}/ q$

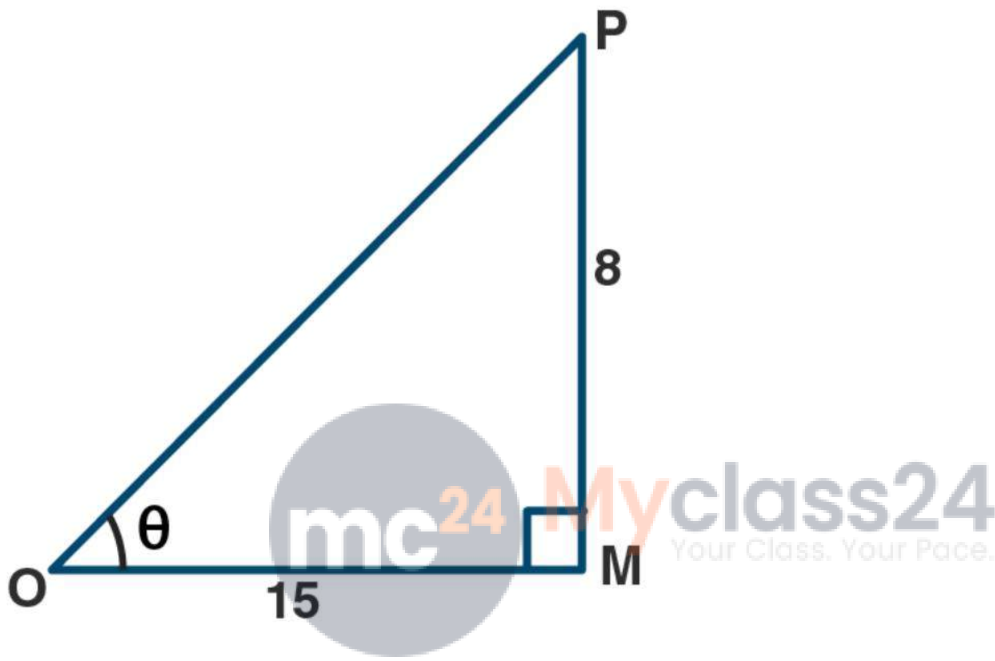
Now,

$\sin θ + \cos θ = p/q + \sqrt{(q^2 – p^2)}/ q$

$= [p + \sqrt{(q^2 – p^2)}]/ q$

15. If θ is an acute angle and tan = 8/15, find the value of sec θ + cosec θ.

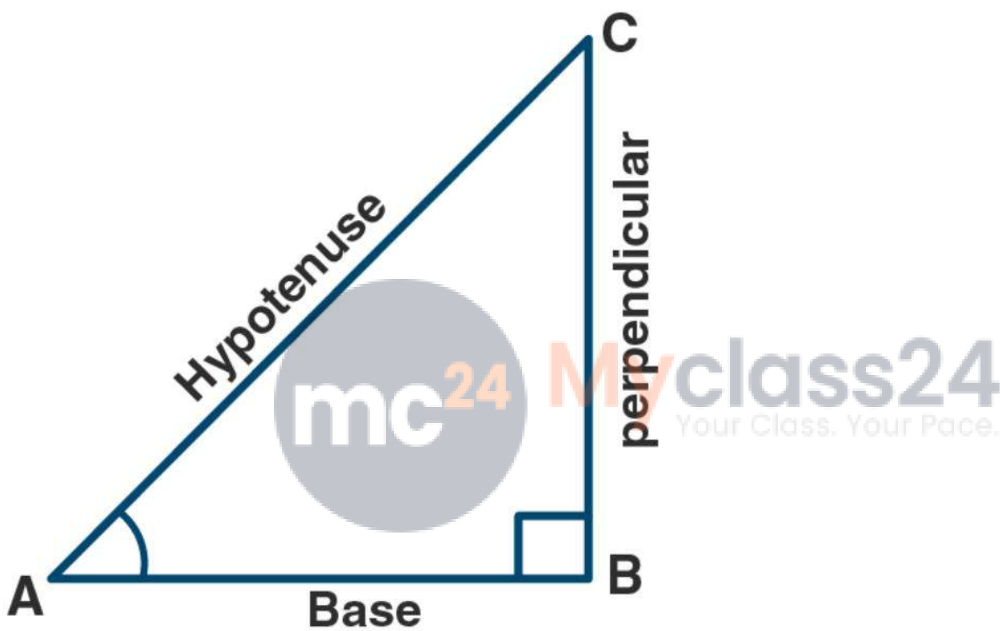
Solution:
Given tan θ = 8/15
θ is an acute angle
in the figure triangle OMP is a right angled triangle,
∠M = 90° and ∠Q = θ



Tan θ = PM/OL = 8/15
Therefore, PM = 8, OM = 15
But $OP^2 = OM^2 + PM^2$ using Pythagoras theorem,
$= 15^2 + 8^2$
= 225 + 64
= 289
$= 17^2$
Therefore, OP = 17
Sec θ = OP/OM
= 17/15
Cosec θ = OP/PM
= 17/8
Now,
Sec θ + cosec θ = (17/15) + (17/8)
= (136 + 255)/ 120

= 391/120
= 3 31/120

16. Given A is an acute angle and 13 sin A = 5, evaluate:
(5 sin A – 2 cos A)/ tan A.
Solution:
Let triangle ABC be a right angled triangle at B and A is an acute angle
Given that 13 sin A = 5
Sin A = 5/13
AB/Ac = 5/13



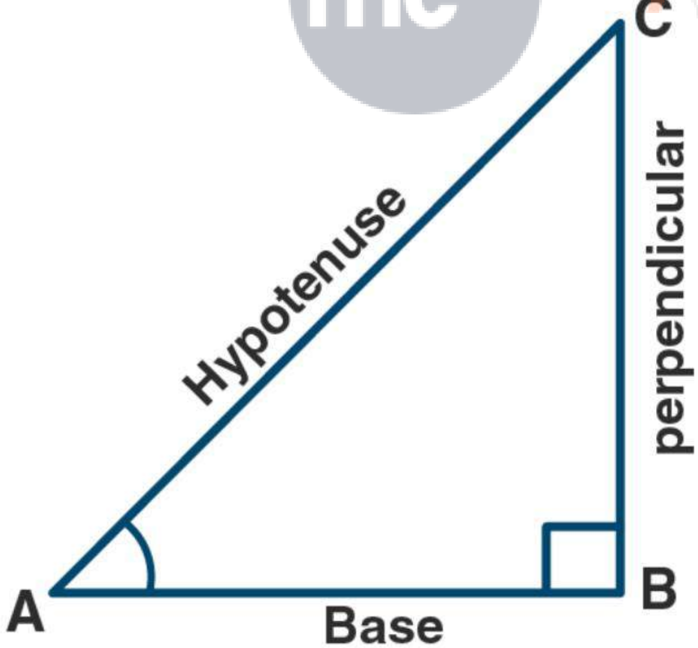
Let AB = 5x
AC = 13 x
In right angled triangle ABC,
Using Pythagoras theorem,
We get
$AC^2 = AB^2 + BC^2$
$BC^2 = AC^2 – BC^2$
$BC^2 = (13x)^2 – (5x)^2$
$BC^2 = 169x^2 – 25x^2$
$BC^2 = 144x^2$
BC = 12x
Sin A = 5/13

Cos A = base/ hypotenuse
= BC/AC
= 12x/ 13x
= 12/13
Tan A = perpendicular/ base
= AB/BC
= 5x/ 12x
= 5/ 12
Now,
(5 sin A – 2 cos A)/ tan A = [(5) (5/13) – (2) (12/13)]/ (5/12)
= (1/13)/ (5/12)
= 12/65
Hence (5 sin A – 2 cos A)/ tan A = 12/65

17. Given A is an acute angle and cosec A = √2, find the value of $(2 \sin^2 A + 3 \cot^2 A)/ (\tan^2 A – \cos^2 A)$.

Solution:

Let triangle ABC be a right angled at B and A is a acute angle.
Given that cosec A = √2

