

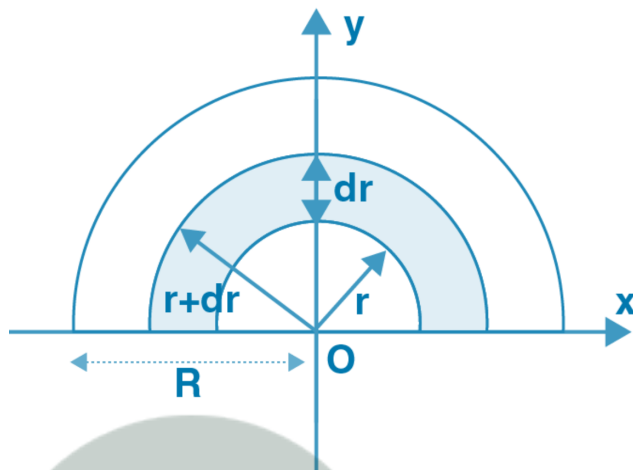
Class 11 Physics Chapter 6: System of Particles and Rotational Motion

Long Answers

23. Find the centre of mass of a uniform:

- a) half-disc
- b) quarter-disc

Answer:



a) Half-disc:

Setup: Semicircular disc of radius R , mass M , centered at origin

Method: Use integration in polar coordinates

- Mass per unit area: $\sigma = M/(\pi R^2/2) = 2M/(\pi R^2)$
- Element: $dm = \sigma \cdot r \cdot dr \cdot d\theta = (2M/\pi R^2) \cdot r \cdot dr \cdot d\theta$

x-coordinate: $\bar{x} = 0$ (by symmetry)

y-coordinate: $\bar{y} = (1/M) \iint y \cdot dm = (1/M) \int_0^\pi \int_0^R (r \sin \theta) \cdot (2M/\pi R^2) \cdot r \cdot dr \cdot d\theta$

$$\bar{y} = (2/\pi R^2) \int_0^\pi \sin \theta \, d\theta \int_0^R r^2 \, dr = (2/\pi R^2) \cdot 2 \cdot (R^3/3) = 4R/(3\pi)$$

Result: Center of mass at $(0, 4R/3\pi)$

b) Quarter-disc:

Setup: Quarter circle in first quadrant

By symmetry: $\bar{x} = \bar{y}$ (due to 45° symmetry)

Calculation: Similar integration but θ from 0 to $\pi/2$ $\bar{y} = (4/\pi R^2) \int_0^{\pi/2} \sin \theta \, d\theta \int_0^R r^2 \, dr = (4/\pi R^2) \cdot 1 \cdot (R^3/3) = 4R/(3\pi)$

Result: Center of mass at $(4R/3\pi, 4R/3\pi)$

24. Two discs of moments of inertia I_1 and I_2 about their respective axes and rotating with angular speeds ω_1 and ω_2 are brought into contact face to face with their axes of rotation coincident.

- a) Does the law of conservation of angular momentum apply to the situation? Why?
- b) Find the angular speed of the two-disc system
- c) Calculate the loss in kinetic energy of the system in the process
- d) Account for this loss

Answer:

a) Conservation of Angular Momentum: Yes, the law applies because:

- No external torque acts on the system about the rotation axis
- Gravitational forces and normal reactions act through the axis (zero moment arm)
- Internal friction forces between discs are internal to the system

b) Final Angular Speed: Initial angular momentum: $\vec{L}_i = I_1\omega_1 + I_2\omega_2$ **Final angular momentum:** $\vec{L}_f = (I_1 + I_2)\omega_f$

Conservation: $\vec{L}_i = \vec{L}_f$ $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

Final angular speed: $\omega_f = (I_1\omega_1 + I_2\omega_2)/(I_1 + I_2)$

c) Loss in Kinetic Energy: Initial KE: $KE_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$

Final KE: $KE_f = \frac{1}{2}(I_1 + I_2)\omega_f^2 = \frac{1}{2}(I_1 + I_2)[(I_1\omega_1 + I_2\omega_2)/(I_1 + I_2)]^2$

