

SOLUTIONS TO CONCEPTS CHAPTER – 1

1. a) Linear momentum : $mv = [MLT^{-1}]$
 b) Frequency : $\frac{1}{T} = [M^0L^0T^{-1}]$
 c) Pressure : $\frac{\text{Force}}{\text{Area}} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$
2. a) Angular speed $\omega = \theta/t = [M^0L^0T^{-1}]$
 b) Angular acceleration $\alpha = \frac{\omega}{t} = \frac{M^0L^0T^{-2}}{T} = [M^0L^0T^{-2}]$
 c) Torque $\tau = Fr = [MLT^{-2}][L] = [ML^2T^{-2}]$
 d) Moment of inertia $= Mr^2 = [M][L^2] = [ML^2T^0]$
3. a) Electric field $E = F/q = \frac{MLT^{-2}}{[IT]} = [MLT^{-3}I^{-1}]$
 b) Magnetic field $B = \frac{F}{qv} = \frac{MLT^{-2}}{[IT][LT^{-1}]} = [MT^{-2}I^{-1}]$
 c) Magnetic permeability $\mu_0 = \frac{B \times 2\pi a}{I} = \frac{MT^{-2}I^{-1} \times [L]}{[I]} = [MLT^{-2}I^{-2}]$
4. a) Electric dipole moment $P = ql = [IT] \times [L] = [LTI]$
 b) Magnetic dipole moment $M = IA = [I][L^2][L^2I]$
5. $E = h\nu$ where $E =$ energy and $\nu =$ frequency.
 $h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$
6. a) Specific heat capacity $= C = \frac{Q}{m\Delta T} = \frac{[ML^2T^{-2}]}{[M][K]} = [L^2T^{-2}K^{-1}]$
 b) Coefficient of linear expansion $= \alpha = \frac{L_1 - L_2}{L_0\Delta T} = \frac{[L]}{[L][R]} = [K^{-1}]$
 c) Gas constant $= R = \frac{PV}{nT} = \frac{[ML^{-1}T^{-2}][L^3]}{[(mol)][K]} = [ML^2T^{-2}K^{-1}(mol)^{-1}]$
7. Taking force, length and time as fundamental quantity
 a) Density $= \frac{m}{V} = \frac{\text{(force/acceleration)}}{\text{Volume}} = \frac{[F/LT^{-2}]}{[L^3]} = \frac{F}{L^4T^{-2}} = [FL^{-4}T^2]$
 b) Pressure $= F/A = F/L^2 = [FL^{-2}]$
 c) Momentum $= mv$ (Force / acceleration) \times Velocity $= [F/LT^{-2}] \times [LT^{-1}] = [FT]$
 d) Energy $= \frac{1}{2}mv^2 = \frac{\text{Force}}{\text{acceleration}} \times (\text{velocity})^2$
 $= \left[\frac{F}{LT^{-2}} \right] \times [LT^{-1}]^2 = \left[\frac{F}{LT^{-2}} \right] \times [L^2T^{-2}] = [FL]$
8. $g = 10 \frac{\text{metre}}{\text{sec}^2} = 36 \times 10^5 \text{ cm/min}^2$
9. The average speed of a snail is 0.02 mile/hr
 Converting to S.I. units, $\frac{0.02 \times 1.6 \times 1000}{3600} \text{ m/sec}$ [1 mile = 1.6 km = 1600 m] $= 0.0089 \text{ ms}^{-1}$
 The average speed of leopard = 70 miles/hr
 In SI units $= 70 \text{ miles/hour} = \frac{70 \times 1.6 \times 1000}{3600} = 31 \text{ m/s}$

