

# NCERT Solutions for Class-XII Math

## Chapter-1 Exercise- Miscellaneous

1. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = 10x + 7$ . Find the function  $g: \mathbb{R} \rightarrow \mathbb{R}$  such that  $g \circ f = f \circ g = \text{IR}$ .

1. It is given that  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $f(x) = 10x + 7$ .

For one – one

Let  $f(x) = f(y)$ , where  $x, y \in \mathbb{R}$ .

$$\Rightarrow 10x + 7 = 10y + 7$$

$$\Rightarrow x = y$$

$\therefore f$  is a one – one function.

For onto

For  $y \in \mathbb{R}$ , let  $y = 10x + 7$ .

$$y - 7$$

$$\Rightarrow x \in \mathbb{R}$$

$\therefore f$  is a one – one function.

For onto

For  $y \in \mathbb{R}$ , let  $y = 10x + 7$ .

$$\Rightarrow x = \frac{y-7}{10} \in \mathbb{R}$$

Therefore, for any  $y \in \mathbb{R}$ , there exists  $x = \frac{y-7}{10} \in \mathbb{R}$  such that

$$f(x) = f\left(\frac{y-7}{10}\right) = 10\left(\frac{y-7}{10}\right) + 7 = y - 7 + 7 = y$$

$\therefore f$  is onto.

Therefore,  $f$  is one – one and onto.

Thus,  $f$  is an invertible function.

Let us define  $g: \mathbb{R} \rightarrow \mathbb{R}$  as  $g(y) = \frac{y-7}{10}$

Now, we have

$$g \circ f(x) = g(f(x)) = g(10x + 7) = \frac{(10x + 7) - 7}{10} = \frac{10x}{10} = x$$

and

$$f \circ g(y) = f(g(y)) = f\left(\frac{y-7}{10}\right) = 10\left(\frac{y-7}{10}\right) + 7 = y - 7 + 7 = y$$

$\therefore g \circ f = I_{\mathbb{R}}$  and  $f \circ g = I_{\mathbb{R}}$ .

Hence, the required function  $g: \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $g(y) = \frac{y-7}{10}$ .

2. Let  $f: W \rightarrow W$  be defined as  $f(n) = n - 1$ , if  $n$  is odd and  $f(n) = n + 1$ , if  $n$  is even. Show that  $f$  is invertible. Find the inverse of  $f$ . Here,  $W$  is the set of all whole numbers.
2. It is given that  $f: W \rightarrow W$  be defined as

$$f(n) = \begin{cases} n-1, & \text{if } n \text{ is odd} \\ n+1, & \text{if } n \text{ is even} \end{cases}$$

Let  $f(n) = f(m)$

We can see that if  $n$  is odd and  $m$  is even, then we will have  $n - 1 = m + 1$ .

$$\Rightarrow n - m = 2$$

$\Rightarrow$  this is impossible

Similarly, the possibility of  $n$  being even and  $m$  being odd can also be ignored under a similar argument.

Therefore, both  $n$  and  $m$  must be either odd or even.

Now, if both  $n$  and  $m$  are odd, then we get:

$$f(n) = f(m)$$

$$\Rightarrow n - 1 = m - 1$$

$$\Rightarrow n = m$$

Again, if both  $n$  and  $m$  are even, then we get:

$$f(n) = f(m)$$

$$\Rightarrow n + 1 = m + 1$$

$$\Rightarrow n = m$$

$\Rightarrow f$  is one - one.

Now, it is clear that any odd number  $2r + 1$  in co-domain  $N$  is the image of  $2r$  in domain  $N$  and any even number  $2r$  in co - domain  $N$  is the image of  $2r + 1$  in domain  $N$ .

$\Rightarrow f$  is onto.

$\Rightarrow f$  is an invertible function.

