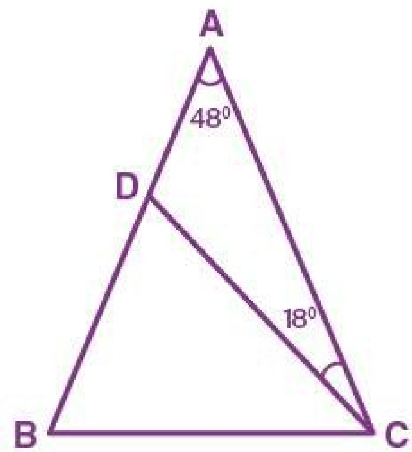
Exercise 10(A)

1.

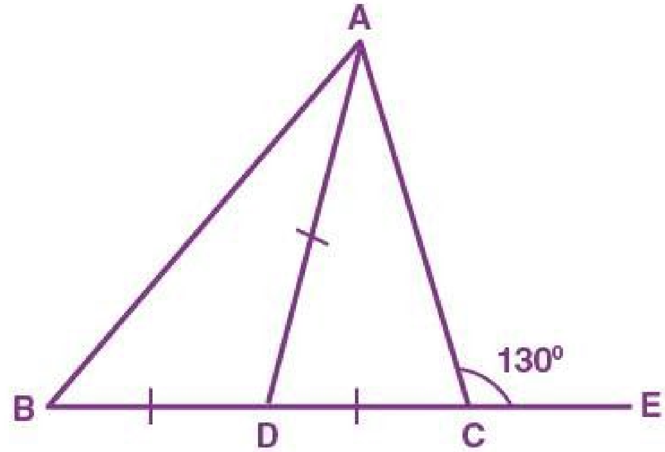
2. In the figure alongside,



AB = AC
$\angle A = 48^o$ and
$\angle ACD = 18^o$.
Show that BC = CD.
Solution:

In ΔABC, we have
$\angle BAC + \angle ACB + \angle ABC = 180^0$
$48^0 + \angle ACB + \angle ABC = 180^0$
But, $\angle ACB = \angle ABC$ [Given, AB = AC]
$2\angle ABC = 180^0 - 48^0$
$2\angle ABC = 132^0$
$\angle ABC = 66^0 = \angle ACB$ ……(i)
$\angle ACB = 66^0$
$\angle ACD + \angle DCB = 66^0$
$18^0 + \angle DCB = 66^0$
$\angle DCB = 48^0$ ………(ii)
Now, In ΔDCB,
$\angle DBC = 66^0$ [From (i), Since $\angle ABC = \angle DBC$]
$\angle DCB = 48^0$ [From (ii)]
$\angle BDC = 180^0 - 48^0 - 66^0$
$\angle BDC = 66^0$
Since $\angle BDC = \angle DBC$
Therefore, BC = CD
Equal angles have equal sides opposite to them.

3. Calculate:
(i) $\angle ADC$
(ii) $\angle ABC$
(iii) $\angle BAC$



Solution:

Given: ∠ACE = 130^0; AD = BD = CD
Proof:
(i) ∠ACD + ∠ACE = 180° [DCE is a straight line]
∠ACD = 180° – 130°
∠ACD = 50°
Now,
CD = AD
∠ACD = ∠DAC = 50° … (i) [Since angles opposite to equal sides are equal]
In ΔADC,
∠ACD = ∠DAC = 50°
∠ACD + ∠DAC + ∠ADC = 180°
50° + 50° + ∠ADC = 180°
∠ADC = 180° – 100°
∠ADC = 80°

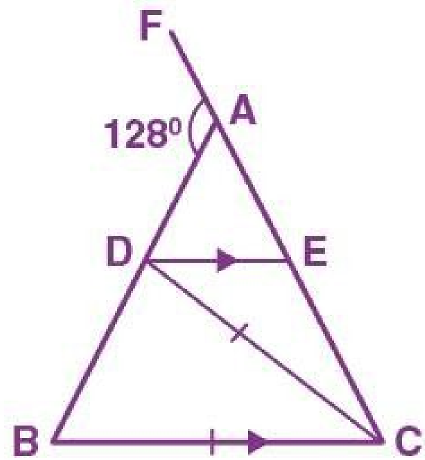
(ii) ∠ADC = ∠ABD + ∠DAB [Exterior angle is equal to sum of opposite interior angles]
But, AD = BD
∴ ∠DAB=∠ABD
80° = ∠ABD + ∠ABD
2∠BD = 80^O
∠ABD = 40^O = ∠DAB … (ii)

(iii) We have,
∠BAC = ∠DAB + ∠DAC
Substituting the values from (i) and (ii),
∠BAC = 40^O + 50^O
Hence, ∠BAC = 90^O

4. In the following figure, AB = AC; BC = CD and DE is parallel to BC. Calculate:
(i) ∠CDE

(ii)

(iii)∠DCE



Solution:

