

# NCERT Solutions for Class-XI Maths

## Chapter-13 Exercise-Miscellaneous

### NCERT Math Class 11

1. Find the derivative of the following functions from first principle:

(i)  $-x$

(ii)  $(-x)^{-1}$

(iii)  $\sin(x+1)$

(iv)  $\cos\left(x - \frac{\pi}{8}\right)$

1. (i) Let  $f(x) = -x$ . Accordingly,  $f(x+h) = -(x+h)$

By first principle,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-(x+h) - (-x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-x - h + x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{h}$$

$$= \lim_{h \rightarrow 0} (-1) = -1$$

(ii) Let  $f(x) = (-x)^{-1} = \frac{1}{-x} = \frac{-1}{x}$ . Accordingly,  $f(x+h) = \frac{-1}{(x+h)}$

By first principle,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{-1}{x+h} - \left( \frac{-1}{x} \right) \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{-1}{x+h} + \frac{1}{x} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{-x + (x+h)}{x(x+h)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{-x + x + h}{x(x+h)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{h}{x(x+h)} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{x(x+h)}$$

$$= \frac{1}{x \cdot x} = \frac{1}{x^2}$$

(iii) Let  $f(x) = \sin(x+1)$ . Accordingly,  $f(x+h) = \sin(x+h+1)$

By first principle,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} [\sin(x+h+1) - \sin(x+1)]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ 2 \cos \left( \frac{x+h+1+x+1}{2} \right) \sin \left( \frac{x+h+1-x-1}{2} \right) \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ 2 \cos \left( \frac{2x+h+2}{2} \right) \sin \left( \frac{h}{2} \right) \right]$$

$$= \lim_{h \rightarrow 0} \left[ \cos \left( \frac{2x+h+2}{2} \right) \cdot \frac{\sin \left( \frac{h}{2} \right)}{\left( \frac{h}{2} \right)} \right]$$

$$= \lim_{h \rightarrow 0} \cos \left( \frac{2x+h+2}{2} \right) \cdot \lim_{\frac{h}{2} \rightarrow 0} \frac{\sin \left( \frac{h}{2} \right)}{\left( \frac{h}{2} \right)} \left[ \text{As } h \rightarrow 0 \Rightarrow \frac{h}{2} \rightarrow 0 \right]$$

$$= \cos \left( \frac{2x+0+2}{2} \right) \cdot 1 \left[ \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \right]$$

$$= \cos(x+1)$$

(iv) Let  $f(x) = \cos \left( x - \frac{\pi}{8} \right)$ . Accordingly,  $f(x+h) = \cos \left( x+h - \frac{\pi}{8} \right)$

By first principle,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \cos \left( x + h - \frac{\pi}{8} \right) - \cos \left( x - \frac{\pi}{8} \right) \right] \\
&= \lim_{h \rightarrow 0} \frac{1}{h} \left[ -2 \sin \left( \frac{x + h - \frac{\pi}{8} + x - \frac{\pi}{8}}{2} \right) \sin \left( \frac{x + h - \frac{\pi}{8} - x + \frac{\pi}{8}}{2} \right) \right] \\
&= \lim_{h \rightarrow 0} \frac{1}{h} \left[ -2 \sin \left( \frac{2x + h - \frac{\pi}{4}}{2} \right) \sin \frac{h}{2} \right] \\
&= \lim_{h \rightarrow 0} \left[ -\sin \left( \frac{2x + h - \frac{\pi}{4}}{2} \right) \frac{\sin \left( \frac{h}{2} \right)}{\left( \frac{h}{2} \right)} \right] \\
&= \lim_{h \rightarrow 0} \left[ -\sin \left( \frac{2x + h - \frac{\pi}{4}}{2} \right) \right] \lim_{\frac{h}{2} \rightarrow 0} \frac{\sin \left( \frac{h}{2} \right)}{\left( \frac{h}{2} \right)} \left[ \text{As } h \rightarrow 0 \Rightarrow \frac{h}{2} \rightarrow 0 \right] \\
&= -\sin \left( \frac{2x + 0 - \frac{\pi}{4}}{2} \right) \cdot 1 \\
&= -\sin \left( x - \frac{\pi}{8} \right)
\end{aligned}$$

2. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $(x + a)$

2. Let  $f(x) = x + a$ . Accordingly,  $f(x + h) = x + h + a$

By first principle,

$$\begin{aligned}
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{x + h + a - (x + a)}{h} = \lim_{h \rightarrow 0} \frac{x + h + a - x - a}{h} \\
&= \lim_{h \rightarrow 0} \frac{h}{h} \\
&= \lim_{h \rightarrow 0} 1 \\
&= 1
\end{aligned}$$

3. Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $(px + q)\left(\frac{r}{x} + s\right)$