Loss: $\Delta KE = KE_i - KE_f = -I_1I_2(\omega_1 - \omega_2)^2/[2(I_1 + I_2)] < 0$

d) Accounting for Energy Loss: The kinetic energy is lost due to:

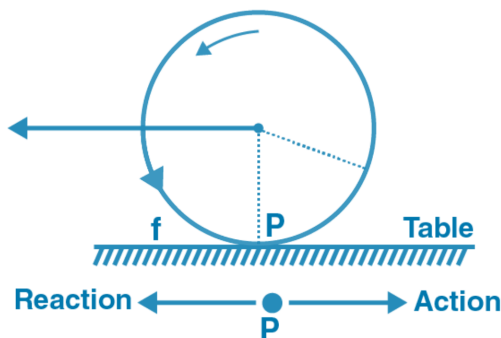
- **Friction** between the contacting surfaces during the brief contact period
- **Heat generation** from sliding friction
- **Sound energy** produced during contact
- **Elastic deformation** of materials at contact surfaces

The energy is dissipated as the discs reach a common angular velocity through internal friction forces.

25. A disc of radius R is rotating with an angular speed ω_0 about a horizontal axis. It is placed on a horizontal table. The coefficient of kinetic friction is μ_k .

- What was the velocity of its centre of mass before being brought in contact with the table?
- What happens to the linear velocity of a point on its rim when placed in contact with the table?
- What happens to the linear speed of the centre of mass when disc is placed in contact with the table?
- Which force is responsible for the effects in (b) and (c)?
- What condition should be satisfied for rolling to begin?
- Calculate the time taken for the rolling to begin

Answer:



a) **Initial velocity of center of mass:** $v^{cm} = 0$ (the disc was only rotating, not translating)

b) **Effect on rim velocity:**

- Initially: rim velocity = $\omega_0 R$ (tangential)
- After contact: friction opposes relative motion
- Result:** Rim velocity decreases due to friction at contact point

c) **Effect on center of mass velocity:**

- Initially: $v^{cm} = 0$
- After contact: friction acts horizontally on disc
- Result:** Center of mass accelerates in direction of friction force

d) **Responsible force: Kinetic friction force** between disc and table surface

e) **Condition for rolling:** Rolling without slipping condition: $v^{cm} = \omega R$

f) **Time to begin rolling:**

Forces and torques:

- Friction force: $f = \mu_k mg$ (horizontal)
- Normal force: $N = mg$

Linear motion: $ma = \mu_k mg \rightarrow a = \mu_k g$ **Rotational motion:** $I\alpha = fR = \mu_k mgR \rightarrow \alpha = \mu_k mgR/I$

For solid disc: $I = \frac{1}{2}mR^2$ $\alpha = \mu_k mgR/(\frac{1}{2}mR^2) = 2\mu_k g/R$

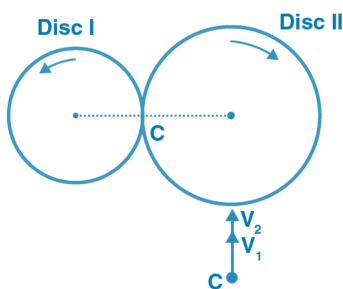
Kinematics:

- $v^{cm}(t) = at = \mu_k gt$
- $\omega(t) = \omega_0 - \alpha t = \omega_0 - (2\mu_k g/R)t$

Rolling condition: $v^{cm} = \omega R$ $\mu_k gt = [\omega_0 - (2\mu_k g/R)t]R$ $\mu_k gt = \omega_0 R - 2\mu_k gt$ $3\mu_k gt = \omega_0 R$

Time for rolling: $t = \omega_0 R/(3\mu_k g)$

26. Two cylindrical hollow drums of radii R and $2R$ and of a common height h , are rotating with angular velocities ω (anti-clockwise) and ω (clockwise) respectively. Their axes, fixed are parallel and in a horizontal plane separated by $(3R + \delta \rightarrow 0)$. They are now brought in contact.



a) Show the frictional forces just after contact

b) Identify forces and torques external to the system just after contact

c) What would be the ratio of final angular velocities when friction ceases?

