Activity 28

Objective

To find analytically $\lim_{x \to c} f(x) = \dfrac{x^2 - c^2}{x - c}$

Material Required

Pencil, white paper, calculator.

Method of Construction

1. Consider the function f given by $f(x) = \dfrac{x^2 - 9}{x - 3}$
2. In this case $c = 3$ and the function is not defined at $x = 3$.

Demonstration

1. Take some values of c less than $c = 3$ and some other values of c more than $c = 3$.
2. In both cases, the values to be taken have to be very close to $c = 3$.
3. Calculate the corresponding values of f at each of the values of c taken close to $c = 3$.

Demonstration : Table 1

1. Write the values of $f(x)$ in the following tables:

Table 1

x	2.9	2.99	2.999	2.9999	2.99999	2.999999
$f(x)$	5.9	5.99	5.999	5.9999	5.99999	5.999999

Table 2

x	3.1	3.01	3.001	3.0001	3.00001	3.000001
$f(x)$	6.1	6.01	6.001	6.0001	6.00001	6.000001

Observation

1. Values of $f(x)$ as $x \to 3$ from the left, as in Table 1 are coming closer and closer to _____.
2. Values of $f(x)$ as $x \to 3$ from the right, as in Table 2 are coming closer and closer to _____ from tables (2) and (3), $\lim_{x\to 3} f(x) = \frac{x^2-9}{x-3} =$ ______.

Application

This activity can be used to demonstrate the concept of a limit $\lim_{x\to c} f(x)$ when $f(x)$ is not defined at $x = c$.

Activity 29

Objective

Verification of the geometrical significance of derivative.

Material Required

Graph sheets, adhesive, hardboard, trigonometric tables, geometry box, wires.

Method of Construction

1. Paste three graph sheets on a hardboard and draw two mutually perpendicular lines representing x-axis and y-axis on each of them.