3. Let  $f(x) = (px + q)\left(\frac{r}{x} + s\right)$

$$f'(x) = (px + q)\left(\frac{r}{x} + s\right)' + \left(\frac{r}{x} + s\right)(px + q)'$$

$$= (px + q)(rx^{-1} + s)' + \left(\frac{r}{x} + s\right)(p)$$

$$= (px + q)(-rx^{-2}) + \left(\frac{r}{x} + s\right)p$$

$$= (px + q)\left(\frac{-r}{x^2}\right) + \left(\frac{r}{x} + s\right)p$$

$$= \frac{-pr}{x} - \frac{qr}{x^2} + \frac{pr}{x} + ps$$

$$= ps - \frac{qr}{x^2}$$

4. Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $(ax + b)(cx + d)^2$

4. Let  $f(x) = (ax + b)(cx + d)^2$

By product rule,

$$f'(x) = (ax + b)\frac{d}{dx}(cx + d)^2 + (cx + d)^2\frac{d}{dx}(ax + b)$$

$$= (ax + b)\frac{d}{dx}(c^2x^2 + 2cdx + d^2) + (cx + d)^2\frac{d}{dx}(ax + b)$$

$$= (ax + b)\left[\frac{d}{dx}(c^2x^2) + \frac{d}{dx}(2cdx) + \frac{d}{dx}d^2\right] + (cx + d)^2\left[\frac{d}{dx}ax + \frac{d}{dx}b\right]$$

$$= (ax + b)(2c^2x + 2cd) + (cx + d)^2 a$$

$$\therefore f'(x) = 2c(ax + b)(cx + d) + a(cx + d)^2$$

5. Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\frac{ax + b}{cx + d}$

5. Let  $f(x) = \frac{ax + b}{cx + d}$

$$\begin{aligned}
 f'(x) &= \frac{(cx+d)\frac{d}{dx}(ax+b) - (ax+b)\frac{d}{dx}(cx+d)}{(cx+d)^2} \\
 &= \frac{(cx+d)(a) - (ax+b)(c)}{(cx+d)^2} \\
 &= \frac{acx + ad - acx - bc}{(cx+d)^2} \\
 &= \frac{ad - bc}{(cx+d)^2}
 \end{aligned}$$

6. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

and s are fixed non-zero constants and m and n are integers):  $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$

6. Let  $f(x) = \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{x-1}{x}}$ , where  $x \neq 0$

By quotient rule

$$f'(x) = \frac{(x-1)\frac{d}{dx}(x+1) - (x+1)\frac{d}{dx}(x-1)}{(x-1)^2}, \quad x \neq 0, 1$$

$$= \frac{(x-1) \times 1 - (x+1) \times 1}{(x-1)^2}, \quad x \neq 0, 1$$

$$= \frac{x-1-x-1}{(x-1)^2}, \quad x \neq 0, 1$$

$$= -\frac{2}{(x-1)^2}, \quad x \neq 0, 1$$

$$\therefore f'(x) = -\frac{2}{(x-1)^2}, \quad x \neq 0, 1$$

7. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r

and s are fixed non-zero constants and m and n are integers):  $\frac{1}{ax^2 + bx + c}$

7. Let  $f(x) = \frac{1}{ax^2 + bx + c}$

By quotient rule,

$$\begin{aligned}
 f'(x) &= \frac{(ax^2 + bx + c) \frac{d}{dx}(1) - \frac{d}{dx}(ax^2 + bx + c)}{(ax^2 + bx + c)^2} \\
 &= \frac{(ax^2 + bx + c)(0) - (2ax + b)}{(ax^2 + bx + c)^2} \\
 &= \frac{-(2ax + b)}{(ax^2 + bx + c)^2}
 \end{aligned}$$

8. Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\frac{ax + b}{px^2 + qx + r}$

8. Let  $f(x) =$

By quotient rule

$$\begin{aligned}
 f'(x) &= \frac{(px^2 + qx + r) \frac{d}{dx}(ax + b) - (ax + b) \frac{d}{dx}(px^2 + qx + r)}{(px^2 + qx + r)^2} \\
 &= \frac{(ax^2 + bx + c) \times a - (ax + b)(2px + q)}{(px^2 + qx + r)^2} \\
 &= \frac{apx^2 + aqx + ar - 2apx^2 - aqx - 2bpx - bq}{(px^2 + qx + r)^2} \\
 \therefore f'(x) &= \frac{-apx^2 - 2bpx + ar - bq}{(px^2 + qx + r)^2}
 \end{aligned}$$

9. Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\frac{px^2 + qx + r}{ax + b}$