Now, let us define  $g: W \rightarrow W$  be defined as

$$g(m) = \begin{cases} m+1, & \text{if } m \text{ is even} \\ m-1, & \text{if } m \text{ is odd} \end{cases}$$

Now, when  $n$  is odd:

$$g \circ f(n) = g(f(n)) = g(n-1) = n - 1 + 1 = n \quad (\text{when } n \text{ is odd, then } n-1 \text{ is even})$$

And when  $n$  is even:

$$g \circ f(n) = g(f(n)) = g(n+1) = n + 1 - 1 = n \quad (\text{when } n \text{ is even, then } n+1 \text{ is odd})$$

Similarly, when  $m$  is odd:

$$f \circ g(m) = f(g(m)) = f(m-1) = m-1+1 = m$$

And when  $n$  is even:

$$f \circ g(m) = f(g(m)) = f(m+1) = m+1-1 = m$$

Therefore,  $\text{gof} = I_W$  and  $\text{fog} = I_W$

**Therefore,  $f$  is invertible and the inverse of  $f$  is given by  $f^{-1} = g$ , which is the same as  $f$ .**

**Thus, the inverse of  $f$  is  $f$  itself.**

3. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined by  $f(x) = x^2 - 3x + 2$ , find  $f(f(x))$ .

3. It is given that  $f: \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $f(x) = x^2 - 3x + 2$ .

$$f(f(x)) = f(x^2 - 3x + 2)$$

$$= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$$

$$= (x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2) + (-3x^2 + 9x - 6) + 2 = x^4 - 6x^3 + 10x^2 - 3x$$

4. Show that function  $f: \mathbb{R} \rightarrow \{x \in \mathbb{R} : -1 < x < 1\}$  defined by  $f(x) = \frac{x}{1+|x|}$ ,  $x \in \mathbb{R}$  is one – one and onto function.

4. It is given that  $f: \mathbb{R} \rightarrow \{x \in \mathbb{R} : -1 < x < 1\}$  defined by  $f(x) = \frac{x}{1+|x|}$ ,  $x \in \mathbb{R}$

Now, suppose that  $f(x) = f(y)$ , where  $x, y \in \mathbb{R}$ .

$$\Rightarrow \frac{x}{1+|x|} = \frac{y}{1+|y|}$$

We can see that if  $x$  is positive and  $y$  is negative, then we get:

$$\frac{x}{1+x} = \frac{y}{1+y} \Rightarrow 2xy = x - y$$

Since,  $x$  is positive and  $y$  is negative.

Then,  $2xy \neq x - y$ .

Thus, the case of  $x$  being positive and  $y$  being negative can be ruled out.

Similarly,  $x$  being negative and  $y$  being positive can also be ruled out.

Therefore,  $x$  and  $y$  have to be either positive or negative.

When  $x$  and  $y$  are both positive, we get:

$$f(x) = f(y)$$

$$\Rightarrow \frac{x}{1+x} = \frac{y}{1+y} \Rightarrow x + xy = y + xy \Rightarrow x = y$$

And when  $x$  and  $y$  are both negative, we get:

$$f(x) = f(y)$$

$$\Rightarrow \frac{x}{1-x} = \frac{y}{1-y} \Rightarrow x - xy = y - xy \Rightarrow x = y$$

$\Rightarrow f$  is one- one.

Now, let  $y \in \mathbb{R}$  such that  $-1 < y < 1$ .

If  $y$  is negative, then there exists  $x = \frac{y}{1+y} \in \mathbb{R}$ , such that

$$f(x) = f\left(\frac{y}{1+y}\right) = \frac{\left(\frac{y}{1+y}\right)}{1 + \left(\frac{y}{1+y}\right)} = \frac{\frac{y}{1+y}}{1 + \frac{y}{1+y}} = \frac{y}{1+y+y} = y$$

$\Rightarrow f$  is onto.