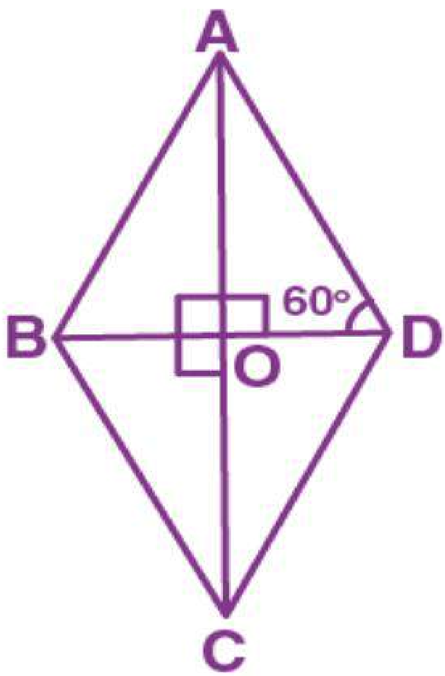


Which implies,
AC/BC = √2/1

Let AC = √2x
Then BC = x
In right angled triangle ABC
By using Pythagoras theorem,
We get
$AC^2 = AB^2 + BC^2$
$(\sqrt{2}x)^2 = AB^2 + x^2$
$AB^2 = 2x^2 - x^2$
AB = x
Sin A = perpendicular/ hypotenuse
= BC/AC
= 1/ √2
Cot A = base/ perpendicular
= x/x
= 1
Tan A = perpendicular/ base
= BC/AB
= x/x
= 1
Cos A = base/ hypotenuse
= AB/AC
= x/ √2x
= 1/√2
Substituting these values we get
$2 \sin^2 A + 3 \cot^2 A/ (\tan^2 A - \cos^2 A) = 8$

18. The diagonals AC and BD of a rhombus ABCD meet at O. If AC = 8 cm and BD = 6 cm, find sin ∠OCD.
Solution:

It is given that
Diagonals AC and BD of rhombus ABCD meet at O
AC = 8 cm and BD = 6 cm
O is the mid point of AC



We know that

AO = OC = AC/2 = 8/2 = 4 cm

O is the mid point of BD

BO = OD = BD/2 = 6/2 = 3 cm

In right angled ΔCOD

$CD^2 = OC^2 + OD^2$

Substituting the values

$CD^2 = 4^2 + 3^2$

So we get

$CD^2 = 16 + 9 = 25$

$CD^2 = 5^2$

CD = 5 cm

In right angled ΔCOD

sin ∠OCD = perpendicular/ hypotenuse

So we get

sin ∠OCD = OD/CD = 3/5

19. If tan θ = 5/12, find the value of (cos θ + sin θ)/ (cos θ – sin θ).

Solution:

Consider ΔABC be right angled at B and ∠ACB = θ
It is given that
tan θ = AB/BC = 5/12

Take AB = 5x then BC = 12x
In right angled ΔABC,
Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
Substituting the values
$AC^2 = (5x)^2 + (12x)^2$
By further calculation
$AC^2 = 25x^2 + 144x^2 = 169x^2$
So we get
$AC^2 = (13x)^2$
AC = 13x
In right angled ΔABC
cos θ = base/hypotenuse
cos θ = BC/AC
Substituting the values
cos θ = 12x/13x = 12/13

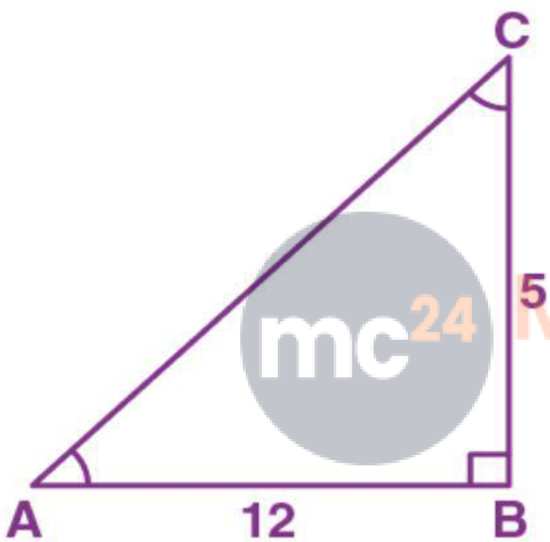
In right angled ΔABC
sin θ = perpendicular/hypotenuse
sin θ = AB/AC
Substituting the values
sin θ = 5x/13x = 5/13

Here
(cos θ + sin θ)/ (cos θ – sin θ) = [12/13 + 5/13]/ [12/13 – 5/13]
Taking LCM
= [(12 + 5)/ 13]/[(12 – 5)/ 13]
So we get
= 17/13/ 7/13
= 17/13 × 13/7
= 17/7

Therefore, (cos θ + sin θ)/ (cos θ – sin θ) = 17/7 = 2 3/7.

20. Given 5 cos A – 12 sin A = 0, find the value of (sin A + cos A)/ (2 cos A – sin A).
Solution:

It is given that
5 cos A – 12 sin A = 0
We can write it as
5 cos A = 12 sin A
So we get
sin A/ cos A = 5/12
We know that sin A/ cos A = tan A
tan A = 5/12



Consider ΔABC right angled at B and ∠A is acute angle
Here
tan A = BC/AB = 5/12
Take BC = 5x then AB = 12x
In right angled ΔABC
Using Pythagoras theorem
$AC^2 = BC^2 + AB^2$
Substituting the values
$AC^2 = (5x)^2 + (12x)^2$
$AC^2 = 25x^2 + 144x^2 = 169x^2$
So we get
$AC^2 = (13x)^2$
AC = 13x

In right angled ΔABC
sin A = perpendicular/hypotenuse
So we get
sin A = BC/AC = 5x/13x = 5/13

In right angled ΔABC
cos A = base/hypotenuse
So we get
cos A = AB/AC = 12x/13x = 12/13

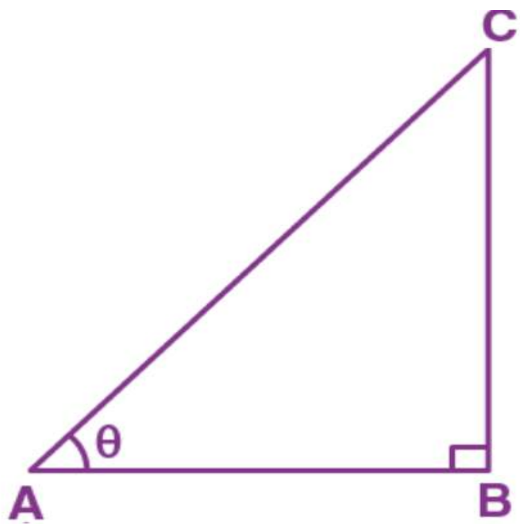
Here
(sin A + cos A)/ (2 cos A – sin A) = [5/13 + 12/13]/ [2 × 12/13 – 5/13]
By further calculation
= [(5 + 12)/13]/ [24/13 – 5/13]
So we get
= [(5 + 12)/13]/[(24 – 5)/13]
= 17/13/ 19/13
= 17/13 × 13/19
= 17/19

Therefore, (sin A + cos A)/ (2 cos A – sin A) = 17/19

21. If tan θ = p/q, find the value of (p sin θ – q cos θ)/ (p sin θ + q cos θ).
Solution:

It is given that
tan θ = p/q
Consider ΔABC be right angled at B and ∠BCA = θ
tan θ = BC/AB = p/q

BC = px then AB = qx



In right angled ΔABC

Using Pythagoras theorem

$AC^2 = BC^2 + AB^2$

Substituting the values

$AC^2 = (px)^2 + (qx)^2$

$AC^2 = p^2x^2 + q^2x^2$

$AC^2 = x^2 (p^2 + q^2)$

So we get

$AC = \sqrt{x^2 (p^2 + q^2)}$

$AC = x(\sqrt{p^2 + q^2})$

In right angled ΔABC

sin θ = perpendicular/hypotenuse

sin θ = BC/AC

Substituting the values

$\sin \theta = px/ x(\sqrt{p^2 + q^2})$

So we get

$\sin \theta = p/ (\sqrt{p^2 + q^2})$

In right angled ΔABC

cos θ = base/hypotenuse

cos θ = AB/AC

Substituting the values

$\cos \theta = qx/ x(\sqrt{p^2 + q^2})$

So we get

$\cos \theta = q/ (\sqrt{p^2 + q^2})$

Here

$$\frac{psin\Theta - qcos\Theta}{psin\Theta + qcos\Theta} = \frac{p(\frac{p}{\sqrt{p^2+q^2}}) - q(\frac{q}{\sqrt{p^2+q^2}})}{p(\frac{p}{\sqrt{p^2+q^2}}) + q(\frac{q}{\sqrt{p^2+q^2}})}$$