9. Let  $f(x) = \frac{px^2 + qx + r}{ax + b}$

By quotient rule,

$$f'(x) = \frac{(ax + b) \frac{d}{dx}(px^2 + qx + r) - (px^2 + qx + r) \frac{d}{dx}(ax + b)}{(ax + b)^2}$$

$$\begin{aligned}
&= \frac{(ax + b)(2px + q) - (px^2 + qx + r)(a)}{(ax + b)^2} \\
&= \frac{2apx^2 + aqx + 2bpx + bq - apx^2 - aqx - ar}{(ax + b)^2} \\
&= \frac{apx^2 + 2bpx + bq - ar}{(ax + b)^2}
\end{aligned}$$

10. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\frac{a}{x^4} - \frac{b}{x^2} + \cos x$

10. Let  $f(x) =$

$$\begin{aligned}
f'(x) &= \frac{d}{dx} \left( \frac{a}{x^4} \right) - \frac{d}{dx} \left( \frac{b}{x^2} \right) - \frac{d}{dx} (\cos x) \\
&= a \frac{d}{dx} (x^{-4}) - b \frac{d}{dx} (x^{-2}) + \frac{d}{dx} (\cos x) \\
&= a \times (-4x^{-5}) - b \times (-2x^{-3}) + (-\sin x) \\
&= -\frac{4a}{x^5} + \frac{2b}{x^3} - \sin x \\
\therefore f'(x) &= -\frac{4a}{x^5} + \frac{2b}{x^3} - \sin x
\end{aligned}$$

11. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $4\sqrt{x} - 2$

11. Let  $f(x) = 4\sqrt{x} - 2$

$$\begin{aligned}
f'(x) &= \frac{d}{dx} (4\sqrt{x} - 2) = \frac{d}{dx} (4\sqrt{x}) - \frac{d}{dx} (2) \\
&= 4 \frac{d}{dx} \left( x^{\frac{1}{2}} \right) - 0 = 4 \left( \frac{1}{2} x^{\frac{1}{2}-1} \right) \\
&= \left( 2x^{\frac{1}{2}} \right) = \frac{2}{\sqrt{x}}
\end{aligned}$$

12. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $(ax + b)^n$

12. Let  $f(x) = (ax + b)^n$   
By first principle

$$\begin{aligned}
 f'(x) &= \frac{d}{dx}(ax + b)^n \\
 &= n(ax + b)^{n-1} \frac{d}{dx}(ax + b) \\
 &= n(ax + b)^{n-1} \left[ \frac{d}{dx}(ax) + \frac{d}{dx}(b) \right] \\
 &= n(ax + b)^{n-1} (a + 0) \\
 \therefore f'(x) &= na (ax + b)^{n-1}
 \end{aligned}$$

13. Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $(ax + b)^n (cx + d)^m$

13. Let  $f(x) = (ax + b)^n (cx + d)^m$

$$f'(x) = (ax + b)^n \frac{d}{dx}(cx + d)^m + (cx + d)^m \frac{d}{dx}(ax + b)^n$$

Now, let  $f_1(x) = (cx + d)^m$

$$f_1(x + h) = (cx + ch + d)^m$$

$$f_1'(x) = \lim_{h \rightarrow 0} \frac{f_1(x + h) - f_1(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(cx + ch + d)^m - (cx + d)^m}{h}$$

$$= (cx + d)^m \lim_{h \rightarrow 0} \frac{1}{h} \left[ \left( 1 + \frac{ch}{cx + d} \right)^m - 1 \right]$$

$$= (cx + d)^m \lim_{h \rightarrow 0} \frac{1}{h} \left[ \left( 1 + \frac{mch}{(cx + d)} + \frac{m(m-1)}{2} \frac{(c^2h^2)}{(cx + d)^2} + \dots \right) - 1 \right]$$

$$= (cx + d)^m \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{mch}{(cx + d)} + \frac{m(m-1)c^2h^2}{2(cx + d)^2} + \dots \text{( Terms containing higher degrees of } h \text{)} \right]$$

$$= (cx + d)^m \lim_{h \rightarrow 0} \left[ \frac{mc}{(cx + d)} + \frac{m(m-1)c^2h}{2(cx + d)^2} + \dots \right]$$

$$= (cx + d)^m \left[ \frac{mc}{cx + d} + 0 \right]$$

$$= \frac{mc(cx + d)^m}{(cx + d)}$$

$$= mc(cx + d)^{m-1}$$

$$\frac{d}{dx}(cx + d)^m = mc(cx + d)^{m-1}$$

Similarly,  $\frac{d}{dx}(ax + b)^n = na(ax + b)^{n-1}$

Therefore, from (1), (2), and (3), we obtain

$$\begin{aligned} f'(x) &= (ax + b)^n \{ mc(cx + d)^{m-1} \} + (cx + d)^m \{ na(ax + b)^{n-1} \} \\ &= (ax + b)^{n-1} (cx + d)^{m-1} [ mc(ax + b) + na(cx + d) ] \end{aligned}$$