**Therefore,  $f$  is one – one and onto.**

5. Show that the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = x^3$  is injective.

5.  $f: \mathbb{R} \rightarrow \mathbb{R}$  is given as  $f(x) = x^3$ . For one – one

Suppose  $f(x) = f(y)$ , where  $x, y \in \mathbb{R}$ .

$$\Rightarrow x^3 = y^3 \dots\dots\dots (1)$$

Now, we need to show that  $x = y$ .

Suppose  $x \neq y$ , their cubes will also not be equal.

$$\Rightarrow x^3 \neq y^3$$

However, this will be a contradiction to (1).

$$\therefore x = y$$

Hence,  $f$  is injective.

6. Give examples of two functions  $f: \mathbb{N} \rightarrow \mathbb{Z}$  and  $g: \mathbb{Z} \rightarrow \mathbb{Z}$  such that  $g \circ f$  is injective but  $g$  is not injective.

(Hint: Consider  $f(x) = x$  and  $g(x) = |x|$ )

6. Define  $f: \mathbb{N} \rightarrow \mathbb{Z}$  as  $f(x) = x$  and  $g: \mathbb{Z} \rightarrow \mathbb{Z}$  as  $g(x) = |x|$

Now, we can see that

$$g(-1) = |-1| = 1$$

$$g(1) = |1| = 1$$

$$\Rightarrow g(-1) = g(1), \text{ but } -1 \neq 1$$

$\Rightarrow g$  is not injective.

Now,  $g \circ f: \mathbb{N} \rightarrow \mathbb{Z}$  is defined as  $g \circ f(x) = g(f(x)) = g(x) = |x|$

Let  $x, y \in \mathbb{N}$  such that  $g \circ f(x) = g \circ f(y)$ .

$$\Rightarrow |x| = |y|$$

$$\Rightarrow x = y$$

**Therefore,  $\text{gof}$  is injective.**

7. Given examples of two functions  $f: \mathbb{N} \rightarrow \mathbb{N}$  and  $g: \mathbb{N} \rightarrow \mathbb{N}$  such that  $g \circ f$  is onto but  $f$  is not onto.

(Hint: Consider  $f(x) = x + 1$  and  $g(x) = \begin{cases} x-1, & \text{if } x > 1 \\ 1, & \text{if } x = 1 \end{cases}$

7. Define  $f: \mathbb{N} \rightarrow \mathbb{N}$  by  $f(x) = x + 1$

and  $g: \mathbb{N} \rightarrow \mathbb{N}$  by  $g(x) = \begin{cases} x-1, & \text{if } x > 1 \\ 1, & \text{if } x = 1 \end{cases}$

We first show that  $g$  is not onto.

For this, consider element 1 in co-domain  $\mathbb{N}$ . It is clear that this element is not an image of any of the elements in domain  $\mathbb{N}$ .

$\therefore f$  is not onto.

Now,  $\text{gof}: \mathbb{N} \rightarrow \mathbb{N}$  is defined by

$$\text{gof}(x) = g(f(x)) = g(x + 1) = x + 1 - 1 = x \quad [x \in \mathbb{N} \Rightarrow x + 1 > 1]$$

Then, it is clear that for  $y \in \mathbb{N}$ , there exists  $x = y \in \mathbb{N}$  such that  $\text{gof}(x) = y$ .

Hence,  $\text{gof}$  is onto.

8. Given a non-empty set  $X$ , consider  $P(X)$  which is the set of all subsets of  $X$ . Define the relation  $R$  in  $P(X)$  as follows:

For subsets  $A, B$  in  $P(X)$ ,  $A R B$  if and only if  $A \subset B$ . Is  $R$  an equivalence relation on  $P(X)$ ? Justify your answer.

8. We know that every set is a subset of itself,  $A R A$  for all  $A \in P(X)$ .

$\Rightarrow R$  is reflexive.

This cannot be implied to  $B \subset A$ .

So, if  $A = \{1, 2\}$  and  $B = \{1, 2, 3\}$ , then it cannot be implied that  $B$  is related to  $A$ .