Given, ∠FAB = 128^O
∠BAC + ∠FAB = 180^O [As FAC is a straight line]
∠BAC = 180^O – 128^O
∠BAC = 52^O

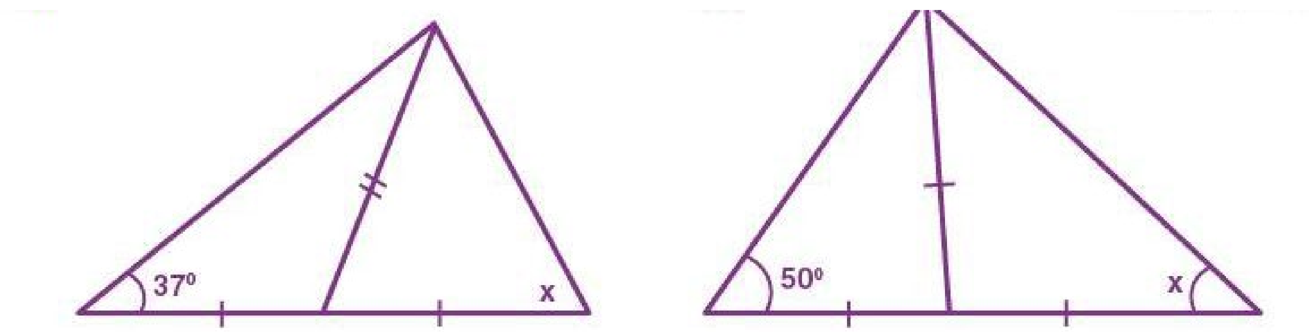
In ΔABC, we have
∠A = 52^O
∠B = ∠C [Given AB = AC and angles opposite to equal sides are equal]
Now, by angle sum property
∠A + ∠B + ∠C =180^O
∠A + ∠B + ∠B = 180^O
52^O+ 2∠B = 180^O
2∠B = 128^O
∠B = 64^O = ∠C… (i)
∠B = ∠ADE [Given DE ll BC]

(i) Now, ∠ADE + ∠CDE + ∠B = 180^O [As ADB is a straight line]
64^O + ∠CDE + 64^O= 180^O
∠CDE = 180^O+ 128^O
∠CDE = 52^O

(ii) Given DE ll BC and DC is the transversal
∠CDE = ∠DCB = 52^o… (ii)
Also, ∠ECB = 64^o… [From (i)]
But,
∠ECB = ∠DCE +∠DCB
64^o = ∠DCE + 52^o
∠DCE = 64^o - 52^o

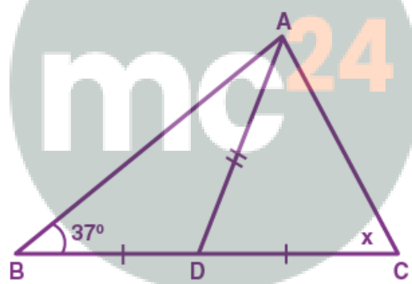
$\angle DCE = 12^\circ$

5. Calculate x:



Solution:

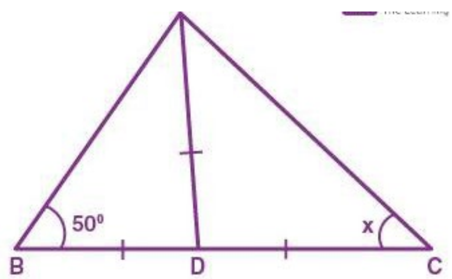
(i) Let the triangle be ABC and the altitude be AD.



In $\triangle ABD$, we have
$\angle DBA = \angle DAB = 37^\circ$ [Given BD = AD and angles opposite to equal sides are equal]
Now,
$\angle CDA = \angle DBA + \angle DAB$ [Exterior angle is equal to the sum of opposite interior angles]
$\angle CDA = 37^\circ + 37^\circ$
$\therefore \angle CDA = 74^\circ$

Now, in $\triangle ADC$, we have
$\angle CDA = \angle CAD = 74^\circ$ [Given CD = AC and angels opposite to equal sides are equal]
Now, by angle sum property
$\angle CAD + \angle CDA + \angle ACD = 180^\circ$
$74^\circ + 74^\circ + x = 180^\circ$
$x = 180^\circ - 148^\circ$
$x = 32^\circ$

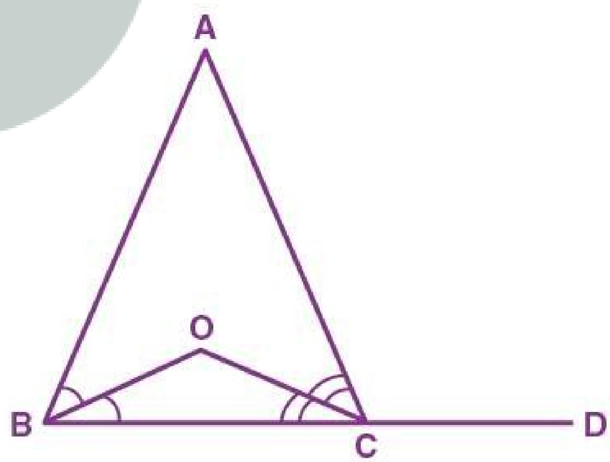
(ii) Let triangle be ABC and altitude be AD.