**14.** Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\sin(x+a)$

**14.** Let  $f(x) = \sin(x + a)$ , accordingly  $f(x + h) = \sin(x + h + a)$

By first principle,

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} [ \sin(x + h + a) - \sin(x + a) ]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ 2 \cos \left( \frac{x + h + a + x + a}{2} \right) \sin \left( \frac{x + h + a - x - a}{2} \right) \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ 2 \cos \left( \frac{2x + h + 2a}{2} \right) \sin \left( \frac{h}{2} \right) \right] = \lim_{h \rightarrow 0} \cos \left( \frac{2x + h + 2a}{2} \right) \times \lim_{h \rightarrow 0} \frac{\sin \left( \frac{h}{2} \right)}{\frac{h}{2}}$$

$$= \cos \left( \frac{2x + 0 + 2a}{2} \right) \times 1$$

$$= \cos(x + a)$$

**15.** Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\operatorname{cosec}x \cot x$

15. Let  $f(x) = \operatorname{cosec}x \cot x$

By product rule,

$$f'(x) = \operatorname{cosec}x(\cot x)' + \cot x(\operatorname{cosec}x)'$$

Let  $f_1(x) = \cot x$ . Accordingly,  $f_1(x+h) = \cot(x+h)$

By first principle,

$$\begin{aligned} f_1'(x) &= \lim_{h \rightarrow 0} \frac{f_1(x+h) - f_1(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cot(x+h) - \cot x}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos x}{\sin x} \right) \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin x \cos(x+h) - \cos x \sin(x+h)}{\sin x \sin(x+h)} \right] \end{aligned}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin(x-x-h)}{\sin x \sin(x+h)} \right] \\ &= \frac{1}{\sin x} \cdot \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin(-h)}{\sin(x+h)} \right] \\ &= \frac{-1}{\sin x} \cdot \left( \lim_{h \rightarrow 0} \frac{\sin h}{h} \right) \left( \lim_{h \rightarrow 0} \frac{1}{\sin(x+h)} \right) \\ &= \frac{-1}{\sin x} \cdot 1 \cdot \left( \frac{1}{\sin(x+0)} \right) \end{aligned}$$

$$= \frac{-1}{\sin 2x}$$

$$= -\operatorname{cosec}^2 x$$

$$\therefore (\cot x)' = -\operatorname{cosec}^2 x$$

Now, let  $f_2(x) = \operatorname{cosec}x$ . Accordingly,  $f_2(x+h) = \operatorname{cosec}(x+h)$

By first principle,

$$\begin{aligned} f_2'(x) &= \lim_{h \rightarrow 0} \frac{f_2(x+h) - f_2(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \operatorname{cosec}(x+h) - \operatorname{cosec}x \right] = \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{1}{\sin(x+h)} - \frac{1}{\sin x} \right] \end{aligned}$$

From (1), (2), and (3), we obtain

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin x - \sin(x+h)}{\sin x \sin(x+h)} \right] \\
&= \frac{1}{\sin x} \cdot \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{2 \cos\left(\frac{x+x+h}{2}\right) \sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)} \right] \\
&= \frac{1}{\sin x} \cdot \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{2 \cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{-h}{2}\right)}{\sin(x+h)} \right] \\
&= \frac{1}{\sin x} \cdot \lim_{h \rightarrow 0} \left[ \frac{-\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \cdot \frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)} \right] \\
&= \frac{-1}{\sin x} \cdot \lim_{h \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)} \cdot \lim_{h \rightarrow 0} \frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)} \\
&= \frac{-1}{\sin x} \cdot 1 \cdot \frac{\cos\left(\frac{2x+0}{2}\right)}{\sin(x+0)} \\
&= \frac{-1}{\sin x} \cdot \frac{\cos x}{\sin x} \\
&= -\operatorname{cosec} x \cdot \cot x
\end{aligned}$$

$$\therefore (\operatorname{cosec} x)' = -\operatorname{cosec} x \cdot \cot x$$

$$\begin{aligned}
f'(x) &= \operatorname{cosec} x (-\operatorname{cosec}^2 x) + \cot x (-\operatorname{cosec} x \cot x) \\
&= -\operatorname{cosec}^3 x - \cot^2 x \operatorname{cosec} x
\end{aligned}$$

16. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\frac{\cos x}{1 + \sin x}$

16. Let  $f(x) = \frac{\cos x}{1 + \sin x}$

By quotient rule

$$f'(x) = \frac{(1 + \sin x) \frac{d}{dx}(\cos x) - \cos x \frac{d}{dx}(1 + \sin x)}{(1 + \sin x)^2}$$

$$\begin{aligned}
&= \frac{(1 + \sin x)(-\sin x) - (\cos x)(\cos x)}{(1 + \sin x)^2} = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2} \\
&= \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1 + \sin x)^2} = \frac{-\sin x - 1}{(1 + \sin x)^2} \\
&= -\frac{(1 + \sin x)}{(1 + \sin x)^2} \\
\therefore f'(x) &= -\frac{1}{(1 + \sin x)}
\end{aligned}$$

17. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\frac{\sin x + \cos x}{\sin x - \cos x}$

17. Let  $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$

By quotient rule,

$$\begin{aligned}
f'(x) &= \frac{(\sin x - \cos x) \frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x) \frac{d}{dx}(\sin x - \cos x)}{(\sin x - \cos x)^2} \\
&= \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2} \\
&= \frac{-(\sin x - \cos x)^2 - (\sin x + \cos x)^2}{(\sin x - \cos x)^2} \\
&= \frac{-[\sin^2 x + \cos^2 x - 2\sin x \cos x + \sin^2 x + \cos^2 x + 2\sin x \cos x]}{(\sin x - \cos x)^2} \\
&= \frac{-[1 + 1]}{(\sin x - \cos x)^2} \\
&= \frac{-2}{(\sin x - \cos x)^2}
\end{aligned}$$

18. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\frac{\sec x - 1}{\sec x + 1}$

18. Let  $f(x) = \frac{\sec x - 1}{\sec x + 1}$

$$f(x) = \frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1} = \frac{1 - \cos x}{1 + \cos x}$$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{(1 + \cos x) \frac{d}{dx}(1 - \cos x) - (1 - \cos x) \frac{d}{dx}(1 + \cos x)}{(1 + \cos x)^2} \\ &= \frac{(1 + \cos x)(\sin x) - (1 - \cos x)(-\sin x)}{(1 + \cos x)^2} = \frac{\sin x + \cos x \sin x + \sin x - \sin x \cos x}{(1 + \cos x)^2} \\ &= \frac{2 \sin x}{(1 + \cos x)^2} = \frac{2 \sin x}{\cos^2 x \left(1 + \frac{1}{\cos x}\right)^2} = \frac{2 \tan x \sec x}{(\sec x + 1)^2} \\ \therefore f'(x) &= \frac{2 \tan x \sec x}{(\sec x + 1)^2} \end{aligned}$$

19. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\sin^n x$

19. Let  $y = \sin^n x$ .

Accordingly, for  $n = 1, y = \sin x$ .

$$\therefore \frac{dy}{dx} = \cos x, \text{ i.e., } \frac{d}{dx} \sin x = \cos x$$

For  $n = 2, y = \sin^2 x$ .

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sin x \sin x)$$

$$= (\sin x)' \sin x + \sin x (\sin x)'$$

$$= \cos x \sin x + \sin x \cos x$$

$$= 2 \sin x \cos x$$

[By Leibnitz product rule]

For  $n = 3, y = \sin^3 x$ .

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sin x \sin^2 x)$$

$$= (\sin x)' \sin^2 x + \sin x (\sin^2 x)' \quad [\text{By Leibnitz product rule}]$$

$$\begin{aligned}
&= \cos x \sin^2 x + \sin x (2 \sin x \cos x) \quad [\text{Using (1)}] \\
&= \cos x \sin^2 x + 2 \sin^2 x \cos x \\
&= 3 \sin^2 x \cos x
\end{aligned}$$

We assert that  $\frac{d}{dx}(\sin^n x) = n \sin^{(n-1)} x \cos x$

Let our assertion be true for  $n = k$ .

$$\text{i.e., } \frac{d}{dx}(\sin^k x) = k \sin^{(k-1)} x \cos x$$

Consider

$$\begin{aligned}
&\frac{d}{dx}(\sin^{k+1} x) \\
&= \frac{d}{dx}(\sin x \sin^k x) \\
&= (\sin x)' \sin^k x + \sin x (\sin^k x)' \quad [\text{By Leibnitz product rule}] \\
&= \cos x \sin^k x + \sin x (k \sin^{(k-1)} x \cos x) \\
&= \cos x \sin^k x + k \sin^k x \cos x \\
&= (k+1) \sin^k x \cos x
\end{aligned}$$

Thus, our assertion is true for  $n = k + 1$ .