$\Rightarrow R$  is not symmetric.

So, if  $A R B$  and  $B R C$ , then  $A \subset B$  and  $B \subset C$ .

$\Rightarrow A \subset C$

$\Rightarrow R$  is transitive.

**Therefore,  $R$  is not an equivalence relation since it is not symmetric.**

9. Given a non-empty set  $X$ , consider the binary operation  $*$ :  $P(X) \times P(X) \rightarrow P(X)$  given by  $A * B = A \cap B \forall A, B$  in  $P(X)$  is the power set of  $X$ . Show that  $X$  is the identity element for this operation and  $X$  is the only invertible element in  $P(X)$  with respect to the operation  $*$ .

9. It is given the binary operation  $*$ :

$P(X) \times P(X) \rightarrow P(X)$  given by  $A * B = A \cap B \forall A, B$  in  $P(X)$

We know that  $A \cap X = A = X \cap A$  for all  $A \in P(X)$

$A * X = A = X * A$  for all  $A \in P(X)$

Thus,  $X$  is the identity element for the given binary operation  $*$ .

Now, an element  $A \in P(X)$  is invertible if there exists  $B \in P(X)$  such that

$A * B = X = B * A$  [As  $X$  is the identity element]

or

$A \cap B = X = B \cap A$

This case is possible only when  $A = X = B$ .

Thus,  $X$  is the only invertible element in  $P(X)$  with respect to the given operation  $*$ .

Hence, the given result is proved.

- 10.** Find the number of all onto functions from the set  $\{1, 2, 3, \dots, n\}$  to itself.
- 10.** Onto function from the set  $\{1, 2, 3, \dots, n\}$  to itself is simply a permutation on  $n$  symbols  $1, 2, 3, \dots, n$ .
- Therefore, the total number of onto maps from  $\{1, 2, 3, \dots, n\}$  to itself is the same as the total number of permutations on  $n$  symbols  $1, 2, 3, \dots, n$ , which is  $n!$
- 11.** Let  $S = \{a, b, c\}$  and  $T = \{1, 2, 3\}$ . Find  $F^{-1}$  of the following functions  $F$  from  $S$  to  $T$ , if it exists.
- (i)  $F = \{(a, 3), (b, 2), (c, 1)\}$                       (ii)  $F = \{(a, 2), (b, 1), (c, 1)\}$
- 11.**  $S = \{a, b, c\}$ ,                       $T = \{1, 2, 3\}$
- (i)  $F: S \rightarrow T$  is defined as  $F = \{(a, 3), (b, 2), (c, 1)\}$   
 $\Rightarrow F(a) = 3, F(b) = 2, F(c) = 1$   
Therefore,  $F^{-1}: T \rightarrow S$  is given by  $F^{-1} = \{(3, a), (2, b), (1, c)\}$ .
- (ii)  $F: S \rightarrow T$  is defined as  $F = \{(a, 2), (b, 1), (c, 1)\}$   
Since  $F(b) = F(c) = 1$ ,  $F$  is not one – one.  
Hence,  $F$  is not invertible i.e.,  $F^{-1}$  does not exist.
- 12.** Consider the binary operations  $*$ :  $R \times R \rightarrow R$  and  $\circ$ :  $R \times R \rightarrow R$  defined as  $a * b = |a - b|$  and  $a \circ b = a, \forall a, b \in R$ . Show that  $*$  is commutative but not associative,  $\circ$  is associative but not commutative. Further, show that  $\forall a, b, c \in R, a*(b \circ c) = (a * b) \circ (a * c)$ . [If it is so, we say that the operation  $*$  distributes over the operation  $\circ$ ]. Does  $\circ$  distribute over  $*$ ? Justify your answer.
- 12.** It is given that  $*$ :  $R \times R \rightarrow R$  and  $\circ$ :  $R \times R \rightarrow R$  defined as  $a * b = |a - b|$  and  $a \circ b = a, \forall a, b \in R$ .  
For  $a, b \in R$ , we get:  
 $a * b = |a - b|$

$$b * a = |b - a| = |-(a - b)| = |a - b|$$

Therefore,  $a * b = b * a$

⇒ the operation  $*$  is commutative.