By further calculation

$$= \frac{\frac{p^2}{\sqrt{p^2+q^2}} - \frac{q^2}{\sqrt{p^2+q^2}}}{\frac{p^2}{\sqrt{p^2+q^2}} + \frac{q^2}{\sqrt{p^2+q^2}}}$$

$$= \frac{\frac{p^2-q^2}{\sqrt{p^2+q^2}}}{\frac{p^2+q^2}{\sqrt{p^2+q^2}}}$$

So we get

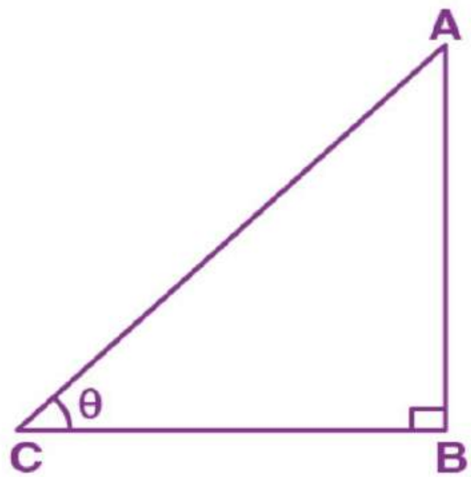
$$= \frac{p^2 - q^2}{\sqrt{p^2 + q^2}} \times \frac{\sqrt{p^2 + q^2}}{p^2 + q^2}$$

$$= \frac{p^2 - q^2}{p^2 + q^2}$$

$$Therefore,\ \frac{psin\Theta - qcos\Theta}{psin\Theta + qcos\Theta} = \frac{p^2 - q^2}{p^2 + q^2}$$

22. If 3 cot θ = 4, find the value of (5 sin θ – 3 cos θ)/ (5 sin θ + 3 cos θ).
Solution:

It is given that
3 cot θ = 4
cot θ = 4/3



Consider ΔABC be right angled at B and ∠ACB = θ

cot θ = BC/AB = 4/3

Take BC = 4x then AB = 3x

In right angled ΔABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = (3x)^2 + (4x)^2$

$AC^2 = 9x^2 + 16x^2 = 25x^2$

So we get

$AC^2 = (5x)^2$

AC = 5x

In right angled ΔABC

sin θ = perpendicular/hypotenuse

sin θ = AB/AC

Substituting the values

sin θ = 3x/5x = 3/5

In right angled ΔABC

cos θ = base/hypotenuse

cos θ = BC/AC

Substituting the values

cos θ = 4x/5x = 4/5

$$\frac{5sin\Theta - 3cos\Theta}{5sin\Theta + 3cos\Theta} = \frac{5 \times \frac{3}{5} - 3 \times \frac{4}{5}}{5 \times \frac{3}{5} + 3 \times \frac{4}{5}}$$

By further calculation

$$= \frac{\frac{15}{5} - \frac{12}{5}}{\frac{15}{5} + \frac{12}{5}}$$

$$= \frac{\frac{15-12}{5}}{\frac{15+12}{5}}$$

So we get

$$= \frac{\frac{3}{5}}{\frac{27}{5}}$$

$$= \frac{3}{5} \times \frac{5}{27}$$

$$= \frac{3}{27}$$

$$= \frac{1}{9}$$

$$Therefore,\ \frac{5sin\Theta - 3cos\Theta}{5sin\Theta + 3cos\Theta} = \frac{1}{9}$$

23. (i) If 5 cos θ – 12 sin θ = 0, find the value of (sin θ + cos θ)/(2 cos θ - sin θ).
(ii) If cosec θ = 13/12, find the value of (2 sin θ – 3 cos θ)/(4 sin θ – 9 cos θ).
Solution:

(i) It is given that
5 cos θ – 12 sin θ = 0
We can write it as
5 cos θ = 12 sin θ
sin θ/ cos θ = 5/12
tan θ = 5/12

Dividing both numerator and denominator by cos θ

$$\frac{sin\Theta + cos\Theta}{2cos\Theta - sin\Theta} = \frac{\frac{sin\Theta}{cos\Theta} + \frac{cos\Theta}{cos\Theta}}{\frac{2cos\Theta}{cos\Theta} - \frac{sin\Theta}{cos\Theta}}$$

$$= \frac{tan\Theta + 1}{2 - tan\Theta}$$

Substituting the values

$$= \frac{\frac{5}{12} + 1}{2 - \frac{5}{12}}$$

Taking LCM

$$= \frac{\frac{5+12}{12}}{\frac{24-5}{12}}$$

$$= \frac{\frac{17}{12}}{\frac{19}{12}}$$

So we get

$$= \frac{17}{12} \times \frac{12}{19}$$

$$= \frac{17}{19}$$

(ii) It is given that

cosec θ = 13/12

We know that cosec θ = 1/ sin θ

1/sin θ = 13/12

sin θ = 12/13

Here $\cos^2 \theta = 1 - \sin^2 \theta$

Substituting the values

$= 1 - (12/13)^2$

By further calculation

= 1 – 144/169

Taking LCM

= (169 – 144)/ 169

= 25/169

So we get

$= (5/13)^2$

cos θ = 5/13

Here

$$\frac{2sin\Theta - 3cos\Theta}{4sin\Theta - 9cos\Theta} = \frac{2(\frac{12}{13}) - 3(\frac{5}{13})}{4(\frac{12}{13}) - 9(\frac{5}{13})}$$

$By\ further\ calculation$

$$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}}$$

$So\ we\ get$

$$= \frac{\frac{24-15}{13}}{\frac{48-45}{13}}$$

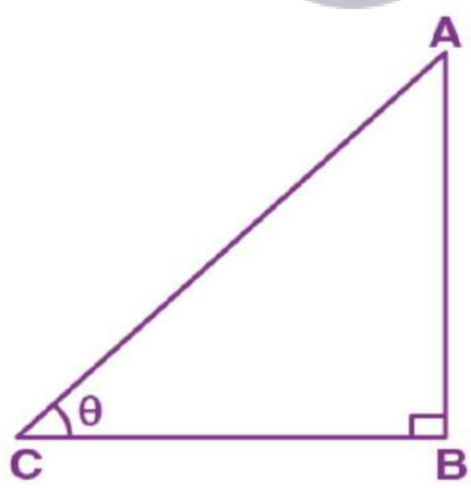
$$= \frac{\frac{9}{13}}{\frac{3}{13}}$$

$$= \frac{9}{13} \times \frac{13}{3}$$

$$= 3$$

24. If 5 sin θ = 3, find the value of (sec θ – tan θ)/ (sec θ + tan θ).

Solution:



Consider ΔABC be right angled at B and ∠ACB = θ

It is given that

5 sin θ = 3

sin θ = AB/AC = 3/5

Take AB = 3x then AC = 5x
In right angled ΔABC
Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
$BC^2 = AC^2 – AB^2$
Substituting the values
$BC^2 = (5x)^2 – (3x)^2$
So we get
$BC^2 = 25x^2 – 9x^2 = 16x^2$
$BC^2 = (4x)^2$
BC = 4x

In right angled ΔABC
sec θ = hypotenuse/base
sec θ = AC/BC = 5x/4x = 5/4

In right angled ΔABC
tan θ = perpendicular/base
tan θ = AB/BC = 3x/4x = ¾

$$\frac{sec\Theta - tan\Theta}{sec\Theta + tan\Theta} = \frac{\frac{5}{4} - \frac{3}{4}}{\frac{5}{4} + \frac{3}{4}}$$

By further calculation

$$= \frac{\frac{5-3}{4}}{\frac{5+3}{4}}$$

So we get

$$= \frac{\frac{2}{4}}{\frac{8}{4}}$$

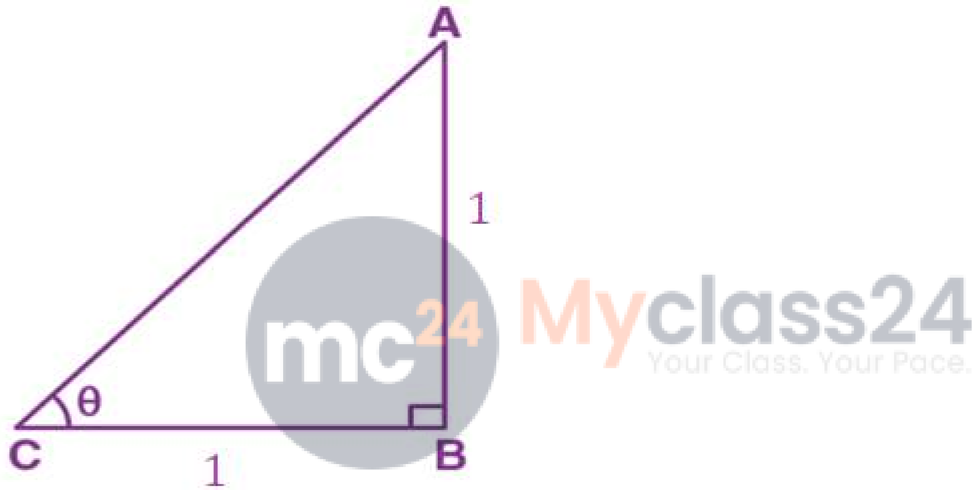
$$= \frac{2}{4} \times \frac{4}{8}$$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

$$Therefore, \frac{sec\Theta - tan\Theta}{sec\Theta + tan\Theta} = \frac{1}{4}$$

25. If θ is an acute angle and sin θ = cos θ, find the value of 2 $\tan^2$ θ + $\sin^2$ θ – 1.
Solution:



Consider ΔABC be right angled at B and ∠ACB = θ
It is given that
sin θ = cos θ
sin θ/cos θ = 1
tan θ = AB/BC = 1

Take AB = x then BC = x
In right angled ΔABC
Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
$AC^2 = x^2 + x^2 = 2x^2$
So we get
$AC = \sqrt{2x^2}$
$AC = (\sqrt{2})x$

In right angled ΔABC
sin θ = perpendicular/hypotenuse
sin θ = AB/AC = x/√2x = 1/√2

Here
$2 \tan^2 \theta + \sin^2 \theta - 1 = 2 \times (1)^2 + (1/\sqrt{2})^2 - 1$
By further calculation
= 2 × 1 + ½ - 1
= 2 + ½ - 1
= 1+ ½
Taking LCM
= (2 + 1)/2
= 3/2

Therefore, $2 \tan^2 \theta + \sin^2 \theta - 1 = 3/2$.