Hence, by mathematical induction,  $\frac{d}{dx}(\sin^n x) = n \sin^{(n-1)} x \cos x$

- 20.** Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\frac{a + b \sin x}{c + d \cos x}$

**20.** Let  $f(x) = \frac{a + b \sin x}{c + d \cos x}$

By quotient rule,

$$\begin{aligned}
f'(x) &= \frac{(c + d \cos x) \frac{d}{dx}(a + b \sin x) - (a + b \sin x) \frac{d}{dx}(c + d \cos x)}{(c + d \cos x)^2} \\
&= \frac{(c + d \cos x)(b \cos x) - (a + b \sin x)(-d \sin x)}{(c + d \cos x)^2} \\
&= \frac{cb \cos x + bdc \cos^2 x + ad \sin x + bds \sin^2 x}{(c + d \cos x)^2} = \frac{bc \cos x + ad \sin x + bd(\cos^2 x + \sin^2 x)}{(c + d \cos x)^2}
\end{aligned}$$

$$\therefore f'(x) = \frac{bc \cos x + ad \sin x + bd}{(c + d \cos x)^2}$$

21. Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers):  $\frac{\sin(x+a)}{\cos x}$

21. Let  $f(x) = \frac{\sin(x+a)}{\cos x}$

By quotient rule,

$$f'(x) = \frac{\cos x \frac{d}{dx} [\sin(x+a)] - \sin(x+a) \frac{d}{dx} \cos x}{\cos^2 x}$$

$$f'(x) = \frac{\cos x \frac{d}{dx} [\sin(x+a)] - \sin(x+a)(-\sin x)}{\cos^2 x}$$

Let  $g(x) = \sin(x+a)$ . Accordingly,  $g(x+h) = \sin(x+h+a)$

By first principle,

$$g'(x) = \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} [\sin(x+h+a) - \sin(x+a)]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ 2 \cos \left( \frac{x+h+a+x+a}{2} \right) \sin \left( \frac{x+h+a-x-a}{2} \right) \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ 2 \cos \left( \frac{2x+2a+h}{2} \right) \sin \left( \frac{h}{2} \right) \right]$$

$$= \lim_{h \rightarrow 0} \left[ \cos \left( \frac{2x+2a+h}{2} \right) \left\{ \frac{\sin \left( \frac{h}{2} \right)}{\left( \frac{h}{2} \right)} \right\} \right]$$

$$= \lim_{h \rightarrow 0} \cos \left( \frac{2x+2a+h}{2} \right) \cdot \lim_{\frac{h}{2} \rightarrow 0} \left\{ \frac{\sin \left( \frac{h}{2} \right)}{\left( \frac{h}{2} \right)} \right\} \left[ \text{As } h \rightarrow 0 \Rightarrow \frac{h}{2} \rightarrow 0 \right]$$

$$= \left( \cos \frac{2x+2a}{2} \right) \times 1 \left[ \lim_{h \rightarrow 0} \frac{\sin h}{h} = 1 \right]$$

$$= \cos(x+a)$$

From (i) and (ii), we obtain

$$\begin{aligned}
 f'(x) &= \frac{\cos x \cdot \cos(x+a) + \sin x \sin(x+a)}{\cos^2 x} \\
 &= \frac{\cos(x+a-x)}{\cos^2 x} \\
 &= \frac{\cos a}{\cos^2 x}
 \end{aligned}$$

22. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $x^4(5\sin x - 3\cos x)$

22. Let  $f(x) = x^4(5\sin x - 3\cos x)$

By product rule,

$$\begin{aligned}
 f'(x) &= x^4 \frac{d}{dx}(5\sin x - 3\cos x) + (5\sin x - 3\cos x) \frac{d}{dx}(x^4) \\
 &= x^4 \times \left[ 5 \frac{d}{dx}(\sin x) - 3 \frac{d}{dx}(\cos x) \right] + (5\sin x - 3\cos x) \frac{d}{dx}(x^4) \\
 &= x^4 [5\cos x - 3(-\sin x)] + (5\sin x - 3\cos x)(4x^3) \\
 \therefore f'(x) &= x^3 [5x\cos x + 3x\sin x + 20\sin x - 12\cos x]
 \end{aligned}$$

23. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $(x^2 + 1)\cos x$

23. Let  $f(x) = (x^2 + 1)\cos x$

By product rule,

$$\begin{aligned}
 f'(x) &= (x^2 + 1) \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(x^2 + 1) \\
 &= (x^2 + 1)(-\sin x) + \cos x(2x) \\
 &= -x^2\sin x - \sin x + 2x\cos x
 \end{aligned}$$

24. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $(ax^2 + \sin x)(p + q\cos x)$

24. Let  $f(x) = (ax^2 + \sin x)(p + q\cos x)$

By product rule,

$$\begin{aligned}
 f'(x) &= (ax^2 + \sin x) \frac{d}{dx}(p + q\cos x) + (p + q\cos x) \frac{d}{dx}(ax^2 + \sin x) \\
 &= (ax^2 + \sin x)(-q\sin x) + (p + q\cos x)(2ax + \cos x) \\
 \therefore f'(x) &= (ax^2 + \sin x)(-q\sin x) + (p + q\cos x)(2ax + \cos x)
 \end{aligned}$$

25. Find the derivative of the following functions (it is to be understood that  $a, b, c, d, p, q, r$  and  $s$  are fixed non-zero constants and  $m$  and  $n$  are integers ):  $(x + \cos x)(x - \tan x)$