We can see that

$$(1 * 2) * 3 = (|1 - 2|) * 3 = |1 - 3| = 2$$

$$1 * (2 * 3) = 1 * (|2 - 3|) = 1 * 1 = 1$$

Therefore, the operation  $*$  is not associative.

Now, consider the operation  $\circ$ :

We can observe that  $1 \circ 2 = 1$  and  $2 \circ 1 = 2$ .

$$\Rightarrow 1 \circ 2 \neq 2 \circ 1 \text{ (where } 1, 2 \in \mathbb{R})$$

⇒ the operation  $\circ$  is not commutative.

Let  $a, b, c \in \mathbb{R}$ . Then, we get:

$$(a \circ b) \circ c = a \circ c = a$$

$$a \circ (b \circ c) = a \circ b = a$$

$$\Rightarrow (a \circ b) \circ c = a \circ (b \circ c)$$

⇒ the operation  $\circ$  is associative.

Now, let  $a, b, c \in \mathbb{R}$ , then we have:

$$a * (b \circ c) = a * b = |a - b|$$

$$(a * b) \circ (a * c) = (|a - b|) \circ (|a - c|) = |a - b|$$

$$\text{Thus, } a * (b \circ c) = (a * b) \circ (a * c)$$

Now,

$$1 \circ (2 * 3) = 1 \circ (|2 - 3|) = 1 \circ 1 = 1$$

$$(1 \circ 2) * (1 \circ 3) = 1 * 1 = |1 - 1| = 0$$

Therefore,  $1 \circ (2 * 3) \neq (1 \circ 2) * (1 \circ 3)$  (where  $1, 2, 3 \in \mathbb{R}$ )

Therefore, the operation  $\circ$  does not distribute over  $*$ .

- 13.** Given a non -empty set  $X$ , let  $*$ :  $P(X) \times P(X) \rightarrow P(X)$  be defined as  $A * B = (A - B) \cup (B - A)$ ,  $\forall A, B \in P(X)$ . Show that the empty set  $\Phi$  is the identity for the operation  $*$  and all the elements  $A$  of  $P(X)$  are invertible with  $A^{-1} = A$ .

(Hint:  $(A - \Phi) \cup (\Phi - A) = A$  and  $(A - A) \cup (A - A) = A * A = \Phi$ ).

- 13.** It is given that  $*$ :  $P(X) \times P(X) \rightarrow P(X)$  is defined as  $A * B = (A - B) \cup (B - A) \forall A, B \in P(X)$ .

Let  $A \in P(X)$ . Then, we have

$$A * \Phi = (A - \Phi) \cup (\Phi - A) = A \cup \Phi = A$$

$$\Phi * A = (\Phi - A) \cup (A - \Phi) = \Phi \cup A = A$$

$$\therefore A * \Phi = A = \Phi * A \text{ for all } A \in P(X)$$

Thus,  $\Phi$  is the identity element for the given operation  $*$ .

Now, an element  $A \in P(X)$  will be invertible if there exists  $B \in P(X)$  such that  $A * B = \Phi = B * A$ .

[As  $\Phi$  is the identity element]

Now, we observed that

$$A * A = (A - A) \cup (A - A) = \Phi \cup \Phi = \Phi \text{ for all } A \in P(X).$$

Hence, all the elements  $A$  of  $P(X)$  are invertible with  $A^{-1} = A$ .

14. Define a binary operation  $*$  on the set  $\{0, 1, 2, 3, 4, 5\}$  as

$$a * b = \begin{cases} a + b & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

Show that zero is the identity for this operation and each element  $a \neq 0$  of the set is invertible with  $6 - a$  being the inverse of  $a$ .