2. Sketch the graph of the curve (circle) $x^2 + y^2 = 25$ on one sheet.

3. On the other two sheets sketch the graphs of $(x-3)^2 + y^2 = 25$ and the curve $xy = 4$ (rectangular hyperbola).

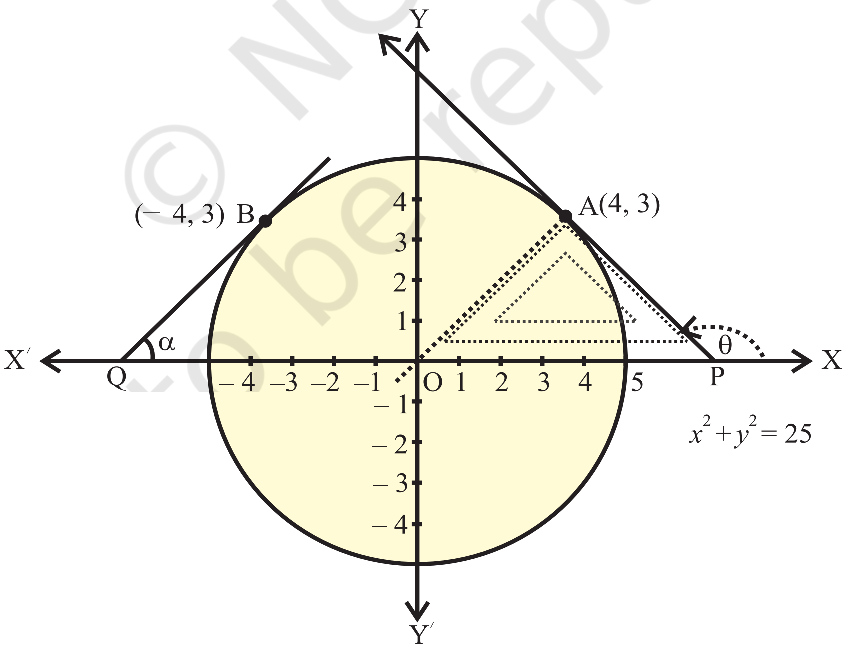

Fig. 29.1

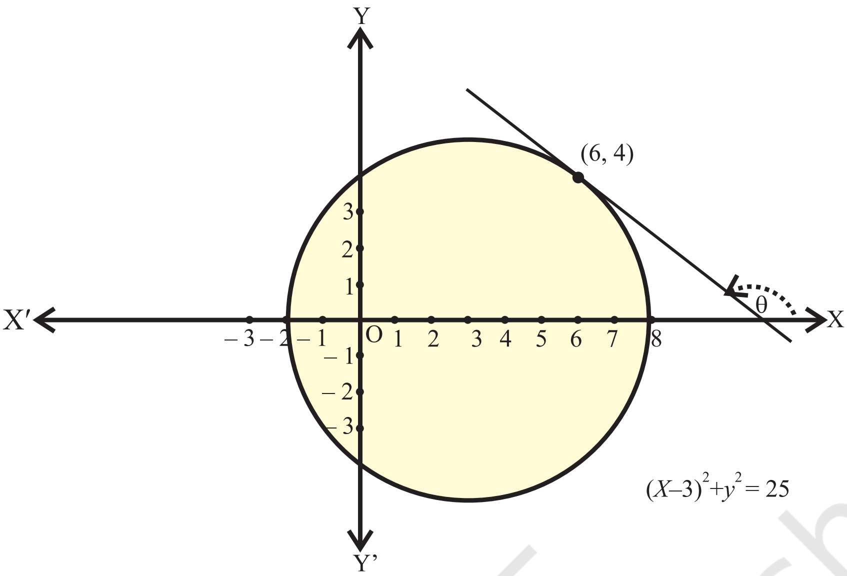

Fig 29.2

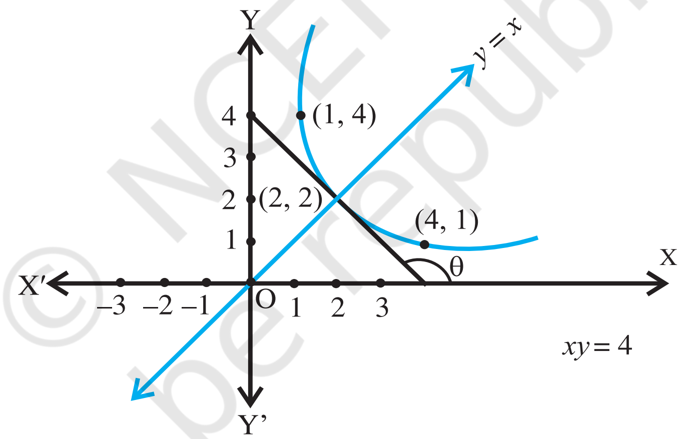

Fig. 29.3

DEMONSTRATION

1. Take first sheet on which, the graph of the circle $x^2 + y^2 = 25$ has been drawn (see Fig.29.1.
2. Take a point A (4, 3) on the circle.
3. With the help of a set square, place a wire in the direction OA and other perpendicular to OA at the point A to meet x-axis at a point (say P).

4. Measure the angle between the wire and the positive direction of x-axis at P (say θ).
5. Then find tan θ (with the help of trigonometric tables)

   Now, $x^2 + y^2 = 25 \Rightarrow y = \sqrt{25 - x^2} \Rightarrow \frac{dy}{dx} = \frac{-x}{\sqrt{25 - x^2}}$.

   Find $\frac{dy}{dx}$ at the point (4, 3) and verify that $\left(\frac{dy}{dx}\right)$ at (4, 3) = tan θ.

6. Similarly, take another point (– 4, 3) on the circle. Verify that $\frac{dy}{dx}$ at (– 4, 3) = tan α where α is the angle made by the tangent to the circle at the point (– 4, 3) with the positive direction of x-axis. (see Fig. 29.1).
7. Take other sheet with the graph of $(x - 3)^2 + y^2 = 25$ and take the point (6,4) on it and repeat the above process using set square and wires as shown in Fig. 29.2, i.e. verify that $\frac{dy}{dx}$ at (6, 4) = tan θ.
8. Now take the third sheet, showing the graph of the curve $xy = 4$. Take the point (2, 2) on it. Place one perpendicular side of set square along the line $y = x$ and a wire along the other side touching the curve at the point (2, 2) and find the angle made by the wire with the positive direction of x-axis as shown in Fig. 29.3. Let it be θ. Verify that $\frac{dy}{dx}$ at (2, 2) = tan θ.