25. Let  $f(x) = (x + \cos x)(x - \tan x)$

By product rule,

$$\begin{aligned} f'(x) &= (x + \cos x) \frac{d}{dx}(x - \tan x) + (x - \tan x) \frac{d}{dx}(x + \cos x) \\ &= (x + \cos x) \left[ \frac{d}{dx}(x) - \frac{d}{dx}(\tan x) \right] + (x - \tan x)(1 - \sin x) \\ &= (x + \cos x) \left[ 1 - \frac{d}{dx} \tan x \right] + (x - \tan x)(1 - \sin x) \end{aligned}$$

Let  $g(x) = \tan x$ . Accordingly,  $g(x + h) = \tan(x + h)$

By first principle,

$$\begin{aligned} g'(x) &= \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \\ &= \lim_{h \rightarrow 0} \left( \frac{\tan(x+h) - \tan x}{h} \right) \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin(x+h)}{\cos(x+h)} - \frac{\sin x}{\cos x} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin(x+h)\cos x - \sin x \cos(x+h)}{\cos(x+h)\cos x} \right] \\ &= \frac{1}{\cos x} \cdot \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin(x+h-x)}{\cos(x+h)} \right] \\ &= \frac{1}{\cos x} \cdot \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin h}{\cos(x+h)} \right] \\ &= \frac{1}{\cos x} \cdot \left( \lim_{h \rightarrow 0} \frac{\sin h}{h} \right) \cdot \left( \lim_{h \rightarrow 0} \frac{1}{\cos(x+h)} \right) \\ &= \frac{1}{\cos x} \cdot 1 \cdot \frac{1}{\cos(x+0)} \\ &= \frac{1}{\cos^2 x} \\ &= \sec^2 x \end{aligned}$$

Therefore, from (i) and (ii), we obtain

$$f'(x) = (x + \cos x)(1 - \sec^2 x) + (x - \tan x)(1 - \sin x)$$

$$= (x + \cos x)(-\tan^2 x) + (x - \tan x)(1 - \sin x)]$$

$$= -\tan^2 x(x + \cos x) + (x - \tan x)(1 - \sin x)$$

26. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\frac{4x + 5 \sin x}{3x + 7 \cos x}$

26. Let  $f(x) = \frac{4x + 5 \sin x}{3x + 7 \cos x}$

By quotient rule,

$$f'(x) = \frac{(3x + 7 \cos x) \frac{d}{dx}(4x + 5 \sin x) - (4x + 5 \sin x) \frac{d}{dx}(3x + 7 \cos x)}{(3x + 7 \cos x)^2}$$

$$= \frac{(3x + 7 \cos x)(4 + 5 \cos x) - (4x + 5 \sin x)(3 - 7 \sin x)}{(3x + 7 \cos x)^2}$$

$$= \frac{12x + 15x \cos x + 28 \cos x + 35 \cos^2 x - 12x + 28x \sin x - 15 \sin x + 35 \sin^2 x}{(3x + 7 \cos x)^2}$$

$$= \frac{15x \cos x + 28 \cos x + 28x \sin x - 15 \sin x + 35(\cos^2 x + \sin^2 x)}{(3x + 7 \cos x)^2}$$

$$\therefore f'(x) = \frac{35 + 15x \cos x + 28 \cos x + 28x \sin x - 15 \sin x}{(3x + 7 \cos x)^2}$$

27. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):

$$\frac{x^2 \cos\left(\frac{\pi}{4}\right)}{\sin x}$$

27. Let  $f(x) = \frac{x^2 \cos\left(\frac{\pi}{4}\right)}{\sin x}$

By quotient rule,

$$f'(x) = \cos \frac{\pi}{4} \cdot \left[ \frac{\sin x \frac{d}{dx}(x^2) - x^2 \frac{d}{dx}(\sin x)}{\sin^2 x} \right]$$

$$= \cos \frac{\pi}{4} \cdot \left[ \frac{\sin x \cdot 2x - x^2 \cos x}{\sin^2 x} \right]$$

$$= \frac{x \cos \frac{\pi}{4} [2 \sin x - x \cos x]}{\sin^2 x}$$

28. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\frac{x}{1 + \tan x}$

28. Let  $f(x) = \frac{x}{1 + \tan x}$

By quotient rule,

$$f'(x) = \frac{(1 + \tan x) \frac{d}{dx}(x) - x \frac{d}{dx}(1 + \tan x)}{(1 + \tan x)^2}$$

$$= \frac{(1 + \tan x) - x \sec^2 x}{(1 + \tan x)^2}$$

$$\therefore f'(x) = \frac{1 + \tan x - x \sec^2 x}{(1 + \tan x)^2}$$

29. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $(x + \sec x)(x - \tan x)$