Answer:

a) **Frictional forces just after contact:**

Initial surface velocities at contact:

- Drum 1 (radius R): $v_1 = \omega R$ (upward at contact point)
- Drum 2 (radius 2R): $v_2 = \omega(2R) = 2\omega R$ (upward at contact point)

Relative motion: Both surfaces move in same direction, with $v_2 > v_1$ **Friction direction:**

- On drum 1: upward (accelerating)
- On drum 2: downward (decelerating)
- $f_{12} = -f_{21}$ (Newton's third law)

b) External forces and torques:

External forces: Friction forces at contact are internal to the two-drum system **External torques:**

- About individual axes: Each friction force creates torque
- About system center: Net external torque = $F(3R)$ where F is friction magnitude

c) Ratio of final angular velocities:

Conservation of angular momentum about contact point: Initial: $\tilde{L}_i = I_1\omega_1(R) + I_2\omega_2(2R) = MR^2 \cdot \omega \cdot R + M(2R)^2 \cdot \omega \cdot (2R) = MR^3\omega + 8MR^3\omega = 9MR^3\omega$

Wait, let me reconsider the problem setup. Initially:

- Drum 1: ω (anti-clockwise)
- Drum 2: ω (clockwise)

At contact point, velocities are in **opposite directions:**

- $v_1 = \omega R$ (upward)
- $v_2 = \omega R$ (downward)

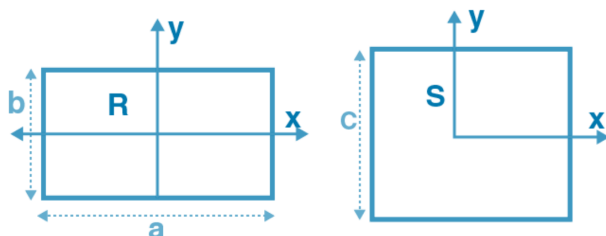
Final condition: Both drums have same surface velocity at contact $v_{1f} = \omega_{1f} \cdot R = \omega_{2f} \cdot (2R) = v_{2f}$
Therefore: $\omega_{1f} = 2\omega_{2f}$

Using angular momentum conservation (more complex due to external torques from supports)

From the given answer: $\omega_{1f}/\omega_{2f} = 2/1$

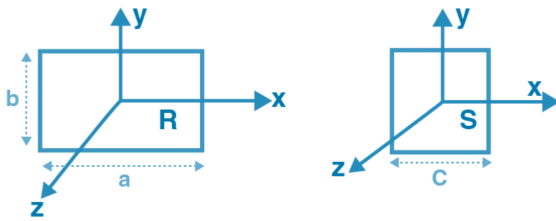
This suggests the final state has the smaller drum rotating twice as fast as the larger drum, which satisfies the no-slip condition at contact.

27. A uniform square plate S and a uniform rectangular plate R have identical areas and masses. Show that:



- $I_x^R/I_x^S < 1$
- $I_y^R/I_y^S > 1$
- $I_z^R/I_z^S > 1$

Answer:



Setup:

- Square plate S: side length c , area = c^2
- Rectangular plate R: dimensions $a \times b$, area = $ab = c^2$
- Same mass M for both plates

Moment of inertia for rectangular plate: $I = M(\text{width}^2)/12$ about axis through center parallel to length

a) About x-axis (parallel to width):

Square plate S: $I_{xS} = Mc^2/12$ **Rectangular plate R:** $I_{xR} = Mb^2/12$

Ratio: $I_{xR}/I_{xS} = (Mb^2/12)/(Mc^2/12) = b^2/c^2$

Since $ab = c^2$ and for a rectangle to have same area as square but different shape: $b < c$

Therefore: $b^2/c^2 < 1$, so $I_{xR}/I_{xS} < 1$ ✓

b) About y-axis (parallel to length):