26. Prove the following:
(i) cos θ tan θ = sin θ
(ii) sin θ cot θ = cos θ
(iii) $\sin^2 \theta / \cos \theta + \cos \theta = 1/ \cos \theta$.
Solution:

(i) cos θ tan θ = sin θ
LHS = cos θ tan θ
We know that tan θ = sin θ/cos θ
= cos θ (sin θ/cos θ)
So we get
= 1 × sin θ/1
= sin θ
= RHS

Therefore, LHS = RHS.

(ii) sin θ cot θ = cos θ
LHS = sin θ cot θ
We know that cot θ = cos θ/sin θ
= sin θ (cos θ/sin θ)

= 1 × cos θ/1
= cos θ
= RHS

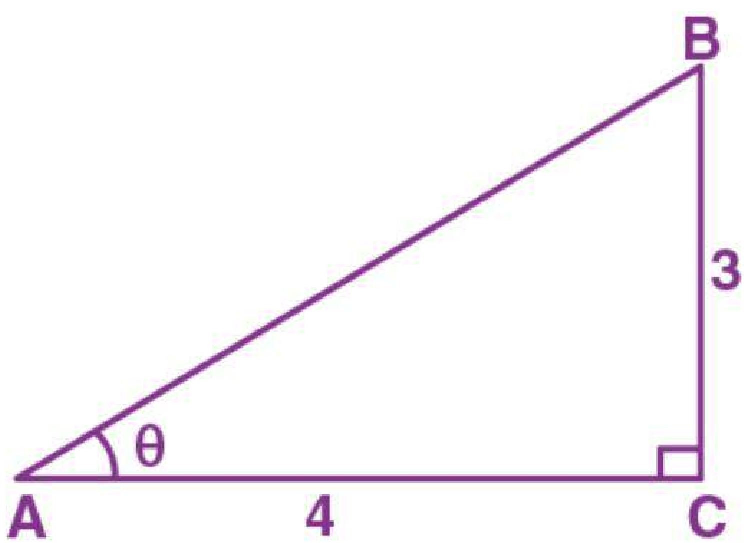
Therefore, LHS = RHS.

(iii) $\sin^2 \theta/\cos \theta + \cos \theta = 1/\cos \theta$
LHS = $\sin^2 \theta/\cos \theta + \cos \theta/1$
Taking LCM
= $(\sin^2 \theta + \cos^2 \theta)/\cos \theta$
We know that $\sin^2 \theta + \cos^2 \theta = 1$
= 1/cos θ
= RHS

Therefore, LHS = RHS.

27. If in ΔABC, ∠C = 90^0 and tan A = ¾, prove that sin A cos B + cos A sin B = 1.
Solution:

It is given that
tan A = BC/AC = ¾



Using Pythagoras theorem
$AB^2 = AC^2 + BC^2$
Substituting the values

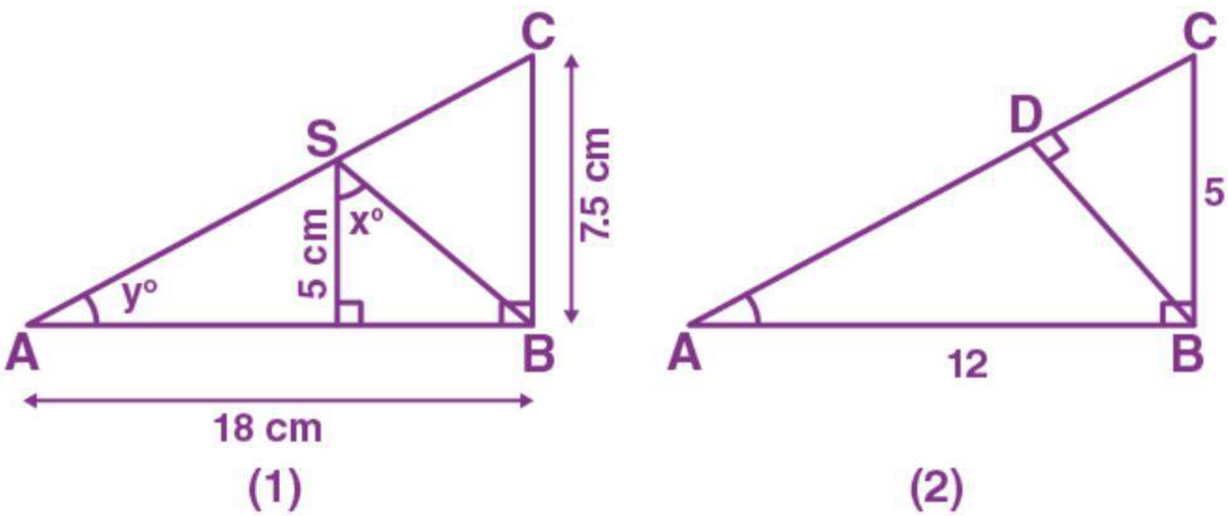
$= 4^2 + 3^2$
= 16 + 9
= 25
$= 5^2$
So we get AB = 5

Here
sin A = BC/AC = 3/5
cos A = AC/AB = 4/5
cos B = BC/AB = 3/5
sin B = AC/AB = 4/5

LHS = sin A cos B + cos A sin B
Substituting the values
= 3/5 × 3/5 + 4/5 × 4/5
By further calculation
= 9/25+ 16/25
= (9 + 16)/ 25
= 25/25
= 1
= RHS

Therefore, LHS = RHS.

28. (a) In figure (1) given below, ΔABC is right-angled at B and ΔBRS is right-angled at R. If AB = 18 cm, BC = 7.5 cm, RS = 5 cm, ∠BSR = x^0 and ∠SAB = y^0, then find:
(i) tan x^0
(ii) sin y^0.
(b) In the figure (2) given below, ΔABC is right angled at B and BD is perpendicular to AC. Find
(i) cos ∠CBD
(ii) cot ∠ABD.



Solution:

(a) ΔABC is right-angled at B, ΔBSC is right-angled at S and ΔBRS is right-angled at R

It is given that

AB = 18 cm, BC = 7.5 cm, RS = 5 cm, ∠BSR = x^0 and ∠SAB = y^0

By Geometry ΔARS and ΔABC are similar

AR/AB = RS/BC

Substituting the values

AR/18 = 5/7.5

By further calculation

AR = (5 × 18)/7.5 = (1 × 18)/1.5

Multiply both numerator and denominator by 10

AR = (18 × 10)/15

AR = (10 × 6)/5

AR = (2 × 6)/1 = 12

So we get

RB = AB – AR

RB = 18 – 12 = 6

In right angled ΔABC

Using Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Substituting the values

$AC^2 = 18^2 + 7.5^2$
By further calculation
$AC^2 = 324 + 56.25 = 380.25$
AC = √380.25 = 19.5 cm

(i) In right angled ΔBSR
tan x^0 = perpendicular/base
tan x^0 = RB/RS = 6/5

(ii) In right angled ΔASR
sin y^0 = perpendicular/hypotenuse
Using Pythagoras theorem
$AS^2 = 12^2 + 5^2$
By further calculation
$AS^2 = 144 + 25 = 169$
AS = √169 = 13 cm

So we get
sin y^0 = RS/AS = 5/13

(b) We know that
ΔABC is right angled at B and BD is perpendicular to AC
In right angled ΔABC
Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
Substituting the values
$AC^2 = 12^2 + 5^2$
By further calculation
$AC^2 = 144 + 25 = 169$
So we get
$AC^2 = (13)^2$
AC = 13