10. Height $h = 75 \text{ cm}$, Density of mercury $= 13600 \text{ kg/m}^3$, $g = 9.8 \text{ ms}^{-2}$ then
 Pressure $= hfg = 10 \times 10^4 \text{ N/m}^2$ (approximately)
 In C.G.S. Units, $P = 10 \times 10^5 \text{ dyne/cm}^2$
11. In S.I. unit $100 \text{ watt} = 100 \text{ Joule/sec}$
 In C.G.S. Unit $= 10^9 \text{ erg/sec}$
12. $1 \text{ micro century} = 10^4 \times 100 \text{ years} = 10^{-4} \times 365 \times 24 \times 60 \text{ min}$
 So, $100 \text{ min} = 10^5 / 52560 = 1.9 \text{ microcentury}$
13. Surface tension of water $= 72 \text{ dyne/cm}$
 In S.I. Unit, $72 \text{ dyne/cm} = 0.072 \text{ N/m}$
14. $K = kI^a \omega^b$ where $k = \text{Kinetic energy of rotating body}$ and $k = \text{dimensionless constant}$
 Dimensions of left side are,
 $K = [ML^2T^{-2}]$
 Dimensions of right side are,
 $I^a = [ML^2]^a$, $\omega^b = [T^{-1}]^b$
 According to principle of homogeneity of dimension,
 $[ML^2T^{-2}] = [ML^2T^{-2}] [T^{-1}]^b$
 Equating the dimension of both sides,
 $2 = 2a$ and $-2 = -b \Rightarrow a = 1$ and $b = 2$
15. Let energy $E \propto M^a C^b$ where $M = \text{Mass}$, $C = \text{speed of light}$
 $\Rightarrow E = KM^a C^b$ ($K = \text{proportionality constant}$)
 Dimension of left side
 $E = [ML^2T^{-2}]$
 Dimension of right side
 $M^a = [M]^a$, $[C]^b = [LT^{-1}]^b$
 $\therefore [ML^2T^{-2}] = [M]^a [LT^{-1}]^b$
 $\Rightarrow a = 1$; $b = 2$
 So, the relation is $E = KMC^2$
16. Dimensional formulae of $R = [ML^2T^{-3}I^{-2}]$
 Dimensional formulae of $V = [ML^2T^3I^{-1}]$
 Dimensional formulae of $I = [I]$
 $\therefore [ML^2T^3I^{-1}] = [ML^2T^{-3}I^{-2}] [I]$
 $\Rightarrow V = IR$
17. Frequency $f = KL^a F^b M^c$ $M = \text{Mass/unit length}$, $L = \text{length}$, $F = \text{tension (force)}$
 Dimension of $f = [T^{-1}]$
 Dimension of right side,
 $L^a = [L]^a$, $F^b = [MLT^{-2}]^b$, $M^c = [ML^{-1}]^c$
 $\therefore [T^{-1}] = K[L]^a [MLT^{-2}]^b [ML^{-1}]^c$
 $M^0 L^0 T^{-1} = KM^{b+c} L^{a+b-c} T^{-2b}$
 Equating the dimensions of both sides,
 $\therefore b + c = 0 \quad \dots(1)$
 $-c + a + b = 0 \quad \dots(2)$
 $-2b = -1 \quad \dots(3)$
 Solving the equations we get,
 $a = -1$, $b = 1/2$ and $c = -1/2$
 \therefore So, frequency $f = KL^{-1} F^{1/2} M^{-1/2} = \frac{K}{L} F^{1/2} M^{-1/2} = \frac{K}{L} = \sqrt{\frac{F}{M}}$

$$18. a) h = \frac{2S \cos \theta}{\rho g}$$

$$\text{LHS} = [L]$$

$$\text{Surface tension} = S = F/l = \frac{MLT^{-2}}{L} = [MT^{-2}]$$

$$\text{Density} = \rho = M/V = [ML^{-3}T^0]$$

$$\text{Radius} = r = [L], g = [LT^{-2}]$$

$$\text{RHS} = \frac{2S \cos \theta}{\rho g} = \frac{[MT^{-2}]}{[ML^{-3}T^0][L][LT^{-2}]} = [M^0L^1T^0] = [L]$$

$$\text{LHS} = \text{RHS}$$

So, the relation is correct

$$b) v = \sqrt{\frac{\rho}{\rho}} \text{ where } v = \text{velocity}$$

$$\text{LHS} = \text{Dimension of } v = [LT^{-1}]$$

$$\text{Dimension of } \rho = F/A = [ML^{-1}T^{-2}]$$

$$\text{Dimension of } \rho = m/V = [ML^{-3}]$$

$$\text{RHS} = \sqrt{\frac{\rho}{\rho}} = \sqrt{\frac{[ML^{-1}T^{-2}]}{[ML^{-3}]}} = [L^2T^{-2}]^{1/2} = [LT^{-1}]$$

So, the relation is correct.

