NCERT Solutions for Class-XII Maths

Chapter-7.10

1. $\int_0^1 \frac{x}{x^2+1}dx$

1. $\int_0^1 \frac{x}{x^2+1}dx$

Let $x^2 + 1 = t \Rightarrow 2x\,dx = dt$

When $x = 0$, $t = 1$ and when $x = 1$, $t = 2$

$$\therefore \int_0^1 \frac{x}{x^2+1}dx = \frac{1}{2}\int_1^2 \frac{dt}{t}$$

$$= \frac{1}{2}\left[\log|t|\right]_1^2$$

$$= \frac{1}{2}\left[\log 2 - \log 1\right]$$

$$= \frac{1}{2}\log 2$$

2. $\int_0^{\frac{x}{2}} \sqrt{\sin\phi}\cos^5\phi\, d\phi$

2. Given : $\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\ \cos^5\phi\ d\phi$

Let $I = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\ \cos^5\phi\ d\phi = \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\ \cos^4\phi\ \cos\phi\ d\phi$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\ (\cos^2\phi)^2\ \cos\phi\ d\phi$$

$$\int_0^{\frac{\pi}{2}} \sqrt{\sin\phi}\ (1-\sin^2\phi)^2\ \cos\phi\ d\phi$$

Also, let $\sin\phi = t \Rightarrow \cos\phi d\phi = dt$

when, $\phi = 0$, $t = 0$ and when $\phi = \frac{\pi}{2}$, $t = 1$

$$\text{so, } I = \int_0^1 \sqrt{t}\ (1-t^2)^2\ dt$$

$$= \int_0^1 t^{\frac{1}{2}}(1+t^4-2t^2)\ dt$$

$$= \int_0^1 (t^{\frac{1}{2}} + t^{\frac{9}{2}} - 2t^{\frac{5}{2}})dt$$

$$=\left[\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\frac{t^{\frac{11}{2}}}{\frac{11}{2}}+\frac{2t^{\frac{7}{2}}}{\frac{7}{2}}\right]_0^1$$

$$=\frac{2}{3}+\frac{2}{11}-\frac{4}{7}$$

$$=\frac{154+42-132}{231}=\frac{64}{231}$$

3. $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

3. Given : $\int_0^2 x\sqrt{x+2}dx$

Let $x + 2 = t^2 \Rightarrow dx = 2t\,dt$

And $x = t^2-2$

when, $x = 0$, $t = \sqrt{2}$ and when $x = 2$, $t = 2$

so, $\int_0^2 x\sqrt{x+2}dx=\int_{\sqrt{2}}^2 \left(t^2-2\right)\sqrt{t^2}\ 2tdt$

$$=2\int_{\sqrt{2}}^2 \left(t^2-2\right)t.tdt$$

$$=2\int_{\sqrt{2}}^2 \left(t^2-2\right)t^2dt$$

$$=2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt$$

$$=2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2$$

$$=2\left[\frac{(2)^5}{5}-\frac{2(2)^3}{3}-\frac{(\sqrt{2})^5}{5}+\frac{2(\sqrt{2})^3}{3}\right]_{\sqrt{2}}^2$$

$$=2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]$$

$$=2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$$

$$=2\left[\frac{16+8\sqrt{2}}{15}\right]$$

$$=\left[\frac{16(2+\sqrt{2}}{15}\right]$$

$$=\frac{16\sqrt{2}(\sqrt{2}+1)}{15}$$

4. $\int_0^2 x\sqrt{x+2}(\text{Put} x+2=t^2)$

4. $\int_0^2 x\sqrt{x+2}dx$

Let x = 2 = $t^2 \Rightarrow dx = 2tdt$

When x = 0, $t=\sqrt{2}$ and when x' = 2, t = 2

$\therefore \int_0^2 x\sqrt{x+2}dx = \int_{\sqrt{2}}^2 (t^2-2)\sqrt{t^2}\,2tdt$

$= 2\int_{\sqrt{2}}^2 (t^2-2)t^2dt$

$= 2\int_{\sqrt{2}}^2 \left(t^4-2t^2\right)dt$

$= 2\left[\frac{t^5}{5}-\frac{2t^3}{3}\right]_{\sqrt{2}}^2$

$= 2\left[\frac{32}{5}-\frac{16}{3}-\frac{4\sqrt{2}}{5}+\frac{4\sqrt{2}}{3}\right]$

$= 2\left[\frac{96-80-12\sqrt{2}+20\sqrt{2}}{15}\right]$

$= 2\left[\frac{16+8\sqrt{2}}{15}\right]$

$= \frac{16\left(2+\sqrt{2}\right)}{15}$

$= \frac{16\sqrt{2}\left(\sqrt{2}+1\right)}{15}$

5. $\int_0^{\frac{x}{2}} \frac{\sin x}{1+\cos^2 x}dx$

5. Given : $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx$

Let cos x = t

$\Rightarrow -\sin x dx = dt$

$\Rightarrow \sin x dx = -dt$

When x = 0, t = 1 and when x = π /2, t = 0

$\Rightarrow \int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}dx = \int_1^0 \frac{-dt}{1+t^2}$