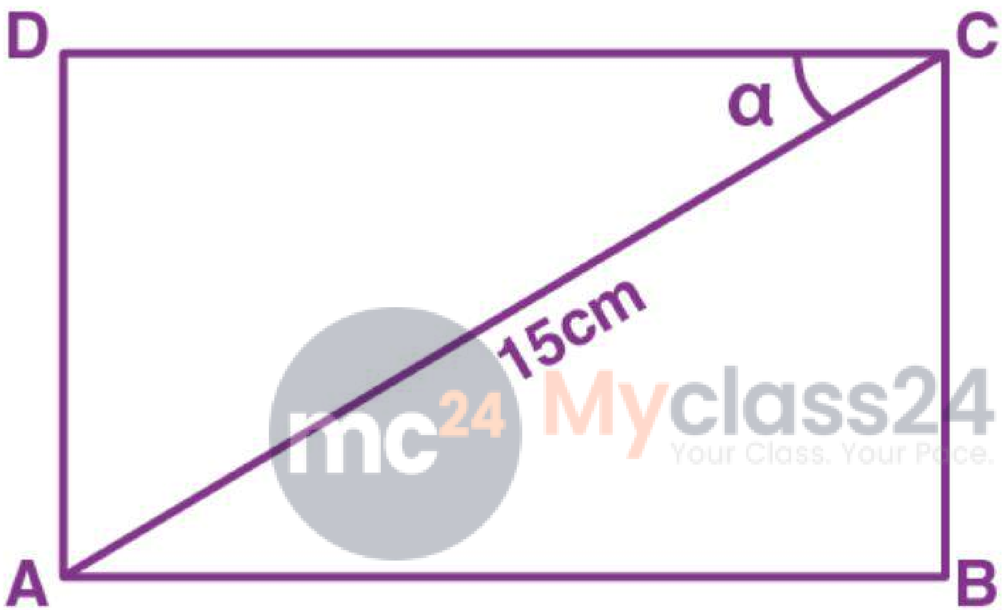
By Geometry ∠CBD = ∠A and ∠ABD = ∠C

(i) cos ∠CBD = cos ∠A = base/hypotenuse
In right angled ΔABC

cos ∠CBD = cos ∠A = AB/AC = 12/13

(ii) cos ∠ABD = cos ∠C = base/perpendicular
In right ΔABC
cos ∠ABD = cos ∠C = BC/AB = 5/12

29. In the adjoining figure, ABCD is a rectangle. Its diagonal AC = 15 cm and ∠ACD = α. If cot α = 3/2, find the perimeter and the area of the rectangle.



Solution:

In right ΔADC
cot α = CD/AD = 3/2

Take CD = 3x then AD = 2x
Using Pythagoras theorem
$AC^2 = CD^2 + AD^2$
Substituting the values
$(15)^2 = (3x)^2 + (2x)^2$
By further calculation
$13x^2 = 225$
$x^2 = 225/13$
So we get

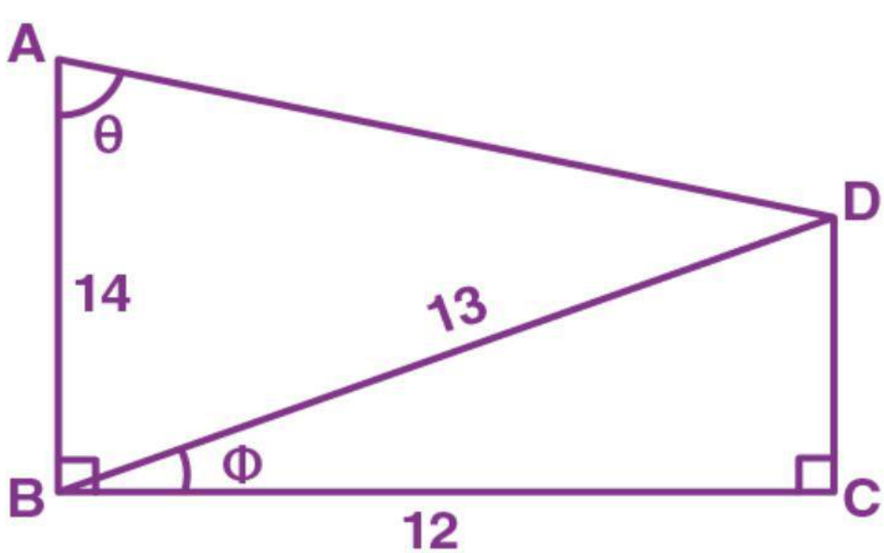
x = √225/13 = 15/√13

Length of rectangle (l) = 3x = (3 × 15)/ √13 = 45/√13 cm
Breadth of rectangle (b) = 2x = (2 × 15)/ √13 = 30/√13 cm

(i) Perimeter of rectangle = 2 (l + b)
Substituting the values of l and b
= 2 (45/√13 + 30/√13)
So we get
= 2 × 75/√13
= 150/√13 cm

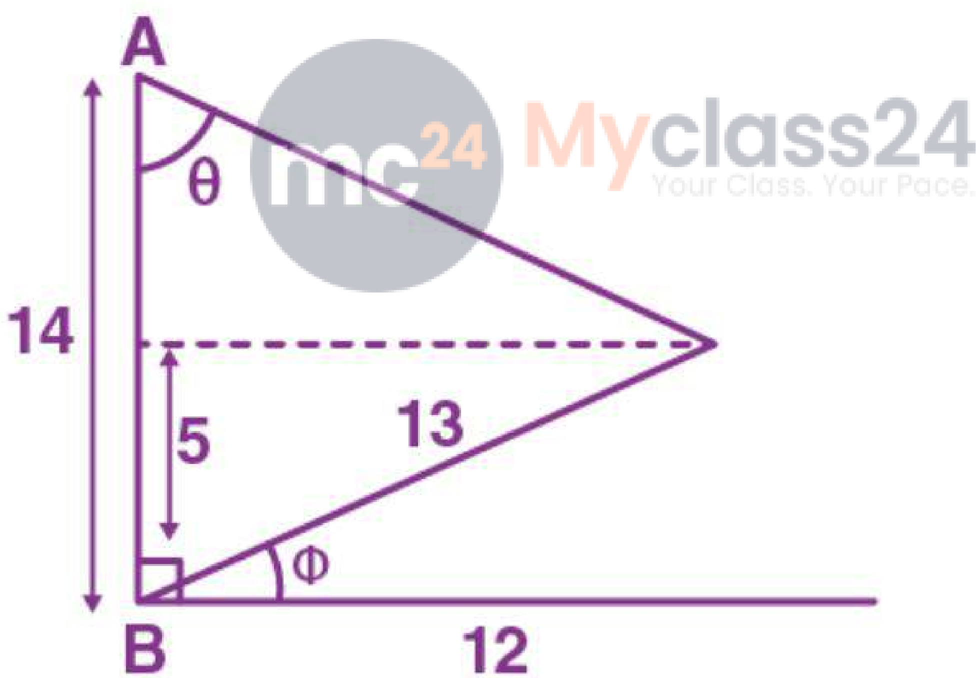
(ii) Area of rectangle = l × b
Substituting the values of l and b
= 45/√13 × 30/√13
So we get
= 1350/13
= 103 11/13 cm^2

30. Using the measurements given in the figure alongside,
(a) Find the values of:
(i) sin φ
(ii) tan θ.
(b) Write an expression for AD in terms of θ.



Solution:

From the figure
BC = 12, BD = 13
In right angled ΔBCD
Using Pythagoras theorem
$BD^2 = BC^2 + CD^2$
It can be written as
$CD^2 = BD^2 – BC^2$
Substituting the values
$CD^2 = (13)^2 – (12)^2$
$CD^2 = 169 – 144 = 25$
So we get
CD = √25 = 5



Construct BE perpendicular to AB
CD = BE = 5 and EA = AE = 14 – 5 = 9

(a) (i) sin φ = perpendicular/hypotenuse
In right angled ΔBCD
sin φ = CD/BD = 5/13

(ii) tan θ = perpendicular/hypotenuse
In right angled ΔAED
tan θ = ED/AE = BC/AE = 12/9 = 4/3 (Since ED = BC)

(b) In right angled ΔAED
sin θ = perpendicular/hypotenuse
cos θ = base/perpendicular
We can write it as
sin θ = ED/AD or cos θ = AE/AD
AD = ED/sin θ or AD = AE/ cos θ
Substituting the values
AD = 12/sin θ or AD = 9/cos θ
Therefore, AD = 12/ sin θ or AD = 9/cos θ.