$$c) V = (\pi r^4 t) / (8 \eta l)$$

$$\text{LHS} = \text{Dimension of } V = [L^3]$$

$$\text{Dimension of } \rho = [ML^{-1}T^{-2}], r^4 = [L^4], t = [T]$$

$$\text{Coefficient of viscosity} = [ML^{-1}T^{-1}]$$

$$\text{RHS} = \frac{\pi r^4 t}{8 \eta l} = \frac{[ML^{-1}T^{-2}][L^4][T]}{[ML^{-1}T^{-1}][L]}$$

So, the relation is correct.

$$d) v = \frac{1}{2\pi} \sqrt{(mgl/l)}$$

$$\text{LHS} = \text{dimension of } v = [T^{-1}]$$

$$\text{RHS} = \sqrt{(mgl/l)} = \sqrt{\frac{[M][LT^{-2}][L]}{[ML^2]}} = [T^{-1}]$$

$$\text{LHS} = \text{RHS}$$

So, the relation is correct.

$$19. \text{ Dimension of the left side} = \int \frac{dx}{\sqrt{(a^2 - x^2)}} = \int \frac{L}{\sqrt{(L^2 - L^2)}} = [L^0]$$

$$\text{Dimension of the right side} = \frac{1}{a} \sin^{-1}\left(\frac{a}{x}\right) = [L^{-1}]$$

$$\text{So, the dimension of } \int \frac{dx}{\sqrt{(a^2 - x^2)}} \neq \frac{1}{a} \sin^{-1}\left(\frac{a}{x}\right)$$

So, the equation is dimensionally incorrect.

20. Important Dimensions and Units :

Physical quantity	Dimension	SI unit
Force (F)	$[M^1L^1T^{-2}]$	newton
Work (W)	$[M^1L^2T^{-2}]$	joule
Power (P)	$[M^1L^2T^{-3}]$	watt
Gravitational constant (G)	$[M^{-1}L^3T^{-2}]$	$N\cdot m^2/kg^2$
Angular velocity (ω)	$[T^{-1}]$	radian/s
Angular momentum (L)	$[M^1L^2T^{-1}]$	$kg\cdot m^2/s$
Moment of inertia (I)	$[M^1L^2]$	$kg\cdot m^2$
Torque (τ)	$[M^1L^2T^{-2}]$	N-m
Young's modulus (Y)	$[M^1L^{-1}T^{-2}]$	N/m^2
Surface Tension (S)	$[M^1T^{-2}]$	N/m
Coefficient of viscosity (η)	$[M^1L^{-1}T^{-1}]$	$N\cdot s/m^2$
Pressure (p)	$[M^1L^{-1}T^{-2}]$	N/m^2 (Pascal)
Intensity of wave (I)	$[M^1T^{-3}]$	$watt/m^2$
Specific heat capacity (c)	$[L^2T^{-2}K^{-1}]$	J/kg-K
Stefan's constant (σ)	$[M^1T^{-3}K^{-4}]$	$watt/m^2\cdot k^4$
Thermal conductivity (k)	$[M^1L^1T^{-3}K^{-1}]$	$watt/m\cdot K$
Current density (j)	$[I^1L^{-2}]$	ampere/ m^2
Electrical conductivity (σ)	$[I^2T^3M^{-1}L^{-3}]$	$\Omega^{-1} m^{-1}$
Electric dipole moment (p)	$[L^1I^1T^1]$	C-m
Electric field (E)	$[M^1L^1I^{-1}T^{-3}]$	V/m
Electrical potential (V)	$[M^1L^2I^{-1}T^{-3}]$	volt
Electric flux (Ψ)	$[M^1T^3I^{-1}L^{-3}]$	volt/m
Capacitance (C)	$[I^2T^4M^{-1}L^{-2}]$	farad (F)
Permittivity (ϵ)	$[I^2T^4M^{-1}L^{-3}]$	$C^2/N\cdot m^2$
Permeability (μ)	$[M^1L^1I^{-2}T^{-3}]$	Newton/ A^2
Magnetic dipole moment (M)	$[I^1L^2]$	N-m/T
Magnetic flux (ϕ)	$[M^1L^2I^{-1}T^{-2}]$	Weber (Wb)
Magnetic field (B)	$[M^1I^{-1}T^{-2}]$	tesla
Inductance (L)	$[M^1L^2I^{-2}T^{-2}]$	henry
Resistance (R)	$[M^1L^2I^{-2}T^{-3}]$	ohm (Ω)

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