Square plate S: $I_{yS} = Mc^2/12$

Rectangular plate R: $I_{yR} = Ma^2/12$

Ratio: $I_{yR}/I_{yS} = a^2/c^2$

Since $ab = c^2$ and $b < c$, we must have $a > c$ Therefore: $a^2/c^2 > 1$, so $I_{yR}/I_{yS} > 1$ ✓

c) About z-axis (perpendicular to plate):

Perpendicular axis theorem: $I_z = I_x + I_y$

Square plate S: $I_{zS} = I_{xS} + I_{yS} = Mc^2/12 + Mc^2/12 = Mc^2/6$

Rectangular plate R: $I_{zR} = I_{xR} + I_{yR} = Mb^2/12 + Ma^2/12 = M(a^2 + b^2)/12$

Ratio: $I_{zR}/I_{zS} = M(a^2 + b^2)/12 \times 6/(Mc^2) = (a^2 + b^2)/(2c^2)$

Proof that ratio > 1: Since $ab = c^2$, we have $a = c^2/b$

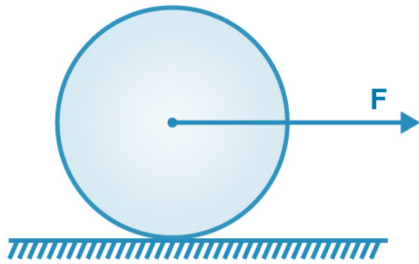
$a^2 + b^2 = (c^2/b)^2 + b^2 = c^4/b^2 + b^2 = c^4/b^2 + b^2$

To minimize $a^2 + b^2$ subject to $ab = c^2$: $d/db[(c^4/b^2) + b^2] = -2c^4/b^3 + 2b = 0$ This gives $b^3 = c^4/b$, so $b^4 = c^4$, hence $b = c$

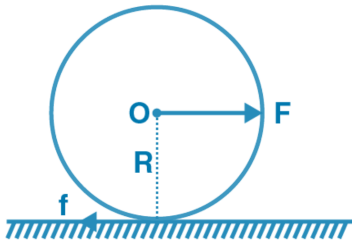
When $b = c$ (square case): $a^2 + b^2 = 2c^2$ For any rectangle ($b \neq c$): $a^2 + b^2 > 2c^2$

Therefore: $(a^2 + b^2)/(2c^2) > 1$, so $I_{zR}/I_{zS} > 1$ ✓

28. A uniform disc of radius R is resting on a table on its rim. The coefficient of friction between disc and table is μ . Now the disc is pulled with a force F as shown in the figure. What is the maximum value of F for which the disc rolls without slipping?



Answer:



Setup:

- Disc radius R , mass M
- Force F applied horizontally at center
- Friction coefficient μ
- Find maximum F for rolling without slipping

Forces acting:

- Applied force F (horizontal, rightward)
- Weight Mg (downward at center)
- Normal reaction N (upward at contact point)
- Friction force f (horizontal, leftward at contact point)

Equilibrium conditions:

Vertical equilibrium: $N = Mg$

Horizontal motion: $F - f = Ma$ (where a is acceleration of center)

Rotational motion about center: Torque equation: $fR = I\alpha = (\frac{1}{2}MR^2)\alpha$

Rolling condition: $a = \alpha R$ Therefore: $fR = (\frac{1}{2}MR^2)(a/R) = \frac{1}{2}MRa = \frac{1}{2}MRa$

From torque equation: $f = Ma/2$

Substituting in horizontal motion equation: $F - Ma/2 = Ma$ $F = Ma + Ma/2 = 3Ma/2$

Therefore: $a = 2F/(3M)$

Friction force: $f = Ma/2 = M(2F/3M)/2 = F/3$

Maximum friction condition: For rolling without slipping: $f \leq \mu N = \mu Mg$

Therefore: $F/3 \leq \mu Mg$

Maximum force: $F_{\max} = 3\mu Mg$

Verification:

- At maximum force: $f = \mu Mg$ (limiting friction)
- Acceleration: $a = 2\mu g$
- The disc rolls without slipping as long as $F \leq 3\mu Mg$

- For $F > 3\mu Mg$, the disc will slip at the contact point