31. Prove the following:
(i) $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$
(ii) $\cot^2 A - 1/\sin^2 A + 1 = 0$
(iii) $1/(1 + \tan^2 A) + 1/(1 + \cot^2 A) = 1$
Solution:

(i) $(\sin A + \cos A)^2 + (\sin A - \cos A)^2 = 2$
LHS = $(\sin A + \cos A)^2 + (\sin A - \cos A)^2$
Using the formula
$(a + b)^2 = a^2 + b^2 + 2ab$ and $(a - b)^2 = a^2 + b^2 - 2ab$
= $[(\sin A)^2 + (\cos A)^2 + 2 \sin A \cos A] + [(\sin A)^2 + (\cos A)^2 - 2 \sin A \cos A]$
By further calculation
= $\sin^2 A + \cos^2 A + 2 \sin A \cos A + \sin^2 A + \cos^2 A - 2 \sin A \cos A$
= $\sin^2 A + \cos^2 A + \sin^2 A + \cos^2 A$
= $2 \sin^2 A + 2 \cos^2 A$
We know that $\sin^2 A + \cos^2 A = 1$
= $2 (\sin^2 A + \cos^2 A)$
= 2 (1)
= 2
= RHS

Therefore, LHS = RHS.

(ii) $\cot^2 A - 1/\sin^2 A + 1 = 0$
LHS = $\cot^2 A - 1/\sin^2 A + 1$
We know that
$1/\sin A = \text{cosec } A$
= $\cot^2 A - \text{cosec}^2 A + 1$
= $(1 + \cot^2 A) - \text{cosec}^2 A$
We know that $1 + \cot^2 A = \text{cosec}^2 A$
= $\text{cosec}^2 A - \text{cosec}^2 A$
= 0
= RHS

Therefore, LHS = RHS.
(iii) $1/(1 + \tan^2 A) + 1/(1 + \cot^2 A) = 1$
LHS = $1/(1 + \tan^2 A) + 1/(1 + \cot^2 A)$
We know that
$\sec^2 A - \tan^2 A = 1$
$\sec^2 A = 1 + \tan^2 A$
$\text{cosec}^2 A - \cot^2 A = 1$
$\text{cosec}^2 A = 1 + \cot^2 A$
So we get
= $1/\sec^2 A + 1/\text{cosec}^2 A$
Here $1/\sec A = \cos A$ and $1/\text{cosec } A = \sin A$
= $\cos^2 A + \sin^2 A$
= 1
= RHS

Therefore, LHS = RHS.

32. Simplify

$$\sqrt{\frac{1 - sin^2\Theta}{1 - cos^2\Theta}}$$

Solution:

$$\sqrt{\frac{1 - sin^2\Theta}{1 - cos^2\Theta}} = \sqrt{\frac{sin^2\Theta + cos^2\Theta - sin^2\Theta}{sin^2\Theta + cos^2\Theta - cos^2\Theta}}$$

We know that $1 = \sin^2 \theta + \cos^2 \theta$

$= \sqrt{\cos^2 \theta / \sin^2 \theta}$
$= \cos \theta / \sin \theta$
Here $\cos \theta / \sin \theta = \cot \theta$
$= \cot \theta$

Therefore, $\sqrt{\frac{1 - sin^2\Theta}{1 - cos^2\Theta}}$ = cot θ.

33. If sin θ + cosec θ = 2, find the value of $\sin^2 \theta + \text{cosec}^2 \theta$.
Solution:

It is given that
sin θ + cosec θ = 2
sin θ + 1/sin θ = 2
By further calculation
$\sin^2 \theta + 1 = 2 \sin \theta$
$\sin^2 \theta - 2 \sin \theta + 1 = 0$
So we get
$(\sin \theta - 1)^2 = 0$
sin θ – 1 = 0
sin θ = 1

Here
$\sin^2 \theta + \text{cosec}^2 \theta = \sin^2 \theta + 1/\sin^2 \theta$
Substituting the values
$= 1^2 + 1/1^2$
= 1 + 1/1
= 1 + 1
= 2

34. If x = a cos θ + b sin θ and y = a sin θ – b cos θ, prove that $x^2 + y^2 = a^2 + b^2$.
Solution:

It is given that
x = a cos θ + b sin θ …. (1)
y = a sin θ – b cos θ …. (2)

By squaring and adding both the equations

$x^2 + y^2 = (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2$

Using the formula

$(a + b)^2 = a^2 + b^2 + 2ab$ and $(a - b)^2 = a^2 + b^2 - 2ab$

$= [(a \cos \theta)^2 + (b \sin \theta)^2 + 2 (a \cos \theta) (b \sin \theta)] + [(a \sin \theta)^2 + (b \cos \theta)^2 - 2 (a \sin \theta) (b \cos \theta)]$

By further calculation

$= a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2 ab \sin \theta \cos \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2 ab \sin \theta \cos \theta$

$= a^2 \cos^2 \theta + b^2 \sin^2 \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta$

So we get

$= a^2 (\cos^2 \theta + \sin^2 \theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$

Here $\sin^2 \theta + \cos^2 \theta = 1$

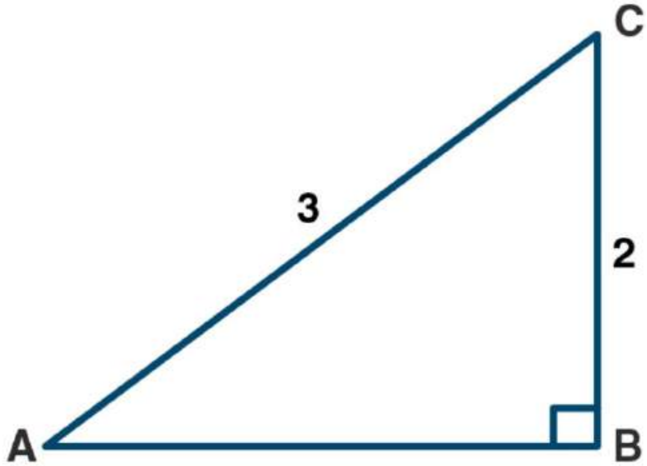
$= a^2 (1) + b^2 (1)$

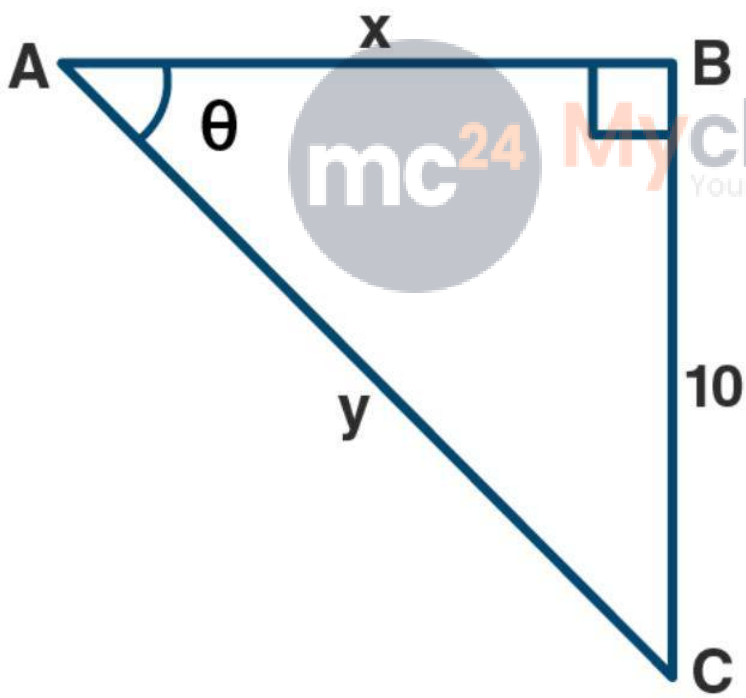
$= a^2 + b^2$

Therefore, $x^2 + y^2 = a^2 + b^2$.