Observation

1. For the curve $x^2 + y^2 = 25$, $\frac{dy}{dx}$ at the point (3, 4) = _________. Value of θ = ______ tan θ = _________ $\frac{dy}{dx}$ at (3, 4) = __________.

2. For the curve $x^2 + y^2 = 25$, $\frac{dy}{dx}$ at $(-4, 3)$ = ________, $\tan \alpha$ = ________

, $\frac{dy}{dx}$ at $(-4, 3)$ = ________.

3. For the curve $(x-3)^2 + y^2 = 25$, $\frac{dy}{dx}$ at $(6, 4)$ = _____ , value of θ = ________,

tan θ = _______, $\frac{dy}{dx}$ at $(6, 4)$ = ________.

4. For the curve $xy = 4$, $\left(\frac{dy}{dx}\right)$ at $(2, 2)$ = ______ ,

θ = ____, $\tan \theta$ = ______.

APPLICATION

Same activity can be used to verify the result that the slope of the tangent at a point is equal to the value of the derivative at that point for other curves.

NOTE

The activity may be repeated by taking point (4, 3) on first sheet, (0, 4) on second sheet and (1, 4) on the third sheet.

Activity 30

Objective

To obtain truth values of compound statements of the type $p \vee q$ by using switch connections in parallel.

Material Required

Switches, electric wires, battery and lamp/bulb.

Method of Construction

1. Connect switches S_1 and S_2 in parallel (See Fig. 30).
2. Connect battery and lamp so as to complete the circuit as shown in the figure.

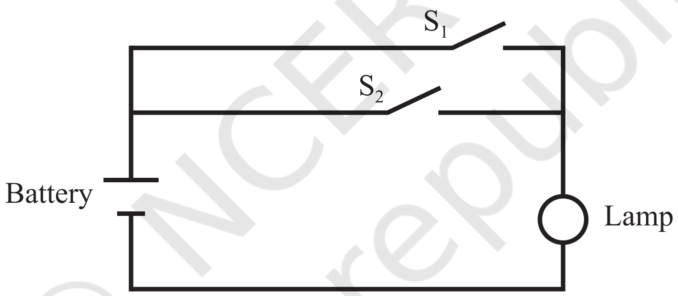

Fig. 30

Demonstration

1. The lamp will glow if atleast one of switches S_1, S_2 is on. This gives the following results:

Switch S_1	Switch S_2	Status of lamp
on	off	glow
off	on	glow
off	off	not glow
on	on	glow

Let p and q represent the statements as follows:

p : S_1 is on, truth value of p is T.

$\sim p$: S_1 is off, truth value of p is F.

q : S_2 is on, truth value of q is T.

$\sim q$: S_2 is off, truth value of q is F.

When the lamp glows, truth value of $p \vee q$ is T. When the lamp does not glow, truth value of $p \vee q$ is F. Thus, from the circuit, the following table gives the truth value of $p \vee q$:

p	q	$p \vee q$
T	F	T
F	T	T
F	F	F
T	T	T

Observation

1. If S_1 is on, truth value of p is _______________.
   If S_1 is off, truth value of p is _______________.
   If S_2 is on, truth value of q is _______________.
   If S_2 is off, truth value of q is _______________.
2. If S_1 is on, S_2 is off, truth value of $p \vee q$ is _________.
   If S_1 is on, S_2 is on, truth value of $p \vee q$ is _________.
   If S_1 is off, S_2 is off, truth value of $p \vee q$ is _________.
   If S_1 is off, S_2 is on, truth value of $p \vee q$ is _________.
   If S_1 is ______, S_2 is _____, truth value of $p \vee q$ is T.

Application

This activity helps in understanding truth values of the statements $p \vee q$ in different cases of the statements p and q.