$= -\int_1^0 \frac{dt}{1+t^2}$

because, $\int \frac{dt}{x^2+a^2} = \frac{1}{a}.\tan^{-1}\frac{x}{a} + C$

$\Rightarrow -\int_1^0 \frac{dt}{1+t^2} = -\left[\frac{1}{1}.\tan^{-1} t\right]_1^0$

$= -[\tan^{-1} 0 - \tan^{-1} 1]$

$= -\left[0 - \frac{\pi}{4}\right]$

$= -\left[-\frac{\pi}{4}\right]$

$= \frac{\pi}{4}$

6. $\int_0^2 \frac{dx}{x+4-x^2}$

6. Given : $\int_0^2 \frac{dx}{x+4-x^2}$

$\int_0^2 \frac{dx}{x+4-x^2} = \int_0^2 \frac{dx}{-(x^2-x-4)}$

we can write it as, $\int_0^2 \frac{dx}{-(x^2-x+\frac{1}{4}-\frac{1}{4}-4)}$

$= \int_0^2 \frac{dx}{-\left[\left(x-\frac{1}{2}\right)^2 - \frac{17}{4}\right]}$

$= \int_0^2 \frac{dx}{\left[\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2\right]}$

let $x - \frac{1}{2} = t \Rightarrow dx = dt$

when $x = 0, t = -\frac{1}{2}$ and when $x = 2, t = \frac{3}{2}$

$\Rightarrow \int_0^2 \frac{dx}{\left[\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2\right]} = \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left[\left(\frac{\sqrt{17}}{2}\right)^2 - (t)^2\right]}$

because, $\int \frac{dx}{\left[(a)^2 - (x)^2\right]} = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

$$\Rightarrow \int_{-\frac{1}{2}}^{\frac{3}{2}} \frac{dt}{\left[\left(\frac{\sqrt{17}}{2}\right)^2 - (t)^2\right]} = \left[\frac{1}{2\left(\frac{\sqrt{17}}{2}\right)} \log \frac{\left(\frac{\sqrt{17}}{2} + t\right)}{\frac{\sqrt{17}}{2} - t}\right]_{-\frac{1}{2}}^{\frac{3}{2}}$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\left(\frac{\sqrt{17}}{2} + \frac{3}{2}\right)}{\frac{\sqrt{17}}{2} - \frac{3}{2}} - \log \frac{\left(\frac{\sqrt{17}}{2} - \frac{1}{2}\right)}{\frac{\sqrt{17}}{2} + \frac{1}{2}}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \frac{\left(\sqrt{17}+3\right)}{\sqrt{17}-3} - \log \frac{\left(\sqrt{17}-1\right)}{\sqrt{17}+1}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \left\{\frac{\left(\sqrt{17}+3\right)}{\sqrt{17}-3} \times \frac{\left(\sqrt{17}+1\right)}{\sqrt{17}-1}\right\}\right]$$

$$= \frac{1}{\sqrt{17}}\left[\log \left\{\frac{\left(\sqrt{17}+3\right)\left(\sqrt{17}+1\right)}{(\sqrt{17}-3)(\sqrt{17}-1)}\right\}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{17+3+4\sqrt{17}}{17+3-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{20+4\sqrt{17}}{20-4\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{5+\sqrt{17}}{5-\sqrt{17}}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(5+\sqrt{17}\right)(5+\sqrt{17})}{\left(5-\sqrt{17}\right)(5+\sqrt{17})}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(25+17+10\sqrt{17}\right)}{25-17}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(42+10\sqrt{17}\right)}{8}\right]$$

$$= \frac{1}{\sqrt{17}} \log \left[\frac{\left(21+5\sqrt{17}\right)}{4}\right]$$

7. $\int_{-1}^{1} \frac{dx}{x^2+2x+5}$

7. $\int_{-1}^{1} \frac{dx}{x^2+2x+5} = \int_{-1}^{1} \frac{dx}{(x^2+2x+1)+4} = \int_{-1}^{1} \frac{dx}{(x+1)^2+(2)^2}$

Let x + 1 = t $\Rightarrow$ dx = dt

When x = –1, t = 0 and when x = 1, t = 2

$\therefore \int_{-1}^{1} \frac{dx}{(x+2)^2+(2)^2} = \int_{0}^{2} \frac{dx}{t^2+2^2}$

$= \left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]_0^{-2}$

$= \frac{1}{2}\tan^{-1}1 - \frac{1}{2}\tan^{-1}0$

$= \frac{1}{2}\left(\frac{\pi}{4}\right) = \frac{\pi}{8}$

8. $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$

8. Given : $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx$

Let 2x = t $\Rightarrow$ 2 dx = dt

When x = 1, t = 2 and when x = 2, t = 4

$\Rightarrow \int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x} dx = \int_2^4 \left(\frac{1}{\left(\frac{t}{2}\right)} - \frac{1}{2\left(\frac{t}{2}\right)^2}\right) e^t \left(\frac{dt}{2}\right)$