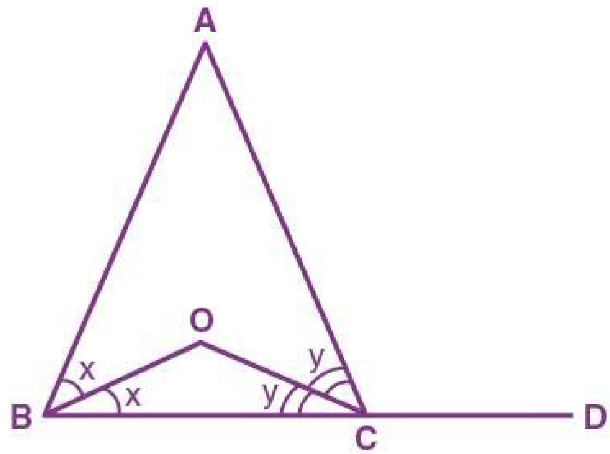
In △ABD, we have
∠DBA = ∠DAB = 50° [Given BD = AD and angles opposite to equal sides are equal]
Now,
∠CDA = ∠DBA + ∠DAB [Exterior angle is equal to the sum of opposite interior angles]
∠CDA = 50° + 50°
∴ ∠CDA = 100°

In △ADC, we have
∠DAC = ∠DCA = x [Given AD = DC and angels opposite to equal sides are equal]
So, by angle sum property
∠DAC + ∠DCA + ∠ADC = 180°
x + x + 100° = 180°
2x = 80°
x = 40°

6. In the figure, given below, AB = AC. Prove that: ∠BOC = ∠ACD.



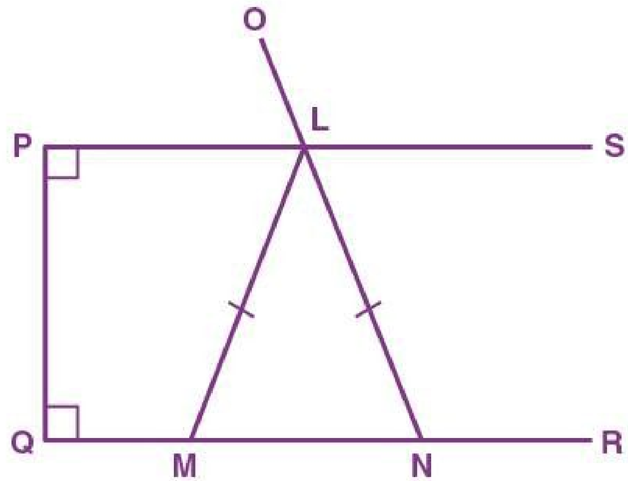
Solution:



Let's assume ∠ABO = ∠OBC = x and ∠ACO = ∠OCB = y
In ABC, we have
∠BAC = 180° - 2x - 2y…(i)
As, ∠B = ∠C [Since, AB = AC]
½ ∠B = ½ ∠C
⇒ x = y
Now,
∠ACD = 2x + ∠BAC [Exterior angle is equal to sum of opposite interior angle]
= 2x + 180° - 2x - 2y [From (i)]
∠ACD = 180° - 2y… (ii)
In ΔOBC, we have
∠BOC = 180° - x - y
∠BOC = 180° - y – y [Since x = y]
∠BOC = 180° - 2y… (iii)

Thus, from (ii) and (iii) we get
∠BOC = ∠ACD

7. In the figure given below, LM = LN; ∠PLN = 110°. Calculate:

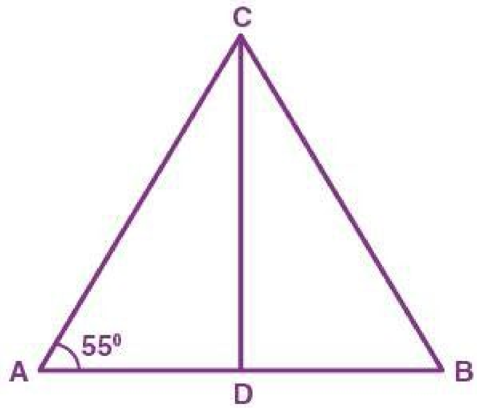


(i) ∠LMN
(ii) ∠MLN
Solution:

Given, LM = LN and ∠PLN = 110°
(i) We know that the sum of the measure of all the angles of a quadrilateral is 360°.
In quad. PQNL,
∠QPL + ∠PLN +LNQ + ∠NQP = 360°
90° + 110° + ∠LNQ + 90° = 360°
∠LNQ = 360° – 290°
∠LNQ = 70°
∠LNM = 70°… (i)
In ∆LMN, we have
LM = LN [Given]
⇒ ∠LNM = ∠LMN [Angles opposite to equal sides are equal]
∠LMN = 70°…(ii) [From (i)]