CHAPTER TEST

1. (a)From the figure (i) given below, calculate all the six t-ratios for both acute………
(b)From the figure (ii) given below, find the values of x and y in terms of t-ratios





Solution:

(a) From right angled triangle ABC,
By Pythagoras theorem, we get
$AC^2 = AB^2 + BC^2$
$AB^2 = AC^2 - BC^2$

$AB^2 = (3)^2 – (2)^2$
$AB^2 = 9 – 4$
$AB^2 = 5$
AB = √5

(i) sin A = perpendicular/ hypotenuse
= BC/AC
= 2/3

(ii) cos A = base/ hypotenuse
= AB/AC
= √5/3

(iii) tan A = perpendicular/ base
= BC/AB
= 2/ √5

(iv) cot A = base/perpendicular
= AB/ BC
= √5/2

(v) sec A = hypotenuse/ base
= AC/AB
= 3/ √5

(vi) cosec A = hypotenuse/perpendicular
= AC/BC
= 3/2

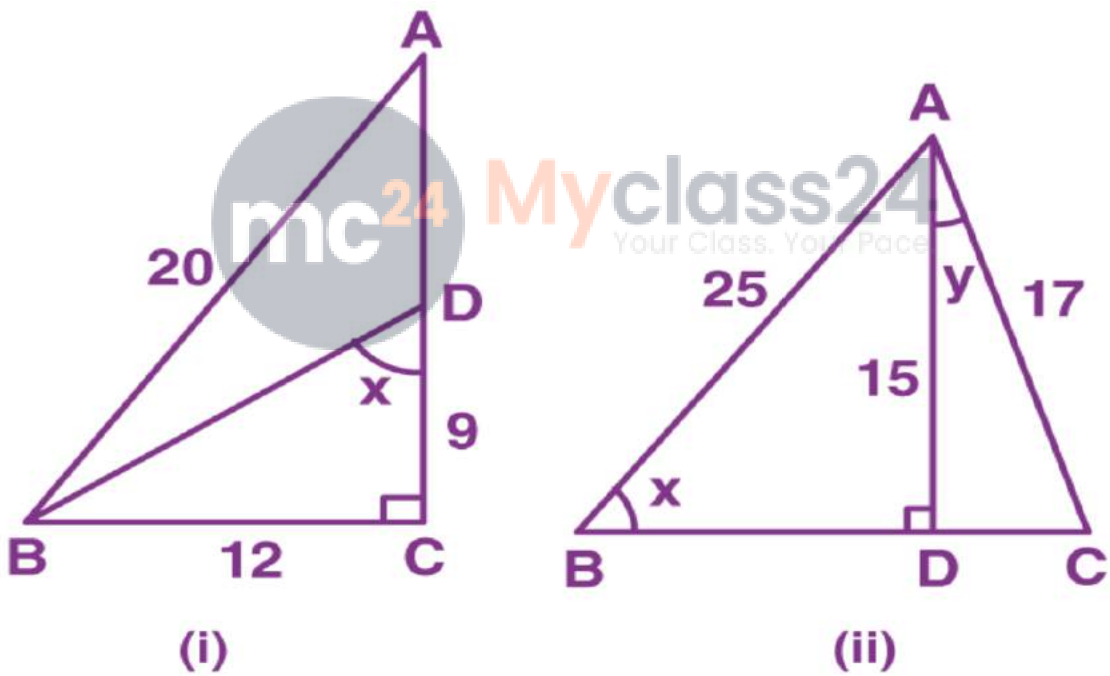
(b) From right angled triangle ABC,

∠BAC = θ
Then we know that,
Cot θ = base/ perpendicular
= AB/ BC
= x/ 10
x = 10 cot θ

also, cosec θ = hypotenuse/ perpendicular
= AC/ BC
= y/ 10
y = 10 cosec θ

Therefore, x = 10 cot θ and y = 10 cosec θ.

2. (a) From the figure (1) given below, find the values of:
(i) sin ∠ABC
(ii) tan x – cos x + 3 sin x.
(b) From the figure (2) given below, find the values of:
(i) 5 sin x
(ii) 7 tan x
(iii) 5 cos x – 17 sin y – tan x.



Solution:

(a) From the figure
BC = 12, CD = 9 and BC = 20

In right angled ΔABC,
Using Pythagoras theorem
$AB^2 = AC^2 + BC^2$

It can be written as
$AC^2 = AB^2 – BC^2$
Substituting the values
$AC^2 = (20)^2 – (12)^2$
By further calculation
$AC^2 = 400 – 144 = 256$
So we get
$AC^2 = (16)^2$
AC = 16

In right angled ΔBCD
Using Pythagoras theorem
$BD^2 = BC^2 + CD^2$
Substituting the values
$BD^2 = 12^2 + 9^2$
By further calculation
$BD^2 = 144 + 81 = 225$
So we get
$BD^2 = (15)^2$
BD = 15

(i) In right angled ΔBCD
sin ∠ABC = perpendicular/hypotenuse
So we get
sin ∠ABC = AC/AB = 16/20 = 4/5

(ii) In right angled ΔBCD
tan x = perpendicular/base
So we get
tan x = BC/CD = 12/9 = 4/3

In right angled ΔBCD
cos x = base/hypotenuse
So we get
cos x = CD/BD = 9/15 = 3/5

In right angled ΔBCD

sin x = perpendicular/hypotenuse
So we get
sin x = BC/BD = 12/15 = 4/5

tan x – cos x + 3 sin x = 4/3 – 3/5 + 3 × 4/5
By further calculation
= 4/3 – 3/5 + 12/5
Taking LCM
= (4 × 5 – 3 × 3 + 12 × 3)/ 15
So we get
= (20 – 9 + 36)/ 15
= (56 – 9)/ 15
= 27/15
= 3 2/15

Therefore, tan x – cos x + 3 sin x = 3 2/15.

(b) In the figure
AC = 17, AB = 25, AD = 15
In right angled ΔACD
Using Pythagoras theorem
$AC^2 = AD^2 + CD^2$
Substituting the values
$(17)^2 = (15)^2 + (CD)^2$
By further calculation
$CD^2 = (17)^2 – (15)^2$
$CD^2 = 289 – 225 = 64$
So we get
$CD^2 = 8^2$
CD = 8

In right angled ΔABD
Using Pythagoras theorem
$AB^2 = AD^2 + BD^2$
Substituting the values
$(25)^2 = (15)^2 + BD^2$
By further calculation

$BD^2 = (25)^2 - (15)^2$
$BD^2 = 625 - 225 = 400$
So we get
$BD^2 = (20)^2$
BD = 20

(i) In right angled ΔABD
5 sin x = 5 (perpendicular/hypotenuse)
So we get
= 5 (AD/AB)
= 5 × 15/25
= 15/5
= 3

(ii) In right angled ΔABD
7 tan x = 7 (perpendicular/base)
So we get
= 7 (AD/AB)
= 7 × 15/20
= 7 × ¾
= 21/4
= 5 ¼

(iii) In right angled ΔABD
cos x = base/hypotenuse
So we get
cos x = BD/AB = 20/25 = 4/5

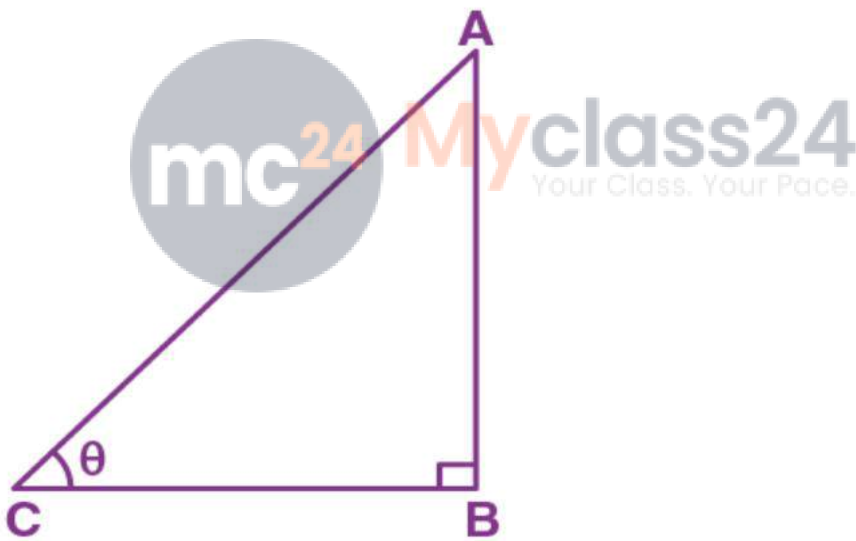
In right angled ΔACD
sin y = perpendicular/hypotenuse
So we get
sin y = CD/AC = 8/17

In right angled ΔABD
tan x = perpendicular/base
So we get
tan x = AD/BD = 15/20 = ¾

5 cos x – 17 sin y – tan x = 5 × 4/5 – 17 × 8/17 – ¾
It can be written as
= 4/1 – 8/1 – ¾
Taking LCM
= (16 – 32 – 3)/4
= (16 – 35)/4
So we get
= - 19/4
= - 4 ¾

Therefore, 5 cos x – 17 sin y – tan x = - 4 ¾.