14. Let  $X = \{0, 1, 2, 3, 4, 5\}$

The operation  $*$  on  $X$  is defined as

$$a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6 & \text{if } a + b \geq 6 \end{cases}$$

An element  $e \in X$  is the identity element for the operation  $*$ ,

If  $a * e = a = e * a \quad \forall a \in X$ .

For  $a \in X$ , we can see that:

$$a * 0 = a + 0 = a \quad [a \in X \Rightarrow a + 0 < 6]$$

$$0 * a = 0 + a = a \quad [a \in X \Rightarrow a + 0 < 6]$$

$$\Rightarrow a * 0 = a = 0 * a \quad \forall a \in X.$$

Therefore,  $0$  is the identity element for the given operation  $*$ .

An element  $a \in X$  is invertible if there exists  $b \in X$  such that

$$a * b = 0 = b * a.$$

$$\Rightarrow \begin{cases} a + b = 0 = b + a, & \text{if } a + b < 6 \\ a + b - 6 = 0 = b + a - 6, & \text{if } a + b \geq 6 \end{cases}$$

$$a = -b \text{ or } b = 6 - a$$

But,  $X = \{0, 1, 2, 3, 4, 5\}$  and  $a, b \in X$ . Then,  $a \neq -b$ .

**Therefore,  $b = 6 - a$  is the inverse of  $a \in X$ .**

**Thus, the inverse of an element  $a \in X$ ,  $a \neq 0$  is  $6 - a$ ,  $a^{-1} = 6 - a$ .**

15. Let  $A = \{-1, 0, 1, 2\}$ ,  $B = \{-4, -2, 0, 2\}$  and  $f, g: A \rightarrow B$  be functions defined by  $f(x) = x^2 - x$ ,  $x \in A$  and  $g(x) = 2 \left\lfloor x - \frac{1}{2} \right\rfloor$ ,  $x \in A$ . Are  $f$  and  $g$  equal? Justify your answer. (Hint:

One may note that two function  $f: A \rightarrow B$  and  $g: A \rightarrow B$  such that  $f(a) = g(a) \quad \forall a \in A$ , are called equal functions).

15. It is given that  $A = \{-1, 0, 1, 2\}$ ,  $B = \{-4, -2, 0, 2\}$ .

Also, it is given that  $f, g: A \rightarrow B$  are defined by

$$f(x) = x^2 - x, x \in A \text{ and } g(x) = 2\left|x - \frac{1}{2}\right| - 1, x \in A$$

It is observed that

$$f(-1) = (-1)^2 - (-1) = 1 + 1 = 2$$

$$\text{and } g(-1) = 2\left|(-1) - \frac{1}{2}\right| - 1 = 2\left(\frac{3}{2}\right) - 1 = 3 - 1 = 2$$

$$\Rightarrow f(-1) = g(-1)$$

$$f(0) = (0)^2 - (0) = 0$$

$$\text{and } g(0) = 2\left|0 - \frac{1}{2}\right| - 1 = 2\left(\frac{1}{2}\right) - 1 = 1 - 1 = 0$$

$$\Rightarrow f(0) = g(0)$$

$$f(1) = (1)^2 - (1) = 1 - 1 = 0$$

$$\text{and } g(1) = 2\left|1 - \frac{1}{2}\right| - 1 = 2\left(\frac{3}{2}\right) - 1 = 1 - 1 = 0$$

$$\Rightarrow f(1) = g(1)$$

$$f(2) = (2)^2 - (2) = 4 - 2 = 2$$

$$\text{And } g(2) = 2\left|2 - \frac{1}{2}\right| - 1 = 2\left(\frac{3}{2}\right) - 1 = 3 - 1 = 2$$

$$\Rightarrow f(2) = g(2)$$

$$\therefore f(a) = g(a) \text{ for all } a \in A$$

Hence, the functions  $f$  and  $g$  are equal.