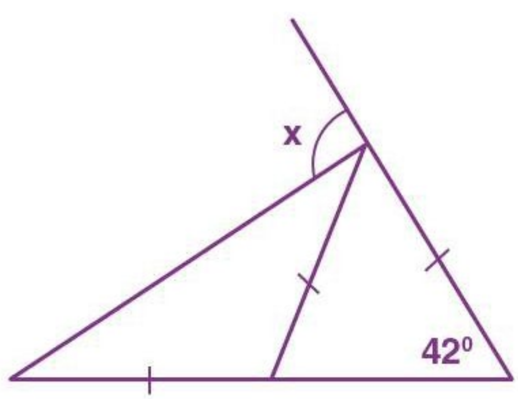
(ii) In ∆LMN, we have
∠LMN + ∠LNM + ∠MLN = 180°
But, ∠LNM = ∠LMN = 70° [From (i) and (ii)]
⇒ 70° + 70° + ∠MLN = 180°
∠MLN = 180° - 140°
∴ ∠MLN = 40°

8. An isosceles triangle ABC has AC = BC. CD bisects AB at D and ∠CAB = 55°.
Find: (i) ∠DCB (ii) ∠CBD.
Solution:



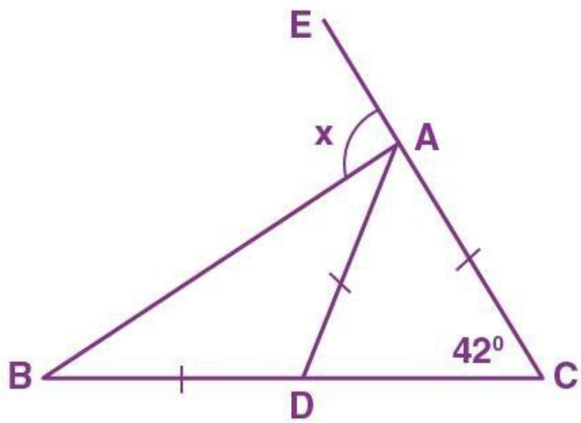
In $\triangle ABC$, we have
AC = BC [Given]
So, $\angle CAB = \angle CBD$ [Angles opposite to equal sides are equal]
$\Rightarrow \angle CBD = 55^o$
In $\triangle ABC$, we have
$\angle CBA + \angle CAB + \angle ACB = 180^o$
But, $\angle CAB = \angle CBA = 55^o$
$55^o + 55^o + \angle ACB = 180^o$
$\angle ACB = 180^o - 110^o$
$\angle ACB = 70^o$
Now,
In $\triangle ACD$ and $\triangle BCD$, we have
AC = BC [Given]
CD = CD [Common]
AD = BD [Given that CD bisects AB]
$\therefore \triangle ACD \cong \triangle BCD$
So, By CPCT
$\angle DCA = \angle DCB$
$\angle DCB = \angle ACB/2 = 70^o/2$
Thus, $\angle DCB = 35^o$

9. Find x:



Solution:

Let's put markings to the figure as following:



In $\triangle ABC$, we have

AD = AC [Given]

$\therefore \angle ADC = \angle ACD$ [Angles opposite to equal sides are equal]

So, $\angle ADC = 42^o$

Now,

$\angle ADC = \angle DAB + DBA$ [Exterior angle is equal to the sum of opposite interior angles]

But,

$\angle DAB = \angle DBA$ [Given: BD = DA]

$\therefore \angle ADC = 2\angle DBA$

$2\angle DBA = 42^o$

$\angle DBA = 21^o$

To find x:

$x = \angle CBA + \angle BCA$ [Exterior angle is equal to the sum of opposite interior angles]

We know that,

$\angle CBA = 21^o$

$\angle BCA = 42^o$

$\Rightarrow x = 21^o + 42^o$

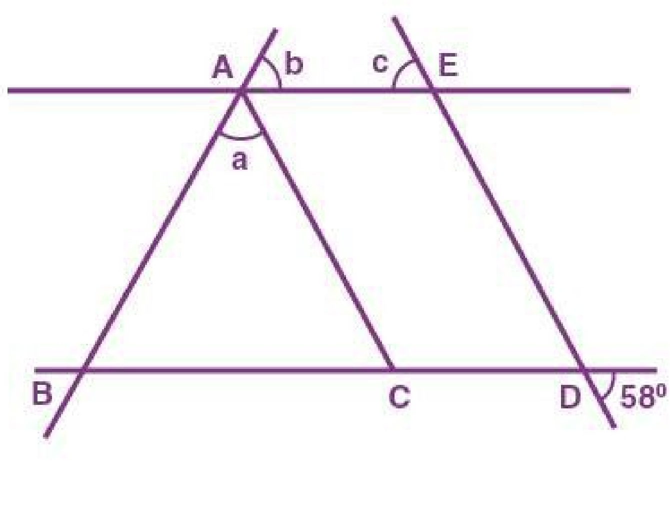
$\therefore x = 63^o$

10. In the triangle ABC, BD bisects angle B and is perpendicular to AC. If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the values of x and y.