29. Let  $f(x) = (x + \sec x)(x - \tan x)$

By product rule,

$$\begin{aligned} f'(x) &= (x + \sec x) \frac{d}{dx}(x - \tan x) + (x - \tan x) \frac{d}{dx}(x + \sec x) \\ &= (x + \sec x) \left[ \frac{d}{dx}(x) - \frac{d}{dx} \tan x \right] + (x - \tan x) \left[ \frac{d}{dx}(x) + \frac{d}{dx} \sec x \right] \\ &= (x + \sec x) \left[ 1 - \frac{d}{dx} \tan x \right] + (x - \tan x) \left[ 1 + \frac{d}{dx} \sec x \right] \end{aligned}$$

Let  $f_1(x) = \tan x, f_2(x) = \sec x$

Accordingly,  $f_1(x+h) = \tan(x+h)$  and  $f_2(x+h) = \sec(x+h)$

$$\begin{aligned} f_1'(x) &= \lim_{h \rightarrow 0} \left( \frac{f_1(x+h) - f_1(x)}{h} \right) \\ &= \lim_{h \rightarrow 0} \left( \frac{\tan(x+h) - \tan x}{h} \right) \\ &= \lim_{h \rightarrow 0} \left[ \frac{\tan(x+h) - \tan x}{h} \right] \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin(x+h) - \sin x}{\cos(x+h) - \cos x} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin(x+h)\cos x - \sin x \cos(x+h)}{\cos(x+h)\cos x} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin(x+h-x)}{\cos(x+h)\cos x} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\sin h}{\cos(x+h)\cos x} \right]$$

$$= \left( \lim_{h \rightarrow 0} \frac{\sin h}{h} \right) \cdot \left( \lim_{h \rightarrow 0} \frac{1}{\cos(x+h)\cos x} \right)$$

$$= 1 \times \frac{1}{\cos^2 x} = \sec^2 x$$

$$\Rightarrow \frac{d}{dx} \tan x = \sec^2 x \quad \text{(ii)}$$

$$f_2'(x) = \lim_{h \rightarrow 0} \left( \frac{f_2(x+h) - f_2(x)}{h} \right)$$

$$= \lim_{h \rightarrow 0} \left( \frac{\sec(x+h) - \sec x}{h} \right)$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{1}{\cos(x+h)} - \frac{1}{\cos x} \right]$$

$$= \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{\cos x - \cos(x+h)}{\cos(x+h)\cos x} \right]$$

$$= \frac{1}{\cos x} \cdot \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{-2\sin\left(\frac{x+x+h}{2}\right) \cdot \sin\left(\frac{x-x-h}{2}\right)}{\cos(x+h)} \right]$$

$$= \frac{1}{\cos x} \cdot \lim_{h \rightarrow 0} \frac{1}{h} \left[ \frac{-2\sin\left(\frac{2x+h}{2}\right) \cdot \sin\left(\frac{-h}{2}\right)}{\cos(x+h)} \right]$$



$$= \frac{1}{\cos x} \cdot \lim_{h \rightarrow 0} \left[ \frac{\sin\left(\frac{2x+h}{2}\right) \left\{ \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \right\}}{\cos(x+h)} \right]$$

$$\frac{\left\{ \lim_{h \rightarrow 0} \sin\left(\frac{2x+h}{2}\right) \right\} \left\{ \lim_{\frac{h}{2} \rightarrow 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \right\}}{\lim_{h \rightarrow 0} \cos(x+h)}$$

$$= \sec x \cdot \frac{\sin x \cdot 1}{\cos x}$$

$$\Rightarrow \frac{d}{dx} \sec x = \sec x \tan x \quad (\text{iii})$$

From (i), (ii), and (iii), we obtain

$$f'(x) = (x + \sec x)(1 - \sec^2 x) + (x - \tan x)(1 + \sec x \tan x)$$

30. Find the derivative of the following functions (it is to be understood that a, b, c, d, p, q, r and s are fixed non-zero constants and m and n are integers):  $\frac{x}{\sin^n x}$

29. Let  $f(x) = \frac{x}{\sin^n x}$

By quotient rule,

$$\begin{aligned} f'(x) &= \frac{\sin^n x \frac{d}{dx} x - x \frac{d}{dx} \sin^n x}{\sin^{2n} x} \\ &= \frac{\sin^n x \times 1 - x \left( n \sin^{(n-1)} x \times \frac{d}{dx} (\sin x) \right)}{\sin^{2n} x} \\ &= \frac{\sin^n x - x \left( \sin^{n-1} x \cos x \right)}{\sin^{2n} x} \\ &= \frac{\sin^{n-1} x (\sin x - nx \cos x)}{\sin^{2n} x} \end{aligned}$$

$$\therefore f'(x) = \frac{\sin x - nx \cos x}{\sin^{n+1} x}$$



**Myclass24**  
Your Class. Your Pace.