$= \frac{1}{2}\int_2^4 \left(\frac{2}{t} - \frac{2}{t^2}\right) e^t dt$

$= \int_2^4 \frac{1}{2}.(2)\left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$

$= \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt$

now, let $1/t = f(t)$

then, $f'(t) = -1/t^2$

$\Rightarrow \int_2^4 \left(\frac{1}{t} - \frac{1}{t^2}\right) e^t dt = \int_2^4 \left(f(t) + f'(t)\right) e^t dt$

because, $\int \left(f(x) + f'(x)\right) e^x dx = e^x f(x) + C$

$\Rightarrow \int_2^4 \left(f(t) + f'(t)\right) e^t dt = \left[e^t f(t)\right]_2^4$

$$=\left[e^t.\frac{1}{t}\right]_2^4$$

$$=\frac{e^4}{4}-\frac{e^2}{2}$$

$$=\frac{e^4-2e^2}{4}$$

$$=\frac{e^2(e^2-2)}{4}$$

9. The value of the integral $\int_{\frac{1}{3}}^{1}\frac{\left(x-x^3\right)^{\frac{1}{3}}}{x^4}dx$ is

(a) 6 (b) 0

(c) 3 (d) 4

9. The correct option is : option (A)

$$\text{Given}: \int_{\frac{1}{3}}^{1}\left(\frac{(x-x^3)^{\frac{1}{3}}}{x^4}\right)dx$$

$$\text{let} I=\int_{\frac{1}{3}}^{1}\left(\frac{(x-x^3)^{\frac{1}{3}}}{x^4}\right)dx$$

Now, let $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

when, $x=\frac{1}{3}, \theta=\sin^{-1}\left(\frac{1}{3}\right)$ and when $x=1, \theta=\pi/2$

$$\Rightarrow I=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\left(\frac{(\sin\theta-\sin^3\theta)^{\frac{1}{3}}}{\sin^4\theta}\right)\cos\theta d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\left(\frac{(\sin\theta)^{\frac{1}{3}}(1-\sin^2\theta)^{\frac{1}{3}}}{\sin^4\theta}\right)\cos\theta d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\left(\frac{(\sin\theta)^{\frac{1}{3}}(\cos^2\theta)^{\frac{1}{3}}}{\sin^4\theta}\right)\cos\theta d\theta$$

$$=\int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}}\left(\frac{(\sin\theta)^{\frac{1}{3}}(\cos\theta)^{\frac{2}{3}}}{\sin^2\theta.\sin^2\theta}\right)\cos\theta d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left(\frac{(\cos\theta)^{\frac{2}{3}+1}}{(\sin\theta)^{2-\frac{1}{3}}} \right) \cdot \frac{1}{\sin^2\theta} d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left(\frac{(\cos\theta)^{\frac{5}{3}}}{(\sin\theta)^{\frac{5}{3}}} \right) .\text{cosec}^2\theta d\theta$$

$$= \int_{\sin^{-1}\left(\frac{1}{3}\right)}^{\frac{\pi}{2}} \left((\cot\theta)^{\frac{5}{3}} \right) .\text{cosec}^2\theta d\theta$$

Now, let $\cot\theta = t \Rightarrow -\text{cosec}^2\theta \, d\theta$

when, $\theta = \sin^{-1}\left(\frac{1}{3}\right)$, $t = 2\sqrt{2}$ and when $\theta = \frac{\pi}{2}$, $t = 0$

$$= \int_{2\sqrt{2}}^{0} -(t)^{\frac{5}{3}} .dt$$

$$= -\left[\frac{(t)^{\frac{5}{3}+1}}{\frac{5}{3}+1} \right]_{2\sqrt{2}}^{0}$$

$$= -\left[\frac{(t)^{\frac{8}{3}}}{\frac{8}{3}} \right]_{2\sqrt{2}}^{0}$$

$$= -\frac{3}{8}\left[(0)^{\frac{8}{3}} - \left(2\sqrt{2}\right)^{\frac{8}{3}} \right]$$

$$= -\frac{3}{8}\left[-\left(\sqrt{8}\right)^{\frac{8}{3}} \right]$$

$$= \frac{3}{8}\left[(8)^{\frac{4}{3}} \right]$$

$$= \frac{3}{8}[16]$$

$$= 6$$

10. If $f(x) = \int_0^x t \sin t \, dt$, then $f'(x)$ is

(a) $\cos x + x \sin x$ (b) $x \sin x$

(c) $x \cos x$ (d) $\sin x + x \cos x$

10. $f(x) = \int_0^x t \sin t \, dt$

Integrating by parts, we obtain

$$f(x) = t\int_0^x \sin t\,dt - \int_0^x \left\{\left(\frac{d}{dt}t\right)\int \sin t\,dt\right\}dt$$

$$= \left[t(-\cos t)\right]_0^x - \int_0^x (-\cos t)\,dt$$

$$= \left[t(-\cos t)\right]_0^x$$

$$= -x\cos x + \sin x$$

$$= x\sin x\ \text{w}$$

Hence, the correct Answer is B.