16. Let  $A = \{1, 2, 3\}$ . Then number of relations containing  $(1, 2)$  and  $(1, 3)$  which are reflexive and symmetric but not transitive is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

16. **The correct option is (A).**

**Explanation:** This is because relation  $R$  is reflexive as  $(1, 1), (2, 2), (3, 3) \in R$ .

Relation  $R$  is symmetric as  $(1, 2), (2, 1) \in R$  and  $(1, 3), (3, 1) \in R$ .

But relation  $R$  is not transitive as  $(3, 1), (1, 2) \in R$  but  $(3, 2) \notin R$ .

Now, if we add any one of the two pairs  $(3, 2)$  and  $(2, 3)$  (or both) to relation  $R$ ,

Then, relation  $R$  will become transitive.

Therefore, the total number of desired relations is one.

17. Let  $A = \{1, 2, 3\}$ . Then number of equivalence relations containing  $(1, 2)$  is  
 (A) 1  
 (B) 2  
 (C) 3  
 (D) 4

17. It is given that  $A = \{1, 2, 3\}$ .

The smallest equivalence relation containing  $(1, 2)$  is given by,

$$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$$

Now, we are left with only four pairs i.e.,  $(2, 3), (3, 2), (1, 3)$ , and  $(3, 1)$ .

If we add any one pair [say  $(2, 3)$ ] to  $R_1$ , then for symmetry we must add  $(3, 2)$ . Also, for transitivity we are required to add  $(1, 3)$  and  $(3, 1)$ .

Hence, the only equivalence relation (bigger than  $R_1$ ) is the universal relation. This shows that the total number of equivalence relations containing  $(1, 2)$  is two. The correct answer is B.

18. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be the Signum Function defined as  $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$

and  $g: \mathbb{R} \rightarrow \mathbb{R}$  be the Greatest Integer Function given by  $g(x) = [x]$ , where  $[x]$  is greatest integer less than or equal to  $x$ . Then does  $f \circ g$  and  $g \circ f$  coincide in  $(0, 1]$ ?

18. It is given that

$f: \mathbb{R} \rightarrow \mathbb{R}$  be the Signum Function defined as

$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

Also,  $g: \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $g(x) = [x]$ , where  $[x]$  is the greatest integer less than or equal to  $x$ .

Now, let  $x \in (0, 1]$

Then, we get,

$$[x] = 1 \text{ if } x = 1 \text{ and } [x] = 0 \text{ if } 0 < x < 1$$

$$\text{Therefore, } f \circ g(x) = f(g(x)) = f([x]) = \begin{cases} f(1), & \text{if } x = 1 \\ f(0), & \text{if } x \in (0, 1) \end{cases} = \begin{cases} 1, & \text{if } x = 1 \\ 0, & \text{if } x \in (0, 1) \end{cases}$$

$$g \circ f(x) = g(f(x))$$

$$= g(1) [x > 0]$$

$$= [1] = 1$$

Then, when  $x \in (0, 1)$ , we get  $f \circ g(x) = 0$  and  $g \circ f(x) = 1$

But  $f \circ g(1) \neq g \circ f(1)$

**Therefore, fog and gof do not coincide in (0,1].**

19. Number of binary operations on the set  $\{a, b\}$  are
- (A) 10
  - (B) 16
  - (C) 20
  - (D) 8
19. A binary operation  $*$  on  $\{a, b\}$  is a function from  $\{a, b\} \times \{a, b\} \rightarrow \{a, b\}$  i.e.,  $*$  is a function from  $\{(a, a), (a, b), (b, a), (b, b)\} \rightarrow \{a, b\}$ .  
Hence, the total number of binary operations on the set  $\{a, b\}$  is  $2^4$  i.e., 16.  
The correct answer is B.



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