Solution:

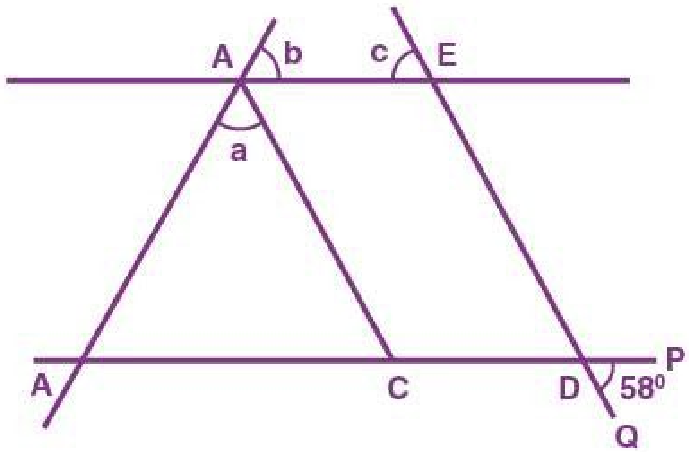
In ∆ABC and ∆DBC, we have
BD = BD [Common]
∠BDA = ∠BDC [Each equal to 90°]
∠ABD = ∠DBC [BD bisects ∠ABC]
∴ ∆ABD ≅ ∆DBC [ASA criterion]
Therefore, by CPCT
AD = DC
x + 1 = y + 2
x = y + 1… (i)
And, AB = BC
3x + 1 = 5y - 2
Substituting the value of x from (i), we get
3(y+1) + 1 = 5y - 2
3y + 3 + 1 = 5y – 2
3y + 4 = 5y – 2
2y = 6
y = 3
Putting y = 3 in (i), we get
x = 3 + 1
∴ x = 4

11. In the given figure; AE // BD, AC // ED and AB = AC. Find ∠a, ∠b and ∠c.



Solution:

Let's assume points P and Q as shown below:

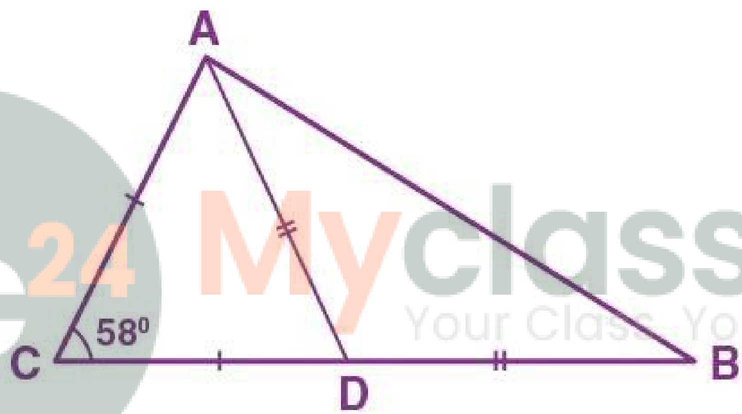


Given, ∠PDQ = 58°
∠PDQ = ∠EDC = 58° [Vertically opposite angles]
∠EDC = ∠ACB = 58° [Corresponding angles ∵ AC ll ED]
In △ABC, we have
AB = AC [Given]
∴ ∠ACB = ∠ABC = 58° [Angles opposite to equal sides are equal]

Now,
∠ACB + ∠ABC + ∠BAC = 180°
58° + 58° + a = 180°
∠a = 180° - 116°
∠a = 64°
Since, AE ll BD and AC is the transversal
∠ABC = ∠b [Corrosponding angles]
∴ ∠b = 58°
Also, since AE ll BD and ED is the transversal
∠EDC = ∠c [Corrosponding angles]
∴ ∠c = 58°

12.

13. In the following figure; AC = CD, AD = BD and ∠C = 58°.



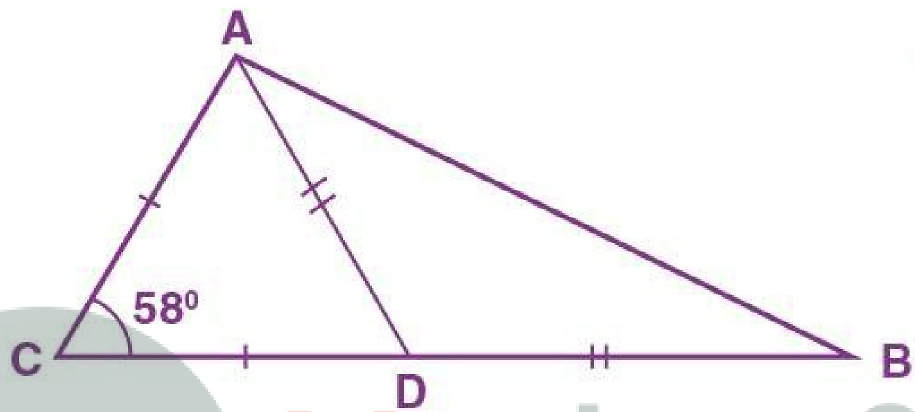
Find ∠CAB.
Solution:

In △ACD, we have
AC = CD [Given]
∴ ∠CAD = ∠CDA [Angles opposite to equal sides are equal]
And,
∠ACD = 58° [Given]
By angle sum property, we have
∠ACD + ∠CDA + ∠CAD = 180°
58° + 2∠CAD = 180°
2∠CAD = 122°
∠CAD = ∠CDA = 61°… (i)
Now,
∠CDA = ∠DAB + ∠DBA [Exterior angles is equal to sum of opposite interior angles]
But,
∠DAB = ∠DBA [Given, AD = DB]
So, ∠DAB + ∠DAB = ∠CDA
2∠DAB = 61°

∠DAB = 30.5º… (ii)

In ∆ABC, we have
∠CAB = ∠CAD + ∠DAB
∠CAB = 61º + 30.5º [From (i) and (ii)]
∴ ∠CAB = 91.5 º

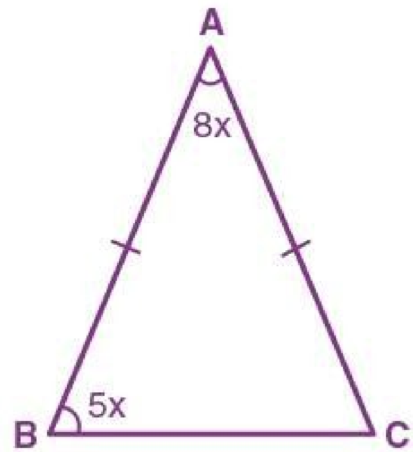
14.
15. In the figure of Q.11 is given above, if AC = AD = CD = BD; find angle ABC.



Solution:

In ∆ACD, we have
AC = AD = CD [Given]
Hence, ACD is an equilateral triangle
∴ ∠ACD = ∠CDA = ∠CAD = 60º
Now,
∠CDA = ∠DAB + ∠ABD [Exterior angle is equal to sum of opposite interior angles]
But,
∠DAB = ∠ABD [Given, AD = DB]
So, ∠ABD + ∠ABD = ∠CDA
2∠ABD = 60º
∴ ∠ABD = ∠ABC = 30º

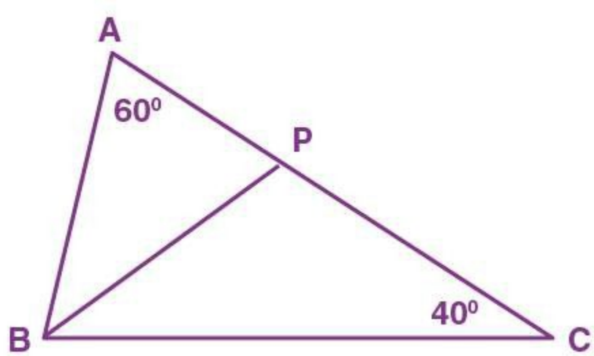
16. In ∆ABC; AB = AC and ∠A: ∠B = 8: 5; find ∠A.
Solution:



Let, ∠A = 8x and ∠B = 5x

Given, In △ABC
AB = AC
So, ∠B = ∠C = 5x [Angles opp. to equal sides are equal]
Now, by angle sum property
∠A + ∠B +C = 180°
8x + 5x + 5x = 180°
18x = 180°
x = 10°
Thus, as ∠A = 8x
∠A = 8 × 10°
∴ ∠A = 80°

17. In triangle ABC; ∠A = 60°, ∠C = 40°, and bisector of angle ABC meets side AC at point P. Show that BP = CP.
Solution:



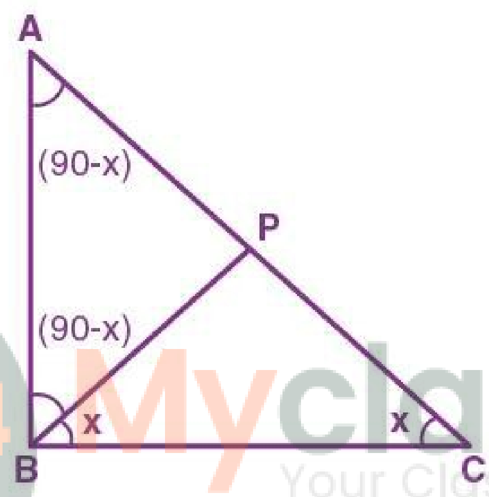
In △ABC, we have
∠A = 60°
∠C = 40°
∴ ∠B = 180° - 60° - 40° [By angle sum property]
∠B = 80°