3. If q cos θ = p, find tan θ – cot θ in terms of p and q.
Solution:



Consider ABC as a triangle right angled at B and ∠ACB = θ
It is given that
q cos θ = p
cos θ = BC/AC = p/q
Take BC = px then AC = qx

In right angled ΔABC
Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
It can be written as

$AB^2 = AC^2 – BC^2$
Substituting the values
$AB^2 = (qx)^2 – (px)^2$
$AB^2 = q^2x^2 – p^2x^2$
Taking out the common terms
$AB^2 = (q^2 – p^2)x^2$
So we get
AB = √$(q^2 – p^2)$ x^2
AB = (√$q^2 – p^2$)x

In right angled ΔABC
tan θ = perpendicular/base
So we get
tan θ = AB/BC = [(√$q^2 – p^2$)x]/px
tan θ = (√$q^2 – p^2$)/p

In right angled ΔABC
cot θ = base/perpendicular
So we get
cot θ = BC/AB = px/[(√$q^2 – p^2$)x]
cot [(√$q^2 – p^2$)x] = p/(√$q^2 – p^2$)

$$tan\Theta - cos\Theta = \frac{\sqrt{q^2-p^2}}{p} - \frac{p}{\sqrt{q^2-p^2}}$$

$Taking\ LCM$

$$= \frac{\sqrt{q^2-p^2}(q^2-p^2) - p \times p}{p(\sqrt{q^2-p^2})}$$

$So\ we\ get$

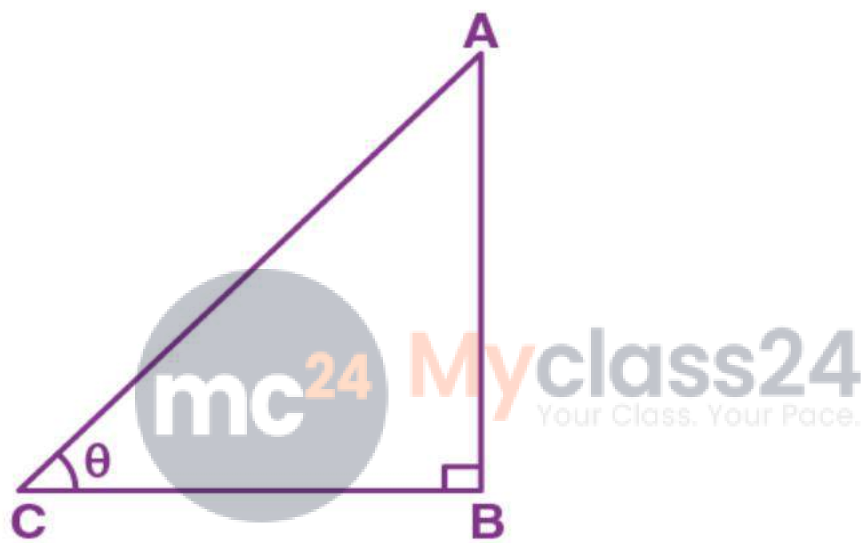
$$= \frac{q^2-p^2-p^2}{p(\sqrt{q^2-p^2})}$$

$$= \frac{q^2-2p^2}{p\sqrt{q^2-p^2}}$$

$$Therefore,\ tan\Theta - cot\Theta = \frac{q^2-2p^2}{p\sqrt{q^2-p^2}}.$$

4. Given 4 sin θ = 3 cos θ, find the values of:
(i) sin θ
(ii) cos θ
(iii) $\cot^2 \theta - \text{cosec}^2 \theta$.
Solution:

It is given that
4 sin θ = 3 cos θ
sin θ/cos θ = ¾
tan θ = ¾



Consider ΔABC right angled at B and ∠ACB = θ
tan θ = perpendicular/base
Substituting the values
¾ = AB/BC
AB/BC = ¾

Take AB = 3x then BC = 4x
In right angled ΔABC
Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
Substituting the values
$AC^2 = (3x)^2 + (4x)^2$
By further calculation
$AC^2 = 9x^2 + 16x^2 = 25x^2$
So we get

$AC^2 = (5x)^2$
AC = 5x

(i) In right angled ΔABC
sin θ = perpendicular/hypotenuse
So we get
sin θ = AB/AC = 3x/5x = 3/5

(ii) In right angled ΔABC
cos θ = base/hypotenuse
So we get
cos θ = BC/AC = 4x/5x = 4/5

(iii) In right angled ΔABC
cot θ = base/perpendicular
So we get
cot θ = BC/AB = 4x/3x = 4/3

In right angled ΔABC
cosec θ = hypotenuse/perpendicular
So we get
cosec θ = AC/AB = 5x/3x = 5/3

Here
$\cot^2 \theta - \csc^2 \theta = (4/3)^2 - (5/3)^2$
By further calculation
= 16/9 – 25/9
= (16 – 25)/9
= -9/9
= - 1

Therefore, $\cot^2 \theta - \csc^2 \theta = -1$.

5. If 2 cos θ = √3, prove that 3 sin θ – 4 $\sin^3$ θ = 1.
Solution:

It is given that

$2 \cos \theta = \sqrt{3}$
$\cos \theta = \sqrt{3}/2$

We know that
$\sin^2 \theta = 1 - \cos^2 \theta$
Substituting the values
$= 1 - (\sqrt{3}/2)^2$
$= 1 - 3/4$
$= 1/4$
$\sin \theta = \sqrt{1/4} = 1/2$

Consider
LHS $= 3 \sin \theta - 4 \sin^3 \theta$
It can be written as
$= \sin \theta (3 - 4 \sin^2 \theta)$
Substituting the values
$= 1/2 (3 - 4 \times 1/4)$
$= 1/2 (3 - 1)$
$= 1/2 \times 1$
$= 1$
= RHS

Therefore, proved.

6. If $(\sec \theta - \tan \theta)/ (\sec \theta + \tan \theta) = 1/4$, find $\sin \theta$.
Solution:

We know that

$$\frac{sec\Theta - tan\Theta}{sec\Theta + tan\Theta} = \frac{\frac{1}{cos\Theta} - \frac{sin\Theta}{cos\Theta}}{\frac{1}{cos\Theta} + \frac{sin\Theta}{cos\Theta}}$$

$Taking\ LCM$

$$= \frac{\frac{1-sin\Theta}{cos\Theta}}{\frac{1+sin\Theta}{cos\Theta}}$$

$So\ we\ get$

$$= \frac{1 - sin\Theta}{cos\Theta} \times \frac{cos\Theta}{1 + sin\Theta}$$

$$= \frac{1 - sin\Theta}{1 + sin\Theta}$$

$Here$

$$\frac{1 - sin\Theta}{1 + sin\Theta} = \frac{1}{4}$$

By cross multiplication
4 – 4 sin θ = 1 + sin θ
We get
4 – 1 = sin θ + 4 sin θ
3 = 5 sin θ
sin θ = 3/5

7. If sin θ + cosec θ = 3 1/3, find the value of sin^2 θ + $cosec^2$ θ.
Solution:

It is given that
sin θ + cosec θ = 3 1/3 = 10/3
By squaring on both sides
$(\sin θ + \text{cosec } θ)^2 = (10/3)^2$
Expanding using formula $(a + b)^2 = a^2 + b^2 + 2ab$
$\sin^2 θ + \text{cosec}^2 θ + 2 \sin θ \text{ cosec } θ = 100/9$
We know that sin θ = 1/cosec θ
$\sin^2 θ + \text{cosec}^2 θ + 2 \sin θ \times 1/ \sin θ = 100/9$
By further calculation
$\sin^2 θ + \text{cosec}^2 θ + 2 = 100/9$

$\sin^2 \theta + \text{cosec}^2 \theta = 100/9 - 2$
Taking LCM
$\sin^2 \theta + \text{cosec}^2 \theta = (100 - 18)/9 = 82/9$
So we get
$\sin^2 \theta + \text{cosec}^2 \theta = 9\ 1/9$

8. In the adjoining figure, AB = 4 m and ED = 3 m.
If sin α = 3/5 and cos β = 12/13, find the length of BD.

Solution:

It is given that
sin α = AB/AC = 3/5
AB = 3 and AC = 5

Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$
Substituting the values
$5^2 = 3^2 + BC^2$
By further calculation
$25 = 9 + BC^2$
$BC^2 = 25 - 9 = 16$
So we get
$BC^2 = 4^2$
BC = 4

We know that

tan α = AB/BC = 4/5
cos β = CD/CE = 12/13
CD = 12 and CE = 13

Using Pythagoras theorem
$CE^2 = CD^2 + ED^2$
Substituting the values
$13^2 = 12^2 + ED^2$
By further calculation
$ED^2 = 13^2 – 12^2$
$ED^2 = 169 – 144 = 25$
So we get
$ED^2 = (5)^2$
ED = 5
tan β = ED/CD = 5/12

From the figure
tan α = AB/BC = 4/BC
So we get
¾ = 4/BC
BC = (4 × 4)/3 = 16/3 m

tan β = ED/CD = 3/CD
5/12 = 3/CD
So we get
CD = (12 × 3)/5 = 36/5 m

Here
BD = BC + CD
Substituting the values
= 16/3 + 36/5
Taking LCM
= (80 + 108)/15
= 188/15 m
= 12 8/15 m