Now, as BP is the bisector of ∠ABC

∴ ∠PBC = ∠ABC/2
∠PBC = 40°
In △PBC, we have
∠PBC = ∠PCB = 40°
∴ BP = CP [Sides opposite to equal angles are equal]

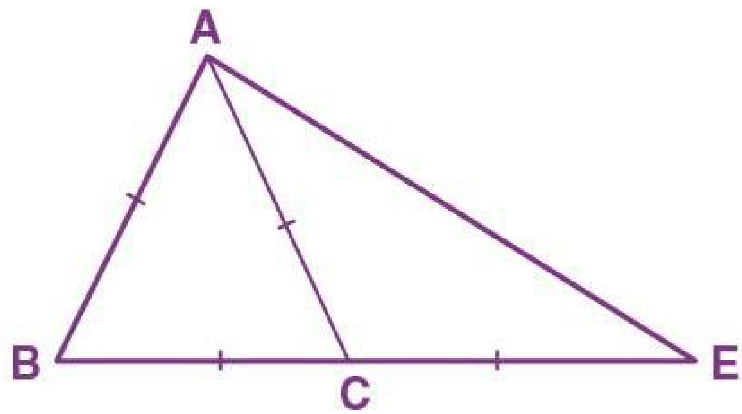
18. In triangle ABC; angle ABC = 90° and P is a point on AC such that ∠PBC = ∠PCB. Show that: PA = PB.

Solution:



Let's assume ∠PBC = ∠PCB = x
In the right-angled triangle ABC,
∠ABC = 90°
∠ACB = x
∠BAC = 180° - (90° + x) [By angle sum property]
∠BAC = (90° - x) …(i)
And
∠ABP = ∠ABC - ∠PBC
∠ABP = 90° – x …(ii)
Thus, in the △ABP from (i) and (ii), we have
∠BAP = ∠ABP
Therefore, PA = PB [sides opp. to equal angles are equal]

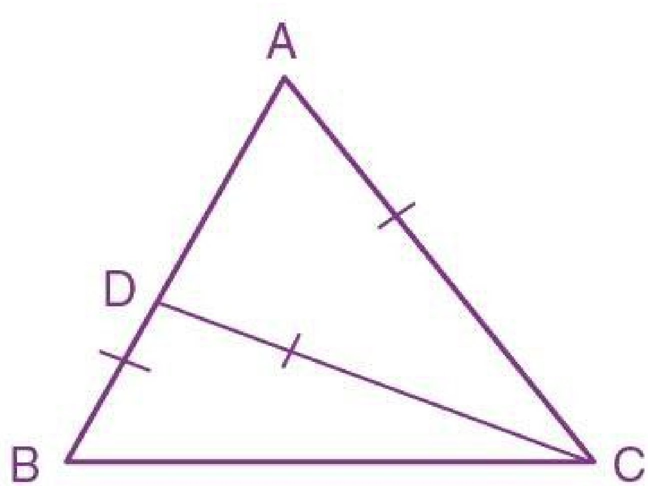
19. ABC is an equilateral triangle. Its side BC is produced upto point E such that C is mid-point of BE. Calculate the measure of angles ACE and AEC.
Solution:



Given, ΔABC is an equilateral triangle
So, AB = BC = AC
∠ABC = ∠CAB = ∠ACB = 60°
Now, as sum of two non-adjacent interior angles of a triangle is equal to the exterior angle
∠CAB + ∠CBA = ∠ACE
60° + 60° = ∠ACE
∠ACE = 120°
Now,
ΔACE is an isosceles triangle with AC = CF
∠EAC = ∠AEC
By angle sum property, we have
∠EAC + ∠AEC + ∠ACE = 180°
2∠AEC + 120° = 180°
2∠AEC = 180° - 120°
∠AEC = 30°

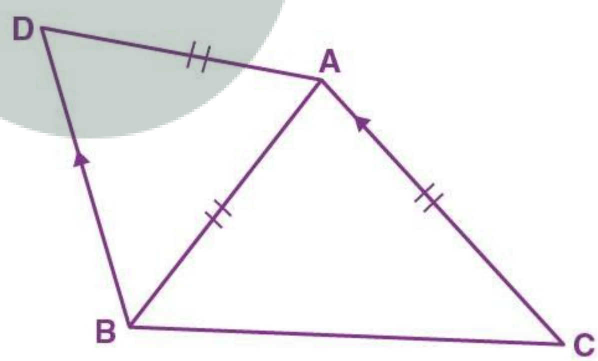
20. In triangle ABC, D is a point in AB such that AC = CD = DB. If ∠B = 28°, find the angle ACD.

Solution:



From given, we get
ΔDBC is an isosceles triangle
⇒ CD = DB
∠DBC = ∠DCB [If two sides of a triangle are equal, then angles opposites to them are equal]
And, ∠B = ∠DBC = ∠DCB = 28°
By angle sum property, we have
∠DCB + ∠DBC + ∠BCD = 180°
28° + 28° + ∠BCD = 180°
∠BCD = 180° - 56°
∠BCD = 124°
As sum of two non-adjacent interior angles of a triangle is equal to the exterior angle, we have
∠DBC + ∠DCB = ∠DAC
28° + 28° = 56°
∠DAC = 56°
Now,
ΔACD is an isosceles triangle with AC = DC
⇒ ∠ADC = ∠DAC = 56°
∠ADC + ∠DAC +∠DCA = 180° [By angle sum property]
56° + 56° + ∠DCA = 180°
∠DCA = 180° - 112°
∠DCA = 64°
Thus, ∠ACD = 64°

21. In the given figure, AD = AB = AC, BD is parallel to CA and ∠ACB = 65°. Find ∠DAC.



Solution:

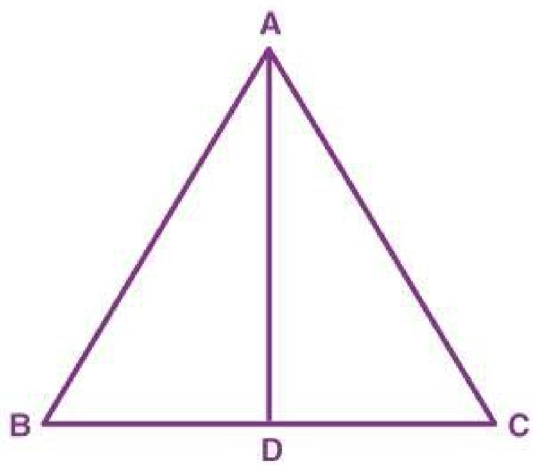
From figure, it's seen that
ΔABC is an isosceles triangle with AB = AC
⇒ ∠ACB = ∠ABC
As ∠ACB = 65° [Given]
∴ ∠ABC = 65°
By angle sum property, we have
∠ACB + ∠CAB + ∠ABC = 180°
65° + 65° + ∠CAB = 180°
∠CAB = 180° - 130°

∠CAB = 50°
As BD is parallel to CA, we have
∠CAB = ∠DBA as they are alternate angles
⇒ ∠CAB = ∠DBA = 50°

Again, from figure, it's seen that
ΔADB is an isosceles triangle with AD = AB.
⇒ ∠ADB = ∠DBA = 50°
By angle sum property, we have
∠ADB + ∠DAB + ∠DBA = 180°
50° + ∠DAB + 50° = 180°
∠DAB = 180° - 100°
∠DAB = 80°
Now,
∠DAC = ∠CAB + ∠DAB
∠DAC = 50° - 80°
∠DAC = 130°

22. Prove that a triangle ABC is isosceles, if:
(i) altitude AD bisects angles BAC, or
(ii) bisector of angle BAC is perpendicular to base BC.
Solution:

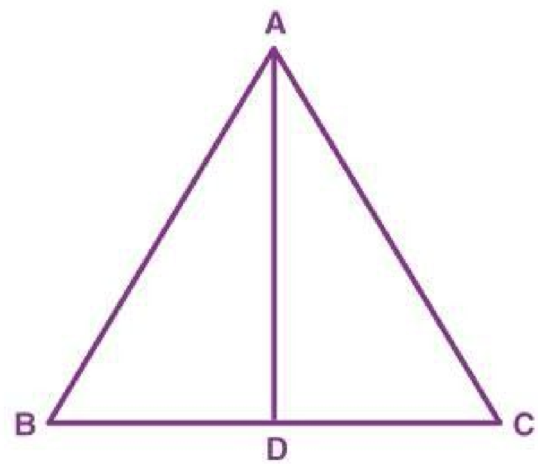
(i) In ΔABC, if the altitude AD bisect ∠BAC.
Then, to prove: ΔABC is isosceles.



In ΔADB and ΔADC, we have
∠BAD = ∠CAD (AD is bisector of ∠BAC)
AD = AD (Common)
∠ADB = ∠ADC (Each equal to 90°)
Therefore, ΔADB ≅ ΔADC by ASA congruence criterion
So, by CPCT

AB = AC
Hence, ΔABC is an isosceles.

(ii) In Δ ABC, the bisector of ∠BAC is perpendicular to the base BC.
Then, to prove: ΔABC is isosceles.



In ΔADB and ΔADC,
∠BAD = ∠CAD (AD is bisector of ∠BAC)
AD = AD (Common)
∠ADB = ∠ADC (Each equal to 90°)
Therefore, ΔADB ≅ ΔADC by ASA congruence criterion
Thus, by CPCT
AB = AC
Hence, ΔABC is an isosceles.

23.

24. In the given figure; AB = BC and AD = EC.
Prove that: